

1. a) k-map

| $\begin{matrix} e & h \\ k \end{matrix}$ | 00 | 01 | 11 | 10 |
|------------------------------------------|----|----|----|----|
| 0                                        | 1  |    |    | 1  |
| 1                                        | 1  |    | 1  |    |

b) PIs:  $\bar{e}h$ ,  $\bar{h}k$ ,  $ehk$ .

c) EPIs:  $\bar{e}h$  ( $\bar{e}hk$  not included in any others),  
 $\bar{h}k$  ( $ehk$  " " " " " " "  
 $ehk$  ( $ehk$  " " " " " " " }

d) Start w/ EPI's -- since these form a cover of  $f$ , we are done:  $f = \bar{e}h + \bar{h}k + ehk$ 

e) Since all PIs used, there is no other min-cost SOP.

5. a)

| $A, A_0$ \ $B, B_0$ |      | no |    | yes |    | abstain |  |
|---------------------|------|----|----|-----|----|---------|--|
|                     |      | 00 | 01 | 11  | 10 |         |  |
| no                  | — 00 | ①  |    | d   |    |         |  |
| yes                 | — 01 |    | 1  | d   |    |         |  |
|                     | 11   | d  | d  | d   | d  |         |  |
| abstain             | — 10 |    |    | d   | 1  |         |  |

b) Find PIs:  $A_1B_1$ ,  $A_0B_0$ ,  $\bar{A}_1\bar{A}_0\bar{B}_1\bar{B}_0$ 

All are essential

So,  $f = A_1B_1 + A_0B_0 + \bar{A}_1\bar{A}_0\bar{B}_1\bar{B}_0$  (min-cost SOP)

2. a).

| ab \ cd | 00 | 01 | 11 | 10 |
|---------|----|----|----|----|
| 00      | 1  | 1  | 1  |    |
| 01      | 1  |    | 1  |    |
| 11      | 1  |    | 1  | 1  |
| 10      | 1  |    | 1  | 1  |

b). PI's (shown above)

$\bar{a}\bar{b}$ ,  $ab$ ,  $ac$ ,  $\bar{b}c$ ,  $\bar{a}\bar{c}\bar{d}$ ,  $b\bar{c}\bar{d}$

c). EPI's:  $\bar{a}\bar{b}$  ( $\bar{a}\bar{b}\bar{c}\bar{d}$  not included in any other PI),  
 $ab$  ( $ab\bar{c}\bar{d}$  " " " " " " " " )

d).

| Choice # | PI                      | Comment                              |
|----------|-------------------------|--------------------------------------|
| 1        | $\bar{a}\bar{b}$        | EPI                                  |
| 2        | $ab$                    | EPI                                  |
| 3        | $ac$                    | 2-literal PI that includes 2 more    |
| 4        | $\bar{a}\bar{c}\bar{d}$ | includes final minterm to form cover |

$f = \bar{a}\bar{b} + ab + ac + \bar{a}\bar{c}\bar{d}$

e). Not unique: • for choice 3, could have used  $\bar{b}c$  instead  
 • for choice 4, could have used  $b\bar{c}\bar{d}$  instead.

3. a).

$X_1 X_2$   
 $X_3 X_4$

|    | 00 | 01 | 11 | 10 |
|----|----|----|----|----|
| 00 | 1  |    | 1  | 1  |
| 01 |    | 1  | 1  |    |
| 11 | 1  | 1  |    | 1  |
| 10 | 1  |    |    | 1  |

b), PIs:  $\overline{X_2} X_3$ ,  $\overline{X_2} \overline{X_4}$ ,  $\overline{X_1} X_3 X_4$ ,  $\overline{X_1} X_2 X_4$ ,  
 $X_2 \overline{X_3} X_4$ ,  $X_1 X_2 \overline{X_3}$ ,  $X_1 \overline{X_3} X_4$

c) EPIs:  $\overline{X_2} X_3$  ( $X_1 \overline{X_2} X_3 X_4$  not included in any others)  
 $\overline{X_2} X_4$  ( $\overline{X_1} \overline{X_2} \overline{X_3} \overline{X_4}$  " " " " " )

| d), Choice # | PI                    | Comment                    |
|--------------|-----------------------|----------------------------|
| 1            | $\bar{X}_2 X_3$       | EPI                        |
| 2            | $\bar{X}_2 \bar{X}_4$ | EPI                        |
| 3            | $\bar{X}_1 X_2 X_4$   | Includes 2 more min terms  |
| 4            | $X_1 X_2 \bar{X}_3$   | Includes final 2 min terms |

e) This is unique. Choices 1 & 2 are required (essential PI's), leaving only 4 minterms not included and only 3-literal PI's. Clearly, it is best to include those 4 minterms with no overlap, and choices 3 and 4 are the only way to do that.

Note: According to procedure, choice 3 could have been  $X_2 \overline{X_3} X_4$ , but then 2 more PIs are required

4.  $f(x_1, x_2, x_3, x_4) = \sum m(0, 3, 4, 5, 7, 9, 11) + \sum d(8, 12, 13, 14)$

a.

| $x_3 \backslash x_4$ | 00 | 01 | 11 | 10 |
|----------------------|----|----|----|----|
| 00                   | 1  | 1  | d  | d  |
| 01                   |    | 1  | d  | 1  |
| 11                   | 1  | 1  |    | 1  |
| 10                   |    |    | d  |    |

b.

$$\begin{aligned} &\bar{x}_3 \cdot \bar{x}_4 && \bar{x}_1 \cdot x_2 \cdot x_4 \\ &x_2 \cdot \bar{x}_3 && x_1 \cdot \bar{x}_2 \cdot x_4 \\ &x_1 \cdot \bar{x}_3 && \bar{x}_1 \cdot x_3 \cdot x_4 \\ &&& \bar{x}_2 \cdot x_3 \cdot x_4 \end{aligned}$$

c. Only  $\bar{x}_3 \cdot \bar{x}_4$ ;  $\bar{x}_1 \cdot \bar{x}_2 \cdot \bar{x}_3 \cdot \bar{x}_4$  not included in any other P.I.

d. First, use the suggestion given in the notes:

| Choice # | PI                              | Comment                                   |
|----------|---------------------------------|-------------------------------------------|
| 1        | $\bar{x}_3 \cdot \bar{x}_4$     | EPI.                                      |
| 2        | $x_2 \cdot \bar{x}_3$           | 2-literal PI that contributes 1 to cover. |
| 3        | $x_1 \cdot \bar{x}_3$           | " " " " " " "                             |
| 4        | $\bar{x}_1 \cdot x_3 \cdot x_4$ | 3-literal " " " 2 " "                     |
| 5        | $x_1 \cdot \bar{x}_2 \cdot x_4$ | 3-literal " " completes the cover         |

$$f = \bar{x}_3 \bar{x}_4 + x_2 \bar{x}_3 + x_1 \bar{x}_3 + \bar{x}_1 x_3 x_4 + x_1 \bar{x}_2 x_4 \quad \text{cost} = 23.$$

Choice 5 could also have been  $\bar{x}_2 x_3 x_4$  - same cost.

OR Choice 4 could have been  $\bar{x}_2 x_3 x_4$ , and then choice 5 could have been either  $\bar{x}_1 x_3 x_4$  or  $\bar{x}_1 x_2 x_4$  - same cost.

But, note that following has less cost:

| Choice # | PI                              | Comment                                   |
|----------|---------------------------------|-------------------------------------------|
| 1        | $\bar{x}_3 \cdot \bar{x}_4$     | EPI                                       |
| 2        | $\bar{x}_1 \cdot x_2 \cdot x_4$ | 3-literal PI that contributes 2 to cover. |
| 3        | $x_1 \cdot \bar{x}_2 \cdot x_4$ | " " " " " " "                             |
| 4        | $\bar{x}_1 \cdot x_3 \cdot x_4$ | " " " completes cover.                    |

$$f = \bar{x}_3 \bar{x}_4 + \bar{x}_1 x_2 x_4 + x_1 \bar{x}_2 x_4 + \bar{x}_1 x_3 x_4 \quad \text{cost} = 20$$

Choice 4 could have been  $\bar{x}_2 x_3 x_4$  - same cost.

e. Not unique - several alternatives given above.

6.  $f(a,b,c) = \sum m(1,2,5,6,7)$

| ab |   | 00 | 01 | 11 | 10 |
|----|---|----|----|----|----|
| c  | 0 | 0  | 1  | 1  | 0  |
|    | 1 | 1  | 0  | 1  | 1  |

a). min-cost SOP:  $f = b\bar{c} + \bar{b}c + ab$  cost =  $3(3) + 1(4) = 13$   
 $\uparrow$  could be  $ac$

b) i) above

ii) POS PIs:  $(b+c), (a+\bar{b}+\bar{c})$

iii)  $(b+c)$ : both  $(\bar{a}+b+c)$  and  $(a+b+c)$  not included by others.

$(a+\bar{b}+\bar{c})$ : this maxterm not included by others.

iv). Simple: POS PIs form POS cover.

$$f = (b+c)(a+\bar{b}+\bar{c}) \quad \text{min-cost POS}$$

c). POS min-cost:  $3+4+3 = 10$  POS is less costly.

7.  $f(a,b,c,d) = \sum m(2,3,4,8,9,10,11)$

a).  $f = a\bar{b} + \bar{b}c + \bar{a}b\bar{c}d$  cost =  $2(3) + 1(5) + 1(4) = 15$

b). i)

|      |    |      |    |    |    |
|------|----|------|----|----|----|
|      |    | $ab$ |    |    |    |
|      |    | 00   | 01 | 11 | 10 |
| $cd$ | 00 | 0    |    | 0  |    |
|      | 01 | 0    | 0  | 0  |    |
|      | 11 |      | 0  | 0  |    |
|      | 10 |      | 0  | 0  |    |

Blank cell = 1

ii) POS PI's:  $(\bar{b} + \bar{d})$ ,  $(\bar{b} + \bar{c})$ ,  $(\bar{a} + \bar{b})$ ,  $(a + b + c)$

iii)  $(\bar{b} + \bar{c})$ :  $(a + \bar{b} + \bar{c} + d)$  not included in any other

$(a + b + c)$ :  $(a + b + c + d)$  " " " " "

$(\bar{a} + \bar{b})$ :  $(\bar{a} + \bar{b} + c + d)$  " " " " "

iv) Start with EPI's, add  $(\bar{b} + \bar{d})$  to complete cover,

$f = (\bar{b} + \bar{c})(\bar{a} + \bar{b})(a + b + c)(\bar{b} + \bar{d})$  min-cost POS

c). POS min-cost =  $3(3) + 1(4) + 1(5) = 18$

So, SOP has lower cost.

8.  $f(W_1, W_0, X_1, X_0) = \sum m(5, 6, 7, 10, 11, 15) + \sum d(0, 1, 2, 3, 4, 8, 12)$

a).  $f = \overline{W}_1 + X_1 X_0 + \overline{W}_0 X_1$ , cost =  $3(2) + 1(4) = 10$

b). i)

|           |    | $W_1 W_0$ |    |    |    |
|-----------|----|-----------|----|----|----|
| $X_1 X_0$ |    | 00        | 01 | 11 | 10 |
|           | 00 | d         | d  | d  | d  |
|           | 01 | d         |    | 0  | 0  |
|           | 11 | d         |    |    |    |
|           | 10 | d         |    | 0  |    |

Blank cell = 1

ii) PoS PIs:  $(\overline{W}_1 + X_1)$ ,  $(W_0 + X_1)$ ,  $(\overline{W}_1 + \overline{W}_0 + X_0)$

iii)  $(\overline{W}_1 + X_1)$ :  $(\overline{W}_1 + \overline{W}_0 + X_1 + \overline{X}_0)$  not included in others.

$(\overline{W}_1 + \overline{W}_0 + X_0)$ :  $(\overline{W}_1 + \overline{W}_0 + X_1 + X_0)$  " " " "

iv). Simple: PoS EPIs form PoS cover

min-cost PoS:  $f = (\overline{W}_1 + X_1)(\overline{W}_1 + \overline{W}_0 + X_0)$

c) PoS min-cost:  $1(3) + 1(4) + 1(3) = 10$

PoS and SoP have same cost.