EECS 863 HW #5 S'08

1. 3.20

360 calls/hr $= 6$ calls/min $= \lambda$ $\quad 1/\mu = 3$ min

$\rho = \lambda/\mu =$ 18 Erlangs

Pr(Block) $= 0.001 = 0.1\%$

M/M/S/S (assume Poisson arrivals & exp. service times)

$\Rightarrow$ Use Erlang B

From table: Can handle 18.21 Erl with 32 trunks

2. 3.20 again, but use Pr(Queue) $= 0.1\%$ Still 18 Erlangs

$\Rightarrow$ Use Erlang C Table: Need 34 trunks

(33 almost enough: handles 17.97 Erl).

3. Same form as $M/M/s/\infty$, but with a finite sum.

$$\pi_0 = \left(1 + \sum_{n=1}^{s-1} \prod_{i=0}^{n-1} \frac{\lambda}{(i+1)\mu} + \sum_{n=s}^{N} \prod_{i=0}^{s-1} \frac{\lambda}{(i+1)\mu} \prod_{i=s}^{n-1} \frac{\lambda}{s\mu}\right)^{-1}$$

$$= \left(\sum_{n=0}^{s-1} \frac{\rho^n}{n!} + \sum_{n=s}^{N} \frac{\rho^n}{s!}\left(\frac{1}{s}\right)^{n-s}\right)^{-1} \qquad \rho = \lambda/\mu.$$

$$= \left(\sum_{n=0}^{s-1} \frac{\rho^n}{n!} + \frac{1}{s!}\rho^s \sum_{k=0}^{N-s} (\rho/s)^k\right)^{-1}$$

$$= \left(\sum_{n=0}^{s-1} \frac{\rho^n}{n!} + \frac{\rho^s}{s!} \cdot \frac{1-(\rho/s)^{N-s+1}}{1-\rho/s}\right)^{-1}$$

$$\pi_n = \pi_0 \frac{\rho^n}{n!} \quad n < s \qquad \pi_n = \pi_0 \frac{\rho^n}{s!\, s^{n-s}}, \quad s \le n \le N$$

$$Pr(Q) = \sum_{n=s}^{N-1} \pi_n = \pi_0 \frac{\rho^s}{s!} \cdot \frac{1-(\rho/s)^{N-s}}{1-\rho/s} \qquad Pr(R) = \pi_N = \pi_0 \frac{\rho^N}{s!\, s^{N-s}}$$

4. a). $\lambda = 75$ calls/hr $= 1.25$ calls/min.
$1/\mu = 2$ min $\Rightarrow \rho = 2.5$ Erlangs.
Erlang B table: 7 phone lines & order-takers.

b). Erlang B table: 4 lines, 1% Block
$\rho = 0.869$ Erlangs $= \lambda/\mu$
$\Rightarrow 1/\mu = 0.869/1.25 = 0.70$ min ≈ 42 sec.

c). $\rho = 1.5$, $N = 4$, Erlang B.

$$P(B) = \frac{(1.5)^4/4!}{1 + 1.5 + \frac{(1.5)^2}{2!} + \frac{(1.5)^3}{3!} + \frac{(1.5)^4}{4!}} = 4.79\%$$

Thruput $= (75)(1-.048) = 71.4$ orders/hour.

d) Use results of problem 1, plus Little's Law.
Note that $\bar{n}_q$ (mean number in queue) is:
(next page)

4 d). $\bar{n}_q = \sum_{n=s+1}^{N} (n-s)\pi_n$

N	π_0	P(Hold)	P(Busy)	Thruput	$\bar{n}_q$
4	0.227	0	4.79%	71.4/hr.	0
5	0.223	4.73%	1.77%	73.7/hr.	0.0177
6	0.222	6.43%	0.66%	74.5/hr.	0.031
∞	0.221	7.13%	0	75.0/hr.	0.045

Last line is Erlang C.

$\bar{n}_q$ for Erlang C: $\pi_0 \sum_{n=s+1}^{\infty} (n-s) \frac{\rho^n}{s!} \left(\frac{1}{s}\right)^{n-s}$

$$= \pi_0 \sum_{k=1}^{\infty} k \frac{\rho^{s+k}}{s!} \left(\frac{1}{s}\right)^k$$

$$= \pi_0 \frac{\rho^s}{s!} \frac{1}{1-\rho/s} \sum_{k=1}^{\infty} k (\rho/s)^k (1-\rho/s)$$

$$\bar{n}_q = \pi_0 \frac{\rho^s}{s!} \frac{\rho/s}{(1-\rho/s)^2} \quad \text{for Erlang C.}$$

P(Hold) increases toward Erlang C value as P(Block) decreases toward 0, as it should!

e). See previous part. With ≤ 1% blocking, P(Hold) is very close to that w/ inf queue.