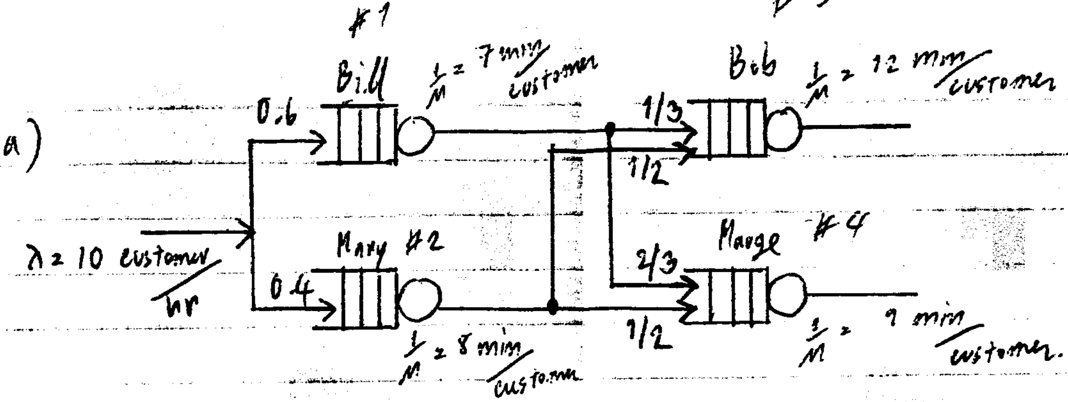
EECS 863 HW12 508.

(1) a)



b) Bill : $\theta_{Bill} = 0.6\lambda = 0.6 \times \frac{10}{60} = 0.1$ customer/min

Mary : $\theta_{Mary} = 0.4\lambda = 0.4 \times \frac{10}{60} = 0.067$ customer/min

Bob : $\theta_{Bob} = \frac{1}{3}\theta_{bill} + \frac{1}{2}\theta_{Mary}$

$= \left(\frac{1}{3}\right)0.1 + \left(\frac{1}{2}\right)0.067 = 0.067$ customer/min

Marge : $\theta_{marge} = \frac{2}{3}\theta_{bill} + \frac{1}{2}\theta_{mary}$

$= \left(\frac{2}{3}\right)0.1 + \left(\frac{1}{2}\right)0.067 = 0.1$ customer/min.

c) $\rho_1 = \frac{\theta_1}{\mu_1} = 0.7$ $\qquad \rho_3 = \frac{\theta_3}{\mu_3} = 0.8$

$\rho_2 = \frac{\theta_2}{\mu_2} = 0.53$ $\qquad \rho_4 = \frac{\theta_4}{\mu_4} = 0.9$

$\bar{Z}_{net} = \frac{1}{\lambda}\sum \frac{\rho_i}{1-\rho_i} = 6\left(\frac{0.7}{0.3} + \frac{0.53}{0.47} + \frac{0.8}{0.2} + \frac{0.9}{0.1}\right)$ min/customer

$= 98.77$ min $= 1$ hr 39 min

$\therefore$ Expect time to finish is 11:39 AM.

d)

9/9

1d) Choose Mary, then Bob $\left(\text{Bill}: \frac{7}{1-0.7} = 23.3 \quad \text{Marge}: 90\,\text{min} \right)$

Expected wait is $\underbrace{\frac{8}{1-0.53}}_{\text{Mary}} + \underbrace{\frac{12}{1-0.8}}_{\text{Bob}} = 17\,\text{min} + 60\,\text{min} = 77\,\text{min}$

Savings: 22 min

Willing to pay $\frac{22}{60}(20) = \$7.33$

3. a) Function to be minimized:

$$Q = \frac{1}{\lambda} \sum_{i=1}^{m} \frac{r_i}{c_i - r_i} + \alpha \left[\left(\sum_{i=1}^{m} c_i \right) - C \right]$$

$$0 = \frac{\partial Q}{\partial c_i} = \frac{-r_i}{\lambda} \frac{1}{(c_i - r_i)^2} + \alpha \Rightarrow \alpha = \frac{r_i}{\lambda} \frac{1}{(c_i - r_i)^2} \quad i = 1, \ldots, m$$

$$0 = \frac{\partial Q}{\partial \alpha} = \left(\sum c_i \right) - C \Rightarrow C = \sum_{i=1}^{m} c_i$$

$$\text{So, } (c_i - r_i)^2 = \frac{r_i}{\alpha \lambda} \Rightarrow c_i = r_i + \frac{\sqrt{r_i}}{\sqrt{\alpha \lambda}}$$

$$\Rightarrow \sum_i c_i = \sum_i r_i + \frac{1}{\sqrt{\alpha \lambda}} \sum_i \sqrt{r_i} = C$$

$$\Rightarrow \frac{1}{\sqrt{\alpha \lambda}} = \frac{\left(C - \sum_i r_i \right)}{\sum_i \sqrt{r_i}}$$

$$\text{So, } c_i = r_i + \frac{\left(C - \sum r_i \right) \sqrt{r_i}}{\sum \sqrt{r_i}} \qquad \text{Algebra: } \alpha = \frac{\left(\sum \sqrt{r_i} \right)^2}{\lambda \left(C - \sum r_i \right)^2}$$

b),
$$I_{net} = \frac{1}{\lambda} \sum_i \frac{r_i}{c_i - r_i} = \frac{1}{\lambda} \sum_i \frac{r_i}{\left(C - \sum_i r_i \right) \cdot \sqrt{r_i}} \cdot \sum_i \sqrt{r_i}$$

$$= \frac{1}{\lambda} \cdot \frac{1}{C(1-\rho)} \cdot \left(\sum_i \sqrt{r_i} \right)^2$$

2. a) $(\lambda + \sum_i \mu_i)\pi_{\underline{n}} = \sum_i \lambda r_{si} \pi_{\underline{n}-1_i} + \sum_i \mu_i r_{id} \pi_{\underline{n}+1_i} +$
$+ \sum_i \sum_j \mu_j r_{ji} \pi_{\underline{n}+1_j-1_i}$

b) $\sum_i (\lambda r_{si}) \pi_{\underline{n}-1_i} = \sum_i \underbrace{\theta_i \pi_{\underline{n}-1_i}}_{\mu_i \pi_{\underline{n}}} - \sum_i \sum_j r_{ji} \underbrace{\theta_j \pi_{\underline{n}-1_i}}_{\mu_j \pi_{\underline{n}-1_i+1_j}}$

$= \underbrace{\sum_i \mu_i \pi_{\underline{n}}}_{\text{Cancels 2nd part of L.H.S.}} - \underbrace{\sum_i \sum_j \mu_j r_{ji} \pi_{\underline{n}-1_i+1_j}}_{\text{Cancels last term of R.H.S.}}$

c) $\lambda \pi_{\underline{n}} = \sum_i \mu_i r_{id} \pi_{\underline{n}+1_i}$ $\quad \mu_i \pi_{\underline{n}+1_i} = \theta_i \pi_{\underline{n}}$

$\lambda \pi_{\underline{n}} = \sum_i r_{id} \theta_i \pi_{\underline{n}}$ or $\lambda = \sum_i r_{id} \theta_i$

Last is certainly true: total flow leaving net.

4)

a)

			Sqrt Assignment		Equal Division		Proportional	
Office	θ_i	r_i	c_i	z_i (msec)	c_i	z_i (msec)	c_i	z_i (msec)
Main	40	40,960	47,199	164	44,544	286	49,231	124
Branch	10	10,240	13,359	328	13,824	286	12,308	495
Sat.	2	2,048	3,443	734	5,632	286	2,461	2,479

$z_{net} = 218$ $\qquad z_{net} = 286$ $\qquad z_{net} = 286$ msec

b) <u>Equal division.</u> $c_i = r_i + \dfrac{(c - \sum_i r_i)}{m} = r_i + \Delta c/m$

$$\bar{z}_{net} = \sum_i \left(\frac{\theta_i}{\lambda}\right)\left(\frac{l_i}{c_i - r_i}\right) = \frac{l}{\lambda}\sum_i \frac{\theta_i}{c_i - r_i} = \frac{l}{\lambda}\sum \frac{\theta_i}{\Delta c/m}$$

$$= \frac{m\,l}{\Delta c\,\cancel{\lambda}}\,\cancel{\sum \theta_i} = \boxed{\frac{m\,l}{\Delta c}}$$

<u>Proportional</u> $\quad c_i = c \cdot \dfrac{r_i}{\sum r_i}$

$$c_i - r_i = \Delta c \frac{r_i}{\sum r_i}$$

$$\bar{z}_{net} = \sum \left(\frac{\theta_i}{\lambda}\right)\left(\frac{l_i}{c_i - r_i}\right) = \frac{1}{\lambda}\sum_i \frac{\theta_i\, l \sum r_i}{\Delta c\, r_i}, \quad (r_i = \theta_i l)$$

$$= \frac{1}{\lambda}\sum_i \frac{\sum r_i}{\Delta c} = \frac{m \sum r_i}{\lambda \Delta c} = \boxed{\frac{m\,l}{\Delta c}} \quad \left(\because \sum r_i = l\lambda\right)$$

Mean delay for Equal division = Mean delay for proportional assignment

5. a). Function to be minimized:

$$Q = \frac{1}{\lambda}\sum_i \frac{r_i}{c_i - r_i} + \alpha\left(\sum_i (d_i c_i + a_i) - D\right)$$

$$0 = \frac{\partial}{\partial c_i} Q = \frac{-r_i}{\lambda}(c_i - r_i)^{-2} + \alpha d_i \Rightarrow \alpha d_i = \frac{r_i}{\lambda (c_i - r_i)^2} \quad (a)$$

$$0 = \frac{\partial}{\partial \alpha} Q = \sum_i (d_i c_i + a_i) - D \Rightarrow D = \sum_i (d_i c_i + a_i) \quad (b)$$

$$(a): \quad (c_i - r_i)^2 = \frac{r_i}{\lambda \alpha d_i}$$

$$c_i = r_i + \frac{\sqrt{r_i}}{\sqrt{\lambda \alpha d_i}}$$

$$a_i + d_i c_i = d_i r_i + \frac{\sqrt{d_i r_i}}{\sqrt{\lambda \alpha}} + a_i$$

Sum over i: $D = \sum_i (d_i r_i + a_i) + \frac{1}{\sqrt{\lambda\alpha}} \sum_i \sqrt{d_i r_i}$

$$\frac{1}{\sqrt{\lambda\alpha}} = \frac{\left[D - \sum (d_i r_i + a_i)\right]}{\sum \sqrt{d_i r_i}} = \frac{D_a}{\sum_i \sqrt{d_i r_i}}$$

5a) (cont).

So, $c_i = r_i + D_a \dfrac{\sqrt{r_i}}{\sqrt{d_i}\sum_i \sqrt{d_i r_i}} = r_i + \dfrac{D_a}{d_i}\dfrac{\sqrt{d_i r_i}}{\sum \sqrt{d_i r_i}}$

b). Let $a_i = 0$ and $d_i = d \ \forall i$

$D_a = D - d \sum r_i$ and $D = d \sum_i c_i = dC$

so $D_a = d(C - \sum_i r_i)$

Then $c_i = r_i + \dfrac{\cancel{d}(C - \sum r_i)}{\cancel{d}} \dfrac{\cancel{\sqrt{d}}\sqrt{r_i}}{\cancel{\sqrt{d}}\sum_i \sqrt{r_i}}$

And so $c_i = r_i + (C - \sum r_i)\dfrac{\sqrt{r_i}}{\sum_i \sqrt{r_i}}$

6)

Min Avg delay

Link	θ_i	r_i (kbps)	d_i ($ per kbps)	$d_i * r_i$	sqrt($d_i * r_i$)	C_i (kbps)	τ_i (msec)
1	3.15	3.15	1023	3222.45	56.77	5.89	365.13
2	3.55	3.55	1566	5559.3	74.56	5.90	425.54
3	0.13	0.13	1028	133.64	11.56	0.69	1801.72
4	3.64	3.64	1011	3680.04	60.66	6.60	337.67
5	0.82	0.82	1085	889.7	29.83	2.18	737.01
6	3.88	3.88	1653	6413.64	80.09	6.27	418.20
7	9.95	9.95	821	8168.95	90.38	15.38	184.04
	25.12	25.12	8187	28067.72	403.85		

Da = 48000-28067.72 = 19932.28

Mean delay = 426.83 msec

7)

Min-Max

Link	r_i (kbps)	C_i (kbps)	τ_i (msec)
1	3.15	5.58	410.74
2	3.55	5.98	410.74
3	0.13	2.56	410.74
4	3.64	6.07	410.74
5	0.82	3.25	410.74
6	3.88	6.31	410.74
7	9.95	12.38	410.74
	25.12		

Mean delay = 538.23 msec

Comparison

	Capacity constraint		Min Avg delay		Min-Max	
Link	C_i (kbps)	τ_i (msec)	C_i (kbps)	τ_i (msec)	C_i (kbps)	τ_i (msec)
1	6.55	294	5.89	365.13	5.58	410.74
2	7.16	277	5.90	425.54	5.98	410.74
3	0.82	1449	0.69	1801.72	2.56	410.74
4	7.29	274	6.60	337.67	6.07	410.74
5	2.55	577	2.18	737.01	3.25	410.74
6	7.65	265	6.27	418.20	6.31	410.74
7	15.99	166	15.38	184.04	12.38	410.74
	τ_{net} (msec) =	326	τ_{net} (msec) =	426.83	τ_{net} (msec) =	538.23