EECS 140/141 Introduction to Digital Logic Design
Spring Semester 2020
Assignment **#6** Due **3 March 2020** Yes, this is the day after Exam 1!

Reading: Sections 4.5 and 4.6 in Brown/Vranesic
All logic networks on this (and every other assignment) *must* be drawn using a logic template. Points will be deducted for failure to do this! You should have had time by now to secure one of these templates.

1. This problem deals with two functions of the variables x, y, and z. The first is function $f(x, y, z) = \Sigma m(1, 2, 5, 6, 7)$, which was given as an example in section II.B.3.a of your lecture notes. That function had two possible minimum-cost SoP syntheses, each with cost=13. The second is function $g(x, y, z) = \Sigma m(0, 2, 5, 6, 7)$.
   a. For this part, consider function g in isolation; that is, only think about function g by itself without thinking about function f at all. Find all possible minimum-cost SoP syntheses for function g, considered by itself, and give the minimum cost amount.
   b. Now design a sum-of-products (SoP) synthesis for the *combined* 3-input, *2-output* circuit, where the 2 outputs are f and g. Simplify this circuit as much as you can using sharing of product terms between functions f and g wherever possible. Give logic expressions for each of f and g, and circle those terms that are shared between the two functions.
   c. Using only AND, OR, and NOT gates, draw the logic diagram for the resulting 3-input, 2 output circuit that you found in the previous part, including the sharing of terms. What is your circuit's cost?
2. This problem deals with two functions of the variables a, b, c, and d. The first is function $s(a, b, c, d) = \Sigma m(1, 7, 9, 10, 11, 12, 13, 15)$. The second is function $t(a, b, c, d) = \Sigma m(1, 8, 9, 10, 12, 13)$.
   a. For this part, consider function s in isolation; that is, only think about function s by itself without thinking about function t at all. Find the minimum-cost SoP syntheses for function s, considered by itself, and give the minimum cost amount. Hint: this minimum-cost synthesis is unique.
   b. Repeat the previous part for function t. Hint: function t's minimum-cost synthesis is also unique.
   c. Now design a sum-of-products (SoP) synthesis for the *combined* 4-input, *2-output* circuit, where the 2 outputs are s and t. Simplify this circuit as much as you can using sharing of product terms between functions s and t wherever possible. Hint: there is one simple and obvious sharing that can be done, but there is a second one that may not be as obvious. Give logic expressions for each of s and t, and circle those terms that are shared between the two functions.
   d. Draw the logic diagram for the resulting 4-input, 2 output circuit that you found in the previous part, including the sharing of terms. What is your circuit's cost? Hint: you should be able to reduce the cost of this *combined* circuit to 29.
3. If you are in EECS 140, use the function $f(a, b, c) = \Sigma m(0, 1, 4, 6, 7)$ for this problem. If you are in EECS 141, use the function $g(w, x, y, z) = \Sigma m(0, 3, 5, 8, 10, 12, 15) + D(1, 4, 13, 14)$.
   a. Find the min-cost SoP synthesis. Don't forget that part of this process is to list all the Prime Implicants (PIs) and then identify which are Essential (EPIs).
   b. Use factoring to produce a simpler (lower-cost), multi-level synthesis compared to the min-cost SoP synthesis. Use factoring multiple times if it will further simplify the synthesis.
   c. Draw the logic diagram corresponding to your *factored* synthesis using AND/OR/NOT gates.
   d. Give the cost, the number of levels, and the maximum fan-in for each of the above two syntheses.

