**EECS-863 Analysis of Communication Networks**
Spring Semester 2008
Assignment **#4** Due **19 February 2008**

Reading (Hayes/Babu): section 3.4
Supplemental Reading: Class notes; Shanmugan/Breipohl section 5.4.2; and Robertazzi sections 2.2, 2.6, 2.8, 2.9, 2.7

1. In an M/M/1 system, evaluate the sums $\Sigma_1$ and $\Sigma_2$ given in class and used for determining recurrence for the three cases: $\rho < 1$, $\rho = 1$, and $\rho > 1$, where $\rho = \lambda/\mu$.

2. This problem is based on one from the Robertazzi book. Consider an M/M/1 system with low, medium, and high loads: specifically, $\rho$ equal to 0.1, 0.5, and 0.9. For each load, calculate the server utilization, the mean number of customers in the system (queue plus server), the variance of the number in the system, and the mean plus 2 standard deviations for the number in the system. Create a graphic (histogram, plot, etc) to show the mean and the mean plus 2 standard deviations for these three load values.

3. Packets arrive at a statistical multiplexer for transmission on a T1 (1.536 Mb/s) transmission link. Note: in communication link speed units, "M" represents $10^6$ (not $2^{20}$) and "b" is bits (not bytes). There is a queue at the statistical multiplexer that can be considered to be infinite. Packets arrive at an average rate of 300 per second. Model the packet arrivals as a Poisson process. The average packet size is 500 bytes, and model the packet size distribution as exponential.

    a. Find the mean service rate, traffic intensity, mean number of packets in the system, mean number in the queue, mean packet delay (queuing plus service), and mean waiting time (queuing only).

    b. What is the probability of finding 4 or more packets in the system (recall that the system is the queue and the server)?

4. Consider an M/M/1/N system.

    a. Under what conditions, if any, can such a system be transient or recurrent null?

    b. Calculate and plot the pmf for the number in an M/M/1/10 system for the following values of $\rho$: 2/3, 1.0, 3/2.

5. In general, results for the blocking probability of a finite queuing system of size $N$ (Pr[$N$ customers in system of size $N$]) can be approximated using results from the corresponding infinite queuing system as Pr[$N$ or more customers in the infinite queuing system]. This problem explores that approximation for $M/M/1$ systems.

    a. Find an expression for Pr[$N$ or more customers in an $M/M/1/\infty$] system.

    b. Compare this approximation to the exact expression for probability of blocking in an $M/M/1/N$ system by plotting both expressions as a function of $N$ for the following values of $\rho$: 0.2, 0.5, 0.8, 1.0.

    c. Discuss the results. In particular, try to explain the reason behind any trends that you see.