EECS-863 Analysis of Communication Networks
Spring Semester 2008
Assignment **#9** Due **8 April 2008**

Reading: Class notes and Hayes/Babu sections 6.1.2, 6.1.3, 6.3 (intro), 6.3.3, Kleinrock supplement on residual life (handout), Hammond and O-Reilly supplement on priority queues (handout)

1. This problem will use M/G/1 results to explore the issue of mean delay (time in system) for various different packet size distributions in a packet transmission system. In all cases, we will take the mean service time to be 2 time units and the mean arrival rate (Poisson) to be 1/3 per time unit.

   a. First find mean packet delay (time in system) if the packet service times are exponentially distributed.

   b. At one extreme of packet service time distributions is deterministic service time. Calculate mean packet delay (time in system) in this case.

   c. For many protocols, there are minimum and maximum packet size limits. Suppose now that the packet service times are uniformly distributed between 1 and 3 time units. Calculate mean packet delay (time in system) in this case.

   d. It was claimed in lecture that some packet length distributions (for example, Ethernet packets) have a bi-modal distribution. To model this case, assume the service times have a parabolic distribution as follows: $f_B(b) = K(b-2)^2$ for $1 < b < 3$, where K must be chosen so that this is a valid pdf. Calculate mean packet delay (time in system) in this case.

   e. An extreme bimodal distribution would be discrete with equal 50% probability for service times of either 1 or 3 time units. Calculate mean packet delay (time in system) in this case.

   f. Summarize all of your results for this problem in a table, and discuss any trends that you see.

2. Use the result for the Laplace transform of time in system in terms of the Laplace transform of service time to derive the pdf for time in system for an M/M/1 system.

3. In this problem, you will derive the result for mean time in the system directly from the result for the Laplace transform of time in system in terms of the Laplace transform of service time.

   a. Show that the first derivative of the Laplace transform of a pdf, evaluated at $s = 0$, is the negative of the mean of the pdf. Also show that the second derivative of the Laplace transform evaluated at $s = 0$ is the second moment of the pdf.

   b. Use the result above to obtain an expression for the mean of the time in system for an M/G/1 system. The procedure will be somewhat similar to the procedure used in lecture to get the expression for the mean number in an M/G/1 system from the MGF of the pmf of the number in system.

4. This problem has to do with results for residual life.

   a. Using the result for the pdf of the residual life, show that when the service time is exponentially distributed, the residual life has exactly the same distribution. This result is expected, of course, due to the memoryless property of the exponential distribution.

b. From the result for the pdf of residual life, find an expression for its Laplace transform in terms of the Laplace transform of the service time pdf.

c. Use the results from the previous part and part (a) of the last problem to show that $E[R] = \frac{E[B^2]}{2E[B]}$, where R is the random variable representing residual life and B is the random variable representing service time.