EECS-863 Analysis of Communication Networks
Spring Semester 2008
Assignment **#6** Due **4 March 2008**

Reading (Hayes/Babu): sections 3.4, 3.2, 4.2
Supplemental Reading: Class notes; Robertazzi section 2.10

1. Complete problem 4(d) from the last assignment by using Little's Law to find the average time a customer spends on hold for each number of phone lines.

2. For an M/M/2/2/3 system (2 servers, no queuing, finite user population of 3), calculate the steady-state system occupancy distribution (probability that n servers are busy) in terms of β/μ, where $1/\beta$ is the mean of the exponential idle time of each user between service requests and $1/\mu$ is the mean of the exponential service time of each of the servers. Get numerical values when $\beta/\mu = 1.0$.

3. This problem is based on one in the Robertazzi text. Consider a system of three users sharing a dual-processor computer. Users submit jobs (programs) to the processors, and a processor runs only one program at a time to completion (mean time $1/\mu$). If both processors are busy when a program is submitted, it is put in queue until one of the processors becomes free. Each user submits only one program at a time and thinks about the results of one program before submitting another (mean time $1/\beta$). We will assume Markovian statistics throughout.

   This situation can be modeled as two queueing systems connected in a cyclic fashion (output of system 1 is input to system 2 and output of system 2 is input to system 1). The dual-processor system would be an M/M/2 with a queue, and the terminal system would be an M/M/3/3. This is almost the same as an M/M/2/2/3 system, but not quite (because of the queue).

   a. Draw the figure of the cyclic queueing system.

   b. Draw and label the state transition diagram (let the state be the number of programs currently running or queued).

   c. Solve for each steady-state processor-system occupancy probability π_n in terms of π_0, β (where $1/\beta$ is the mean time between a user getting a result and submitting another program), and μ.

   d. Let $\beta = \mu = 1.0$ to get numerical values for the last part. Compare these with the values from the previous problem and discuss. Also calculate the mean time for a program to be completed after being submitted (including queuing time).

4. Problem 3.27, p. 110. Resource pooling.

5. The goal of this problem is to prove that the departure process of an M/M/1 system is Poisson with rate λ (same as the arrival rate). This is known as Burke's Theorem, and we will eventually prove it in class using the concept of reversibility of a process. Recall that in an M/M/1 system, the service times are exponentially distributed with parameter μ. Hence, when the server is busy, interdeparture times are exponentially distributed with parameter μ. However, when the server is idle, interdeparture times are the *sum* of two independent random variables: time to wait for an arrival (exponential with parameter λ) and time to serve that arrival (exponential with parameter μ). Use these facts, results from M/M/1 analysis, and basic probability theory to derive the result that the overall (unconditioned) interdeparture time for an M/M/1 system is an exponential random

variable with parameter λ. Since this is the only continuous pdf with the memoryless property, this is equivalent to proving that the departure process is Poisson with parameter λ. Again, explain/justify every step in your derivation.

6. We have derived a general proof of Little's Law in the form: $\bar{n} = \lambda\bar{\tau}$, where $\bar{n}$ is mean number in a "system," $\bar{\tau}$ is the mean time in the "system," and λ is the mean entry rate into the system. For an M/M/1/N *queue* (excluding the server), Little's Law predicts $\bar{n}_q = \lambda \cdot (1 - Pr(Block)) \cdot \bar{\tau}_q$, where $\bar{n}_q$ is the mean number in the queue, $\bar{\tau}_q$ is the mean time in the queue, λ is the mean arrival rate to the queue, and $Pr(Block)$ is the blocking (rejection) probability of the queue. In a manner similar to what we did for an M/M/1/∞ system, verify that Little's Law is correct for this case using basic analytical results for the M/M/1/N case. You *must* explain/justify every step that you use.