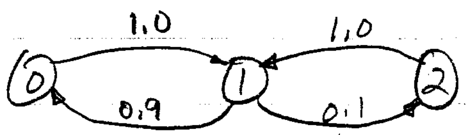
863 HW 2. 5'08

1.



a) irreducible: can get from every state to every other

b) periodic: partition is $S_1 = \{1\}$, $S_2 = \{0, 2\}$.

c). pos recurrent: $r_1(1) = 0$ $\quad r_1(2) = (0.9)(1) + (0.1)(1) = 1.0$

all other $r_1(m) = 0$ $\quad r_1 = 1.0$ $\quad \mu_1 = (2)(1.0) = 2$.

d) period is 2. $\pi(0) = [1\ 0\ 0]$ results in

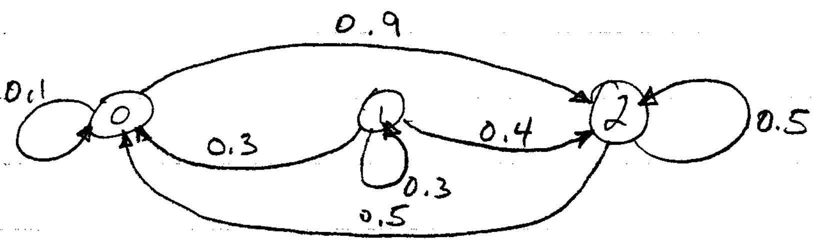
$\pi(n) = [0.9\ \ 0\ \ 0.1]$ for n even & large.

od $\pi(n) = [0\ \ 1\ \ 0\ ]$ " " odd " "

$\pi(0) = [0\ 1\ 0]$ results in $\pi(n) = [0\ 1\ 0]$ n even.

$\pi(n) = [0.9\ 0\ 0.1]$ n odd.

2.



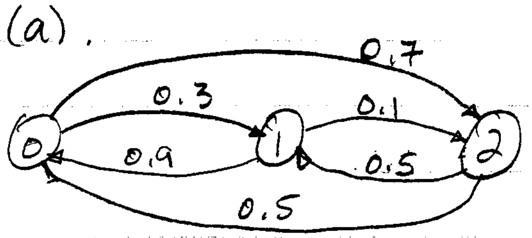
a). not irreducible: $S' = \{0, 2\}$ is absorbing subset.

b). A periodic : at least one $p_{ii} > 0$.

c) N/A.

d). see (a).

3.



a) irreducible: every state reachable from every other

b) A periodic: even though all $p_{ii} = 0$, still no periodic partition

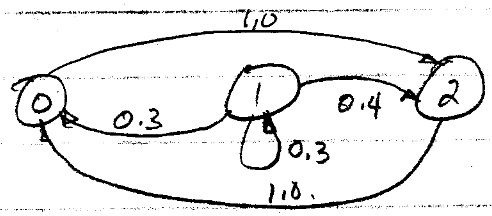
3. c). pos recurrent: Pr(never return to state 0)

$= Pr(\text{cycle forever between } 1 \text{ and } 2)$

$= \lim_{n \to \infty} [(0.1)(0.5)]^{n/2} = 0$

d). Solve $[1\ 0\ 0] = \pi \begin{bmatrix} 1 & -0.3 & -0.7 \\ 1 & 1 & -0.1 \\ 1 & -0.5 & 1 \end{bmatrix}$ $\quad \pi = [0.4077\ \ 0.2790\ \ 0.3133$

4.



a). not irreducible: $S' = \{0, 2\}$ absorbing

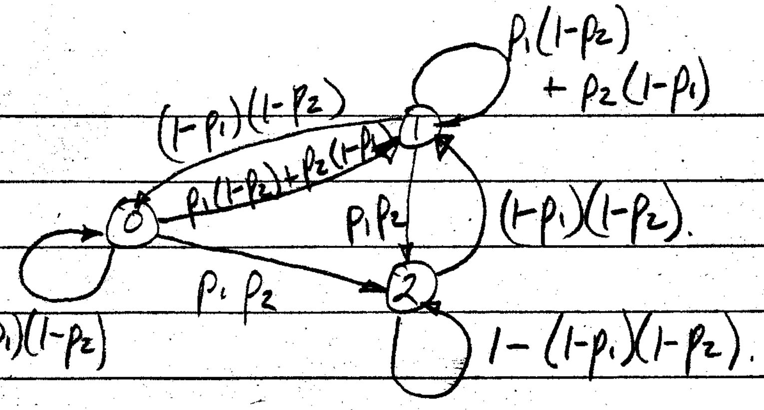
b). asymptotically periodic:
when enters S', becomes periodic.

c). N/A

d). $S' = \{0, 2\}$ absorbing, period is 2 after absorption

5. $p_1 = 0.2 \quad p_2 = 0.4$

a)



$$P = \begin{bmatrix} 0.48 & 0.44 & 0.08 \\ 0.48 & 0.44 & 0.08 \\ 0 & 0.48 & 0.52 \end{bmatrix}$$

Solve $\pi = \pi P \quad \sum_i \pi_i = 1$.

Answer:

$\pi = [0.4114 \quad 0.4457 \quad 0.1429]$.

Book example: $\pi = [0.4140 \ 0.4309 \ 0.1552]$.

Different, but not much so.

b) Packet is dropped in a given (arbitrary) slot with prob $\pi_2 p_1 p_2$ {Buffer is full and 2 arrivals}. This is 0.01143 for this case, compared to 0.01397 for book example.

c) Average cell arrival rate is $p_1 + p_2 = 0.6$ cells/slot, which is the same as the book example.

Departure rate is the arrival rate times $1 - Pr\{\text{packet dropped in arbitrary slot}\}$ (from part (b)).

$\Rightarrow$ Departure rate is $0.6(1-0.01143) = 0.59312$, slightly larger than $0.6(1-0.01397) = 0.59162$ (Book ex.)

6. a) $\lim_{n\to\infty} \pi(n) = \pi(0) \lim_{n\to\infty} P^n = \pi(0) \begin{bmatrix} \pi \\ \pi \\ \vdots \\ \pi \end{bmatrix} = \pi$ since $\sum_i \pi_i(0) = 1$.

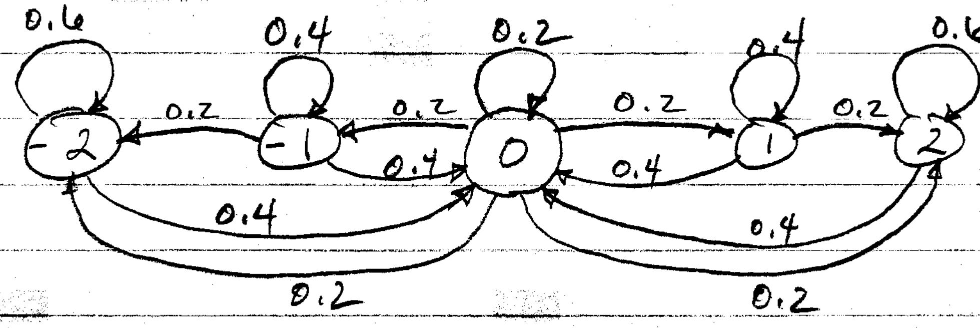
b) If $\lim \pi(n) = \pi$ for all π_0, then true for $\pi(0) = [1 \ 0 \ 0 \cdots 0]$

Then $\pi = \lim \pi(n) = \pi_0 \lim P^n =$ top row of $\lim P^n$

Repeat for $\pi(0) = [0 \ 1 \ 0 \ 0 \cdots 0]$ to show 2^{nd} row $= \pi$

Etc.

7. a).



State Value	-2	-1	0	1	2
State Index	0	1	2	3	4

$$\underline{P} = \begin{bmatrix} 0.6 & 0 & 0.4 & 0 & 0 \\ 0.2 & 0.4 & 0.4 & 0 & 0 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0 & 0 & 0.4 & 0.4 & 0.2 \\ 0 & 0 & 0.4 & 0 & 0.6 \end{bmatrix}$$

b. $p_{ii} > 0 \Rightarrow$ aperiodic

$\pi = \pi \underline{P}$ yields $\pi = \left[2/9 \;\; 1/9 \;\; 1/3 \;\; 1/9 \;\; 2/9 \right]$.

So mean recurrence time to state with value 0 (index = 2) is:

$$\mu_0 = \frac{1}{1/3} = 3.0$$