

1. 4.16 Closed net $N=3$ jobs $M=5$ subsystems.

$\bar{\mu}_j = 0.15 \quad \forall j$ Routing matrix given.

Mean Value Analysis

Solve traffic equations, setting $\theta_1 = 10$

(to get all ratios θ_i/θ_j)

Using Matlab,

$$\theta_1 = 10 \quad \theta_2 = 8.936 \quad \theta_3 = 7.371 \quad \theta_4 = 7.093 \quad \theta_5 = 7.669$$

Ratios:

$$\theta_i/\theta_1 \quad 1.00 \quad 0.894 \quad 0.737 \quad 0.709 \quad 0.767$$

$$\theta_i/\theta_2 \quad 1.119 \quad 1.00 \quad 0.825 \quad 0.794 \quad 0.858$$

$$\theta_i/\theta_3 \quad 1.357 \quad 1.212 \quad 1.00 \quad 0.962 \quad 1.040$$

$$\theta_i/\theta_4 \quad 1.410 \quad 1.260 \quad 1.039 \quad 1.00 \quad 1.081$$

$$\theta_i/\theta_5 \quad 1.304 \quad 1.165 \quad 0.961 \quad 0.925 \quad 1.00$$

$$n=0: \quad \bar{\pi}_j(0) = \theta_j = 1 \dots 5$$

$$n=1: \quad \bar{\pi}_j(1) = 0.15 \quad j=1 \dots 5$$

$$\bar{\pi}_j(1) = \frac{(1)\theta_j}{\theta_j \sum_i \theta_i/\theta_j} = \frac{1}{\theta_j \sum_{i=1}^5 \theta_i/\theta_j}$$

$j:$	1	2	3	4	5
$\bar{\pi}_j(1):$	0.2435	0.2176	0.1795	0.1727	0.1867

$$\text{sum} = 1.0 \quad \checkmark$$

$$n=2: \quad \bar{\pi}_j(2) = 0.1(1 + \bar{\pi}_j(1))$$

$j:$	1	2	3	4	5
$\bar{\pi}_j(2):$	0.1244	0.1218	0.1180	0.1173	0.1187

$$\bar{\pi}_j(2) = \frac{(2)\bar{\pi}_j(2)}{\sum_i \theta_i/\theta_j \bar{\pi}_i(2)}$$

$j:$	1	2	3	4	5
$\bar{\pi}_j(2):$	0.5032	0.4402	0.3578	0.3366	0.3682

sum = 2 ✓

$$n=3: \bar{\pi}_j(3) = 0.1(1 + \bar{\pi}_j(2))$$

$j:$	1	2	3	4	5
$\bar{\pi}_j(3):$	0.1503	0.1440	0.1352	0.1337	0.1368

2. Closed net: $N=4$ $M=5$ CPU is number 5
 $\mu_j:$ 50 20 20 10 50 - "constant" means regardless of terminal.

Traffic Equations:

$$\theta_1 = \theta_2 = \theta_3 = \theta_4 = \theta_5 = \theta$$

$$n=0: \bar{\pi}_j(0) = 0 \quad j = 1, \dots, 5$$

$$n=1: \bar{\pi}_j(1) = \mu_j = 50 \quad 20 \quad 20 \quad 10 \quad 50$$

$\bar{\pi}_j(1):$	0.2985	0.0597	0.0299	0.0149	0.5970
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sum to 1 ✓

$$n=2:$$

$$\bar{\pi}_j(2): 64.9 \quad 21.2 \quad 20.6 \quad 10.1 \quad 79.9$$

$\bar{\pi}_j(2):$	0.5346	0.0873	0.0424	0.0209	1.3149
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sum = 2 ✓

$$n=3:$$

$$\bar{\pi}_j(3): 76.7 \quad 21.7 \quad 20.8 \quad 10.2 \quad 115.7$$

$\bar{\pi}_j(3):$	0.7043	0.0998	0.0478	0.0234	2.1247
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sum to 3 ✓

$$n=4:$$

$$\bar{\pi}_j(4): 85.2 \quad 22.0 \quad 21.0 \quad 10.2 \quad 156.2$$

$\bar{\pi}_j(4):$	0.8184	0.1056	0.0503	0.0246	3.0011
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sum = 4 ✓

Answer.