EECS 863 HW 7 S'08.

1. $$\hat{\sigma}^2(n) = \frac{1}{n-1} \sum_{i=1}^{n} \left[X_i - \frac{1}{n} \sum_{k=1}^{n} X_k \right]^2$$

$$= \frac{1}{n-1} \sum_{i=1}^{n} \left[X_i^2 - \frac{2}{n} X_i \sum_{k=1}^{n} X_k + \frac{1}{n^2} \left(\sum_{k=1}^{n} X_k \right)^2 \right]$$

If $\{X_i\}_n$ are iid, $E[X_i X_j] = E[X_i]\,E[X_j] = \mu^2 \quad i \neq j$

and $E[X_i X_i] = E[X_i^2] = E[X^2] \quad \forall i$

$$E[\hat{\sigma}^2(n)] = \frac{1}{n-1} \sum_{i=1}^{n} \left[E[X^2] - \frac{2}{n}\left(E[X^2] + (n-1)\mu^2 \right) + \frac{1}{n^2}\left[n\,E[X^2] + (n^2-n)\mu^2 \right] \right]$$

$$= \frac{1}{n-1} (\cancel{n}) \left[\frac{n-2+1}{\cancel{n}} E[X^2] + \frac{-2n+2+n-1}{\cancel{n}} \mu^2 \right]$$

$$= \frac{1}{n-1} \left[(n-1) E[X^2] - (n-1)\mu^2 \right]$$

$$= \left[E[X^2] - \mu^2 \right] = \sigma^2$$ Done.

2.

	Sample of $\hat{\mu}(n)$	Sample of $\hat{\sigma}^2(n)$.
100 runs	0.6012	0.0477
40 runs	0.6258	0.0168.

need $t_{99,1-\alpha/2}$ and $t_{39,1-\alpha/2}$: approx with $t_{100,1-\alpha/2}$ and $t_{40,1-\alpha/2}$

	90% CI (use $t_{n-1,0.95}$)	95% CI (use $t_{n-1,0.975}$)
100 runs.	0.6012 ± 0.0363	0.6012 ± 0.0433
40 runs.	0.6258 ± 0.0345	0.6258 ± 0.0414

smaller C.I. width → slightly better estimate