

## EECS-863 Analysis of Communication Networks

Spring Semester 2008

Assignment #12 Due 6 May 2008

Reading: Class notes and Hayes/Babu sections 4.1, 4.3, 4.4, 4.5

1. Customers arrive at a driver's license examining station according to a Poisson process with a mean rate of 10 customers per hour. At this station, each customer must first fill out paperwork and then take a short driving test. When customers arrive, they are met by a greeter who directs each into one of two waiting lines, one for clerk Bill and the other for clerk Mary. They are assigned randomly, with 60% going to Bill's line and 40% to Mary's line. Bill spends an average of 7 minutes filling out paperwork for each customer, and Mary spends an average of 8 minutes (remember, this is a government agency, so there is LOTS of paperwork). Bill randomly assigns his customers to driving examiners Bob and Marge, with twice as many going to Marge. Mary assigns her customers to Bob and Marge randomly, but equally. Bob's driving tests average 12 minutes, and Marge's average 9 minutes. Assume all servicing times are exponentially distributed.
  - a. Draw the queuing network diagram corresponding to this situation, supplying all numerical values.
  - b. Calculate the arrival rates to each clerk or examiner.
  - c. If you arrive at the station at 10 AM, what is the expected time that you will be finished?
  - d. Suppose that you could bribe the greeter and the clerks to influence the people that they assign you to next (please do not construe this as an endorsement of dishonesty!). What assignments would you ask for, and assuming that your requests were granted and your time is worth 20 dollars/hour, how much would you be willing to pay (total) in bribes?
2. This problem will lead you in a proof that the pseudo-local-balance equations (as presented in class) hold for Jackson networks. The method (as outlined in class) is: Start with global balance equation (known to be valid), substitute the pseudo-local-balance equations (to be proved), obtain an equation that is known to be valid, which shows that the substitution was valid.
  - a. Begin by writing the global balance equations for Jackson networks.
  - b. On one side of the global balance equations, there should be a term  $\sum_i \lambda r_{si} \pi_{n-1_i}$ . In this term, first use the appropriate traffic equation to replace the  $\lambda r_{si}$ , and then use Form 1 and Form 3 of the pseudo-local-balance equations to make other substitutions. Work with this expression until you get it in a form that makes it clear that the entire expression is canceled by other terms in the global balance equation.
  - c. With what remains of the global balance equation, substitute Form 2 of the pseudo-local-balance equations to obtain an equation that you know must be valid.
3. Consider the capacity assignment problem in which the constraint is on total link costs and link costs are proportional to capacity, which as noted in class reduces to a constraint on total link capacity.

- a. Show that the solution to the problem of minimizing  $\bar{\tau}_{net}$  under this constraint is:

$$C_i = r_i + (C - \sum_i r_i) \cdot \frac{\sqrt{r_i}}{\sum_i \sqrt{r_i}} \quad \text{and} \quad \alpha = \frac{\left(\sum_i \sqrt{r_i}\right)^2}{\lambda(C - \sum_i r_i)^2}$$

where  $i$  is the link index (from 1 to  $m$ ).

- b. Show that the optimal (minimum) value of  $\bar{\tau}_{net}$  with this solution is:

$$\bar{\tau}_{net}^* = \frac{\left(\sum_i \sqrt{r_i}\right)^2}{\lambda C(1 - \rho)}$$

4. Besides the square root assignment rule derived on the last assignment, two other simple excess capacity assignment rules are:
- Equal division:  $C_i = r_i + (C - \sum r_i)/m$
  - Proportional to traffic:  $C_i = r_i + (C - \sum r_i)r_i/\sum r_i = r_i/\rho$

We will see (if we have not already) that the equal division rule is a special case of the min-max rule when all packets are the same length.

- a. For the bank office example done in class (main, branch, and satellite offices), compare (in a table) the link capacities, delay at each stat mux, and mean network delay for the three excess capacity assignment strategies.
- b. You should have noticed that the equal division rule and proportional to traffic rule yielded the same mean network delay for the example. Show that this is not a coincidence. That is, show that these other two capacity assignment rules will always yield the same mean network delay.
5. Derive the square root capacity assignment rule and the expression for the minimum mean network delay for the following two cases.
- a. The capacity allocation problem with constraint  $D = \sum_i (d_i C_i + a_i)$ , where  $C_i$  is the capacity of link  $i$ ,  $d_i$  is the cost per unit capacity for link  $i$ , and  $a_i$  is the fixed cost (capacity-independent) for link  $i$ .
- b. Show that the expressions you obtained in the last part reduce to the simple total-capacity-constrained expressions (given above) when  $a_i = 0$  and  $d_i = d$  for all  $i$ .
6. Repeat the 7-link example covered in the class handout, but with a total cost constraint instead of a total capacity constraint, as follows. All fixed costs are zero. The link cost is 1 dollar per kb/s per mile. The following are the link mileages (taken from a road atlas).

Link Number	Between Cities	Mileage
1	L.A. and Denver	1023
2	L.A. and Houston	1566
3	Denver and Houston	1028
4	Denver and Chicago	1011
5	Chicago and Houston	1085
6	Houston and N.Y.	1653
7	Chicago and N.Y.	821

The total cost constraint is 48,000 dollars. Minimize mean network-wide delay. Find the link capacities, link delays, and mean network-wide delay. Compare to the capacity-constrained results.

7. Repeat the last problem for min-max delay.