EECS-863 Analysis of Communication Networks
Spring Semester 2008
Assignment **#5** Due **26 February 2008**

Reading (Hayes/Babu): sections 3.4, 4.2
Supplemental Reading: Class notes; Robertazzi sections 2.8, 2.9, 2.7, 2.10

1. Problem 3.20, p. 109. M/M/S/S system. *ALSO*, give the value of the offered load as measured in Erlangs.
2. Repeat the previous problem assuming that blocked calls are put on hold until an outside line becomes available. The requirement will be that the probability of an arriving call being placed in queue is 0.001.
3. Derive the formula for the stationary state pmf for the $M/M/S/N$ case, in which $N > S$. Note that this is an appropriate model for an airline reservation center with S agents and N phone lines, so that if there are S or more but less than N customers in the system, arriving customers are put on hold, but if there are N customers in the system, arriving calls receive a busy signal. Also give the formulas for Pr[Queuing] (first case) and Pr[Rejection] (second case).
4. A phone-order pizza place gets an average of 75 calls per hour on Friday evenings (their busiest time). It takes 2 minutes (on average) for an order-taker to process each order. Assume they plan to have the same number of phone lines and order-takers. You may use your Erlang/Poisson tables for the first couple of parts of this problem.
   a. How many phone lines and order-takers do they need to have $\leq 1\,\%$ probability of a customer receiving a busy signal?
   b. The owner is willing to purchase a computerized order-taking system to speed up the order-taking process. How fast would the order-takers have to be able to process orders to meet their blocking objective with only 4 phone lines (and order-takers)?
   c. From this point on in this problem, perform the exact calculations; do not use your tables. Suppose the best the order-takers can do, even with the computerized system, is 1.2 minutes per order. Find the exact blocking probability (assuming blocked calls cleared) with 4 phone lines (and order-takers). Also, with the given busy-hour offered load of 75 calls per hour, how many orders can the owner expect to have processed per hour on Friday nights?
   d. Still assuming a mean processing time per order of 1.2 minutes and 4 order-takers, the owner wants to know what will happen if he adds more phone lines, keeping the number of employees (order-takers) the same. Callers will be put on hold if no order-takers are available. Adding one phone line at a time, tell the owner what will be the probability that a customer will be put on hold, the probability that a customer will get a busy signal, the owner's mean order throughput (processed orders per hour), and the average time a customer spends on hold. Continue adding additional phone lines until the probability of a customer receiving a busy signal drops below 1%. Discuss the trends that you see.
   e. In the last part, you found the probability of a customer being put on hold when the number of phone lines is sufficient to ensure less than 1% probability of a customer receiving a busy signal. Compare that probability with an estimate for the hold probability obtained assuming an infinite number of phone lines.

5. Problem 3.27, p. 110. Resource pooling.

6. For an M/M/2/2/3 system (2 servers, no queuing, finite user population of 3), calculate the steady-state system occupancy distribution (probability that n servers are busy) in terms of λ/μ. Get numerical values when $\lambda/\mu = 1.0$.

7. This problem is based on one in the Robertazzi text. Consider a system of three users sharing a dual-processor computer. Users submit jobs (programs) to the processors, and a processor runs only one program at a time to completion (mean time $1/\mu$). If both processors are busy when a program is submitted, it is put in queue until one of the processors becomes free. Each user submits only one program at a time and thinks about the results of one program before submitting another. We will assume Markovian statistics throughout.

   This situation can be modeled as two queueing systems connected in a cyclic fashion (output of system 1 is input to system 2 and output of system 2 is input to system 1). The dual-processor system would be an M/M/2 with a queue, and the terminal system would be an M/M/3/3. This is almost the same as an M/M/2/2/3 system, but not quite (because of the queue).

   a. Draw the figure of the cyclic queueing system.

   b. Draw and label the state transition diagram (let the state be the number of programs currently running or queued).

   c. Solve for each steady-state processor-system occupancy probability π_n in terms of π_0, λ (where $1/\lambda$ is the mean time between a user getting a result and submitting another program), and μ.

   d. Let $\lambda = \mu = 1.0$ to get numerical values for the last part. Compare these with the values from the previous problem and discuss. Also calculate the mean time for a program to be completed after being submitted (including queuing time).