EECS 863 HW 6 568

1\.

[4 d)] $\bar{n}_q = \sum_{n=s+1}^{N} (n-s)\pi_n$ and we should use thruput in Little's Law.

from HW5

N	π_0	$P(\text{Hold})$	$P(\text{Busy})$	Thruput	$\bar{n}_q$	$\bar{T}_q = \bar{n}_q / \text{Thruput}$
4	0.227	0	4.79%	71.4/hr.	0	0
5	0.223	4.73%	1.77%	73.7/hr.	0.0177	0.86 sec.
6	0.222	6.43%	0.66%	74.5/hr.	0.031	1.50 sec
∞	0.221	7.13%	0	75.0/hr.	0.045	2.15 sec

Last line is Erlang C.

4\. 3.27

$C_{Total} = 50$ kb/s 5 callers @ 5 pkts/s = 25 pkt/s (assume Poisson)

Pkts exp. distrib w/ mean length $\bar{l} = 1000$ b/pkt

a) Split into five indep channels, each 10 kb/s.

$\lambda = 5$ pkts/s $\mu = 10\text{k b/s} / 1\text{k b/pkt} = 10$ pkt/s.

$\Rightarrow \rho = \lambda/\mu = 0.5$

$\bar{n} = \rho/1-\rho = 0.5/0.5 = 1$ $\bar{n}_q = \rho^2/1-\rho = 0.5$

Total of $5\bar{n} = 5$ pkts in system or $5\bar{n}_q = 2.5$ in queue.

b) Delay (time in system)

$\bar{T} = 1/(\mu-\lambda) = \frac{1}{10-5} = 1/5 = 0.2$ s. $= 200$ ms

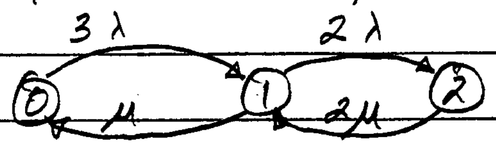
c) Pool resources

$\lambda = 25$ pkts/s $\mu = 50\text{k b/s} / 1\text{k b/pkt} = 50$ pkts/s

$\rho = 25/50 = 0.5$ still, so $\bar{n} = 1$ and $\bar{n}_q = 0.5$ total

$\bar{T} = 1/(\mu-\lambda) = \frac{1}{50-25} = \frac{1}{25} = 0.04 = 40$ ms

2. M/M/2/2/3 $(s=2,\ k=3)$



$3\lambda \pi_0 = \mu \pi_1 \Rightarrow \boxed{\pi_1 = 3\frac{\lambda}{\mu}\pi_0}$

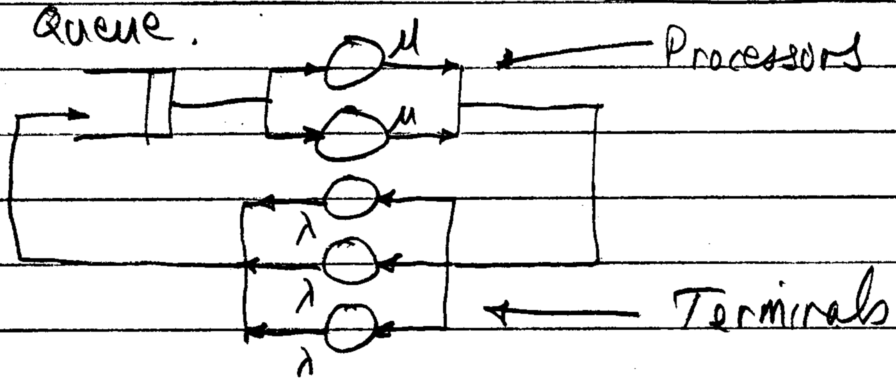
$2\lambda \pi_1 = 2\mu \pi_2 \Rightarrow \pi_2 = \lambda/\mu\, \pi_1 = \boxed{3(\lambda/\mu)^2 \pi_0 = \pi_2}$

Let $\lambda/\mu = \rho$

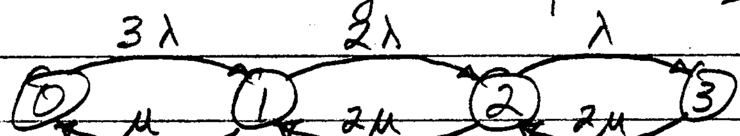
$1 = \sum \pi_i = \pi_0 + 3\rho\pi_0 + 3\rho^2\pi_0 = \pi_0(1+3\rho+3\rho^2)$

$\boxed{\pi_0 = \frac{1}{1+3\rho+3\rho^2}}$ $\boxed{\rho = 1: \pi_0 = \frac{1}{7} = 0.143 \quad \pi_1 = \frac{3}{7} = 0.429 \quad \pi_2 = \frac{3}{7} = 0.429}$

3 a)



b) State = number of jobs in process or in queue.



c) (d)

$3\lambda\pi_0 = \mu\pi_1 \Rightarrow \pi_1 = \boxed{3\rho\pi_0 = \pi_1}$ $\quad \pi_1 = 0.353$

$2\lambda\pi_1 = 2\mu\pi_2 \Rightarrow \pi_2 = \rho\pi_1 = \boxed{3\rho^2\pi_0 = \pi_2}$ $\quad \pi_2 = 0.353$

$\lambda\pi_2 = 2\mu\pi_3 \Rightarrow \pi_3 = 0.5\rho\pi_2 = \boxed{1.5\rho^3\pi_0 = \pi_3}$ $\quad \pi_3 = 0.176$

$\boxed{\pi_0 = 1/(1+3\rho+3\rho^2+1.5\rho^3)}$ $\quad \pi_0 = 0.118$

d) π_0 lower with queue because "blocked" requests not cleared.

$\bar{n} = 1.587$ Thruput $= \pi_1\cdot\mu + \pi_2(2\mu) + \pi_3(2\mu) = 1.411\mu = 1.411$

$\Rightarrow$ Little: Time in sys $= 1.587/1.411 = 1.125$ s.

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6. $\overline{T}_q = 1/\mu \cdot$ (mean number in system given one about to enter). Noting that an arrival cannot enter the system if full, the conditional pmf for this situation is:

$$\frac{\Pr[n \text{ in system and one about to enter}]}{\Pr[\text{one about to enter}]} \stackrel{\text{Markov.}}{=} \frac{\pi_n}{1-\pi_N} \qquad n=0,1,\cdots N-1.$$

$$\text{So } \overline{T}_q = \frac{1}{\mu}\sum_{k=1}^{N-1} k\cdot\frac{\pi_k}{1-\pi_N} = \frac{(1-\rho)}{\mu(1-\pi_N)(1-\rho^{N+1})}\sum_{k=1}^{N-1} k\rho^k.$$

Is this the same as $\frac{\overline{n}_q}{\lambda(1-P(\text{Block}))}$?

Note that pmf of # in queue is

$$q_0 = \pi_0+\pi_1 \quad q_n = \pi_{n+1} \quad n=2,3,\cdots N-1.$$

$$\text{So } \overline{n}_q = \sum_{k=1}^{N-1} k\,\pi_{n+1} \qquad \text{Also } P(\text{Block}) = \pi_N.$$

$$\text{So } \frac{\overline{n}_q}{\lambda(1-P(\text{Block}))} = \frac{(1-\rho)}{\lambda(1-\pi_N)(1-\rho^{N+1})}\sum_{k=1}^{N-1} k\rho^{k+1} = \frac{(1-\rho)}{\mu(1-\pi_N)(1-\rho^{N+1})}\sum_{k=1}^{N-1} k\rho^k$$

And so indeed $\overline{T}_q = \frac{\overline{n}_q}{\lambda(1-P(\text{Block}))}$ as predicted by Little's Law.

5. pdf of sum of RVs is convolution of pdfs:

$$\int_0^t \lambda e^{-\lambda\tau}\mu e^{-\mu(t-\tau)}d\tau = \frac{\lambda\mu e^{-\mu t}}{\mu-\lambda}e^{(\mu-\lambda)\tau}\Big|_0^t = \frac{\lambda\mu}{\mu-\lambda}\left[e^{-\lambda t}-e^{-\mu t}\right]$$

Overall pdf is $\Pr[\text{busy}]\cdot$ busy_pdf + $\Pr[\text{idle}]\cdot$ idle_pdf.

$$= \rho\cdot\mu e^{-\mu t} + \underbrace{(1-\rho)}_{\frac{\mu-\lambda}{\mu}}\frac{\lambda\mu}{\mu-\lambda}\left[e^{-\lambda t}-e^{-\mu t}\right] = \lambda e^{-\lambda t}$$