1. a) irreducible: can get from every state to every other.
   b) periodic: partition is \( S_1 = \{1\} \), \( S_2 = \{0, 2\} \).
   c) pos recurrent: \( \Gamma_0 = 0 \), \( \Gamma_1 = (0.9, 1) + (0.1, 1) = 2 \),
   all other \( \Gamma_i(m) = 0 \), \( \mu_1 = (2)(1) = 2 \).
   d) period is 2, \( \pi(0) = [0.5, 0, 0.5] \) results in
      \[ \pi(n) = \begin{cases} [0.9, 0, 0.1] & \text{for n even} \hat{\text{large}} \cr [0, 0, 1] & \text{for n odd} \end{cases} \]
      \( \pi(0) = [0.5, 0, 0.5] \) results in \( \pi(n) = [0, 1, 0] \) n even.
      \( \pi(n) = [0.9, 0, 0.1] \) n odd.

2. a) not irreducible: \( S' = \{0, 2\} \) is absorbing subset.
   b) A periodic: at least one \( \pi_{ii} > 0 \).
   c) N/A.
   d) see (a).

3. a) irreducible: every state reachable from every other.
   b) A periodic: even though all \( \pi_{ii} = 0 \), still
      no periodic partition.
3. c), pos recurrent: \(\text{Pr(never return to state 0)}\)
   \[= \text{Pr(cycle forever between 1 and 2)}\]
   \[= \lim_{n \to \infty} [(0,1)(0.5)]^n \cdot 0 = 0\]

d), Solve \([1 \ 0 \ 0] = \pi \begin{bmatrix} 1 & -0.3 & -0.7 \\ 1 & 1 & -0.1 \\ 1 & -0.5 & 1 \end{bmatrix} \pi = \begin{bmatrix} 0.4 & 0.7 \ 0.279 \ 0.3 \end{bmatrix}\)

4. 

a), not irreducible: \(S' = \{0, 2\}\) absorbing
b), asymptotically periodic:
   when enters 5', becomes periodic.

c), N/A

d), \(S' = \{0, 2\}\) absorbing, period 2 after absorption
5. \( p_1 = 0.2 \quad p_2 = 0.4 \)

\[ P = \begin{bmatrix} 0.48 & 0.44 & 0.08 \\ 0.48 & 0.44 & 0.08 \\ 0 & 0.48 & 0.52 \end{bmatrix} \]

Solve \( \pi = \pi P \) \( \sum \pi_i = 1 \)

Answer:
\[ \pi = \begin{bmatrix} 0.4114 \\ 0.4457 \\ 0.1429 \end{bmatrix} \]

Book example: \( \pi = \begin{bmatrix} 0.4140 \\ 0.4309 \\ 0.1552 \end{bmatrix} \)

Different, but not much so.

b) Packet is dropped in a given (arbitrary) slot with prob \( p_2 p_1 p_2 \) (Buffer is full and 2 arrivals).
This is 0.01143 for this case, compared to 0.01397 for book example.

c) Average cell arrival rate is \( p_1 p_2 = 0.06 \) cells/slot, which is the same as the book example.
Departure rate is the arrival rate times \( 1 - \sum p_i E \) packet dropped in arbitrary slot? (from part (b))
\[ \Rightarrow \text{Departure rate is 0.6} \times (1 - 0.01143) = 0.59312, \]
slightly larger than 0.6 (1 - 0.01397) = 0.59162 (Book ex.)

6. a) \( \lim_{n \to \infty} \pi(n) = \pi(0) \lim_{n \to \infty} P^n = \pi(0) \pi = \pi \) since \( \sum \pi_i(0) = 1 \).

b) If \( \lim_{n \to \infty} \pi(n) = \pi \) for all \( \pi(0) \), then true for \( \pi(0) = [0 \ 0 \ \cdots \ 0] \)
Then, \( \pi = \lim_{n \to \infty} \pi(n) = \pi(0) \lim_{n \to \infty} P^n = \) top row of \( \lim_{n \to \infty} P^n \) Repeat for \( \pi(0) = [0 \ 0 \ \cdots \ 0] \) to show a\textsuperscript{th} row = \( \pi \)
Etc.
State Value: -2 -1 0 1 2
State Index: 0 1 2 3 4

\[
P = \begin{bmatrix}
0.6 & 0 & 0.4 & 0 & 0 \\
0.2 & 0.4 & 0.4 & 0 & 0 \\
0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\
0 & 0 & 0.4 & 0.4 & 0.2 \\
0 & 0 & 0.4 & 0 & 0.6 \\
\end{bmatrix}
\]

\( b_{ii} > 0 \Rightarrow \) aperiodic
\( \pi = \pi P \) yields \( \pi = \left[ \frac{2}{9}, \frac{1}{9}, \frac{1}{3}, \frac{1}{9}, \frac{2}{9} \right] \).

So mean recurrence time to state with value 0 (index = 2) is '
\[
\mu_0 = \frac{1}{\frac{1}{3}} = 3.0
\]