EECS-863 Analysis of Communication Networks
Spring Semester 2008
Assignment **#10** Due **15 April 2008**

Reading: Class notes and Hayes/Babu sections 6.1.3, 6.3 (intro), 6.3.3, 4.1, Kleinrock supplement on residual life (handout), Hammond and O-Reilly supplement on priority queues (handout)

1. This problem is similar to one in the Robertazzi book. Consider a single-server queue with two classes of customer packets, with class 1 having pre-emptive priority over class 2, as described below. Arrivals are Poisson for both packet classes, but with different mean arrival rates ($\lambda_1 = 500$ packets/second and $\lambda_2 = 300$ packets/second), and services times are exponentially distributed, but again with different mean rates ($\mu_1 = 2000$ packets/second and $\mu_2 = 400$ packets/second).

   Priority 2 packets are allowed to begin service only if no priority 1 packets are in the system (either server or queue). If there are no priority 1 packets in the system and a priority 2 packet is being served, an arriving priority 1 packet will instantly displace the priority 2 packet in the server. The displaced priority 2 packet must re-start its transmission when it is next allowed into the server. Service is FIFO within each class.

   Calculate the mean number of priority 1 packets in the system (queue plus server). Do this *only* for priority 1 packets, not for priority 2 packets. Explain your method of calculation. *Hint*: think how the queuing system treats priority 1 packets.

2. Consider a non-pre-emptive priority queuing system in which the service times of all P priorities are statistically identical.

   a. Find simplified expressions for the mean queuing delay and total delay (queuing plus service time) of each priority class in this particular case.

   b. Find a simple expression for the traffic-weighted mean queuing delay for this particular case given by: $\sum_{i=1}^{P} \lambda'_i W_i$, where $\lambda'_i = \dfrac{\lambda_i}{\sum_{i=1}^{P} \lambda_i}$. Hint: first consider the mean delay of the first two priorities; that is, sum only from $i = 1$ to $i = 2$ and simplify as much as possible. Then, using this result, add in the next term ($i = 3$) and identify a pattern. Then, find a simple expression for the traffic-weighted mean total delay.

3. Suppose that for a certain non-pre-emptive priority queuing system with P priorities, the available capacity is sufficient to handle some of the priorities, but not all. In other words, there exists a J with $1 < J < P$ such that

   $$\sum_{i=1}^{J} \rho_i < 1 \qquad\qquad \sum_{i=1}^{J+1} \rho_i > 1$$

   Find the mean queuing time for each priority class.

4. At a statistical multiplexer, two classes of traffic arrive according to independent Poisson processes as follows. Class 1 traffic arrives an average rate of 100 packets per second. The average arrival rate for class 2 is 150 packets per second. Class 1 packets are uniformly distributed between 32 and 128 bytes. Class 2 packets are exponentially distributed with mean of 128 bytes. In the questions that follow, delay refers to total (queuing plus transmission) delay, and traffic-weighted mean delay is the sum of the two mean delays, each weighted by its proportion of

the total packet arrival rate.

a. Find the mean delay of each class and the traffic-weighted mean over both classes if each class has a dedicated 192 kb/s transmission link.

b. Repeat part (a) if the two classes share a 384 kb/s transmission link on a first-come-first-served basis.

c. Repeat part (a) if the two classes share a 384 kb/s transmission link with class 1 receiving non-preemptive priority.

5. Consider the situation and results of the last problem, parts (a)-(c). Suppose that I tell you that I have developed a non-preemptive, work-conserving queue service discipline that results in the mean delay for class 1 being midway between the class 1 delay in the FCFS system and the class 1 delay in the non-preemptive priority system. What will be the resulting class 2 mean delay and the traffic-weighted mean delay?

6. Could the M/G/1 conservation law be used to derive results for mean waiting times for a 2-class preemptive-resume queue service discipline? If you believe not, explain why not. If you believe so, describe the general procedure you would use (you need not actually do it).

7. Six statistically identical traffic streams each have an arrival rate of 75 packets per second. Every packet is 80 bytes long. Compare mean queuing delay and mean total delay for the following four scenarios: unslotted FDM with each channel having bit rate 64 kb/s; fixed-slot TDM on a channel with bit rate 384 kb/s; statistical multiplexing of all packets on single slotted channel with bit rate 384 kb/s; statistical multiplexing of all packets on single unslotted channel with bit rate 384 kb/s; Present your results as a table or graph.