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2. a). Expressions are not particularly simple compared to general case:

$\rho_i = \lambda_i E[B]$ $\qquad W_0 = \frac{1}{2} E[B^2] \lambda$

where $\lambda = \sum_{i=1}^{P} \lambda_i$

$$W_k = \frac{\frac{1}{2} E[B^2] \lambda}{\left(1 - E[B] \sum_{i=1}^{k} \lambda_i\right)\left(1 - E[B] \sum_{i=1}^{k-1} \lambda_i\right)}$$

b) $$\sum_{k=1}^{P} \lambda_k' W_k = \frac{1}{2} E[B^2] \sum_{k=1}^{P} \frac{\lambda_k}{\left(1 - E[B] \sum_{i=1}^{k} \lambda_i\right)\left(1 - E[B] \sum_{i=1}^{k-1} \lambda_i\right)}.$$

Sum of 1st two terms:

$$\sum_{k=1}^{2} \lambda_k' W_k = \frac{1}{2} E[B^2] \left[\frac{\lambda_1}{1 - E[B]\lambda_1} + \frac{\lambda_2}{[1 - E[B](\lambda_1 + \lambda_2)](1 - E[B]\lambda_1)} \right]$$

$$= \frac{1}{2} E[B^2] \frac{\lambda_1 [1 - E[B](\lambda_1 + \lambda_2)] + \lambda_2}{(1 - E[B]\lambda_1)(1 - E[B](\lambda_1 + \lambda_2))}$$

$$= \frac{1}{2} E[B^2] \frac{(\lambda_1 + \lambda_2)}{1 - E[B](\lambda_1 + \lambda_2)}$$

Pattern continues, and $\sum_{k=1}^{P} \lambda_k$.

$$\sum_{k=1}^{P} \lambda_k' W_k = \frac{1}{2} E[B^2] \frac{\lambda}{1 - E[B]\lambda} = \frac{W_0}{1 - \rho}.$$

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3. Clearly, delay for classes J+1 to ℓ is infinite. Furthermore, classes J+2 to ℓ never receive service because class J+1 queue is never empty.

So, the only priorities that affect the system are priorities 1 to J+1.

But what is effective arrival rate of class J+1?

Call effective traffic intensity of class J+1 ρ'_{J+1}

We then have $\rho'_{J+1} = 1 - \sum_{i=1}^{J} \rho_i$

and effective arrival rate of class J+1 is:

$$\lambda'_{J+1} = \frac{\rho'_{J+1}}{E[B_{J+1}]}$$

Use non-pre-emptive priority model with J+1 priorities, with all statistics of the J+1 classes as in actual system except replace actual λ_{J+1} with λ'_{J+1} above.

1. (2.40 Robertazzi)

High priority customers not affected in any way by low priority customers. So, high priority customers effectively see a single-server queue with arrival rate = 5 and service rate = 20, so

$E[n] = \rho/(1-\rho) = \frac{1}{4}/(1-\frac{1}{4}) = \frac{1}{3}$

[redacted]

EECS 863 HW #10

(a) Each has dedicated link of 192 kbps

4(a) <u>class 1:</u>

$\lambda_1 = 100$ pkts/sec

$L_1 \sim U(32, 128)$

$E[L_1] = \frac{128+32}{2} = 80$ bytes $= 640$ bits

$E[L_1^2] = \frac{(128-32)^2}{12} + 80^2 = 7{,}168$ bytes^2

$E[B_1] = \frac{E[L_1]}{c_1} = \frac{640}{192{,}000} = 3.33$ msec

$E[B_1^2] = \frac{E[L_1^2]}{c_1^2} = 12.44 \times 10^{-6}$ sec^2

$\rho_1 = \lambda_1 E[B_1] = 0.333$

<u>class 2:</u>

$\lambda_2 = 150$ pkts/sec

$L_2 \sim$ expo

$E[L_2] = 128$ bytes $= 1{,}024$ bits

$E[L_2^2] = 2(128^2) = 32{,}768$ bytes^2

$E[B_2] = \frac{E[L_2]}{c_2} = 5.33$ msec

$E[B_2^2] = \frac{E[L_2^2]}{c_2^2} = 56.89 \times 10^{-6}$ sec

$\rho_2 = \lambda_2 E[B_2] = 0.8$

M/G/1

$$E[T_i] = E[B_i] + \frac{\lambda_i E[B_i^2]}{2(1-\rho_i)}$$

Mean delay class 1

$$E[T_1] = 3.33 \times 10^{-3} + \frac{100 \times 12.44 \times 10^{-6}}{2(1-0.333)} = 4.26 \text{ msec}$$ <u>ans</u>

Mean delay class 2

$$E[T_2] = 5.33 \text{ msec} + \frac{150 \times 56.89 \times 10^{-6}}{2(1-0.8)} = 26.66 \text{ msec}$$ <u>ans</u> ✓

traffic-weighted mean delay

$$E[T] = \frac{\lambda_1}{\lambda_1+\lambda_2} E[T_1] + \frac{\lambda_2}{\lambda_1+\lambda_2} E[T_2] = \frac{100}{250}(4.26) + \frac{150}{250}(26.66)$$

$$= 17.7 \text{ msec}$$ <u>ans</u> ✓

4 (b) Share 384 Kbps link.

Class 1:

$$E[B_1] = \frac{640}{384000} = 1.67 \text{ msec}$$

$$E[B_1^2] = 3.11 \times 10^{-6} \text{ sec}^2$$

Class 2:

$$E[B_2] = \frac{1024}{384000} = 2.67 \text{ msec}$$

$$E[B_2^2] = 2(2.67 \text{ msec})^2 = 14.22 \times 10^{-6} \text{ sec}$$

$$E[B] = \frac{\lambda_1}{\lambda_1+\lambda_2} E[B_1] + \frac{\lambda_2}{\lambda_1+\lambda_2} E[B_2] = 2.27 \text{ msec}$$

$$E[B^2] = \frac{\lambda_1}{\lambda_1+\lambda_2} E[B_1^2] + \frac{\lambda_2}{\lambda_1+\lambda_2} E[B_2^2] = 9.78 \times 10^{-6} \text{ sec}^2$$

$$\rho = (\lambda_1+\lambda_2) E[B] = \rho_1 + \rho_2 = 0.167 + 0.4005 = 0.568$$

Mean delay class 1:

$$E[T_1] = 1.67 \text{ msec} + \underbrace{\frac{(100+150)\, 9.78\times 10^{-6}}{2\,(1-0.568)}}_{W = 2.83\times 10^{-3}} \simeq 4.50 \text{ msec} \quad \text{Ans}$$

Mean delay class 2:

$$E[T_2] = 2.67 \text{ msec} + \frac{(100+150)\, 9.78\times 10^{-6}}{2\,(1-0.568)} \simeq 5.50 \text{ msec} \quad \text{Ans}$$

Mean delay

$$E[T] = 2.27 \text{ msec} + \frac{(150+100)\, 9.78\times 10^{-6}}{2\,(1-0.568)}$$

$$\simeq 5.10 \text{ msec} \quad \text{ans}$$

Note: $\sum \rho_i W_i = (\rho_1+\rho_2) W = (0.568)(2.83\times 10^{-3}) = 1.61 \times 10^{-3}$

4 (c) Non-preemption priority.

$$W_0 = \frac{1}{2}\left(\lambda_1 E[B_1^2] + \lambda_2 E[B_2^2]\right) = \frac{1}{2}\left(100 \times 3.11\times 10^{-6} + 150 \times 14.22 \times 10^{-6}\right)$$

$$= 1.22 \text{ msec}$$

$$W_1 = \frac{W_0}{1-\rho_1} = \frac{1.22 \text{ msec}}{1-0.167} = 1.47 \text{ msec}$$

$$W_2 = \frac{W_1}{1-\rho_1-\rho_2} = \frac{1.47 \text{ msec}}{1-0.568} = 3.4 \text{ msec.}$$

Mean delay class 1:

$$E[T_1] = E[B_1] + W_1 = 1.67 + 1.47 = 3.14 \text{ msec} \quad \underline{\text{Ans}}$$

Mean delay class 2:

$$E[T_2] = E[B_2] + W_2 = 2.67 + 3.4 = 6.07 \text{ msec} \quad \underline{\text{Ans}}$$

Mean delay

$$E[T] = \frac{100}{250} E[T_1] + \frac{150}{250} E[T_2]$$

$$= 4.9 \text{ msec} \quad \underline{\text{Ans}}$$

Note: $\Sigma \rho_i W_i = (0.167)(1.47\times 10^{-3}) + (0.4)(3.4\times 10^{-3})$

$= 1.61 \times 10^{-3}$

Same as $\Sigma \rho_i W_i$ for M/G/1 w/o priorities

This is M/G/1 conservation law.

5. M/G/1 Conservation Law: $\sum_i \rho_i W_i = \frac{\rho W_0}{1-\rho}$

From previous solution:

$\rho = 0.568$, $W_0 = 1.22$ ms. $\frac{(0.568)(1.22)}{1-0.568} = 1.61$ ms.

$$\overline{T}_1 = \frac{\overline{T}_{1,fcfs} + \overline{T}_{1,priority}}{2} = \frac{4.5+3.14}{2} = 3.82 \text{ ms}$$

$$W_1 = \overline{T}_1 - E[B_1] = 3.82 - 1.67 = 2.15 \text{ ms}$$

$$\sum_i \rho_i W_i = (0.167)(2.15) + (0.4)(W_2) = 1.61 \Rightarrow W_2 = 3.13 \text{ ms}$$

$$\overline{T}_2 = E[B_2] + W_2 = 2.67 + 3.13 = 5.8 \text{ ms}$$

$$\overline{T} = \frac{\lambda_1}{\lambda}\overline{T}_1 + \frac{\lambda_2}{\lambda}\overline{T}_2 = \frac{100}{250}(3.82) + \frac{150}{250}(5.8) = 5.01 \text{ ms}$$

6. Is it work-conserving? Yes!
Since it is a pre-emptive system, the class 2 traffic is totally transparent to the class 1 traffic. So, class 1 waiting time can be computed using M/G/1 results assuming it is the <u>only</u> traffic. Can then use M/G/1 conservation to get class 2 mean waiting times.