EECS 868 Mathematical Optimization with Applications
Spring Semester 2019
Assignment **#2** Due **5 February 2019**

Reading: Luenberger/Ye: Sections 7.1 - 7.5, Appendix A.4 - A.6

1. Prove the Corollary to Proposition 1 on p. 185. FONC Theory.
2. This problem is purely optional (no points associated) since not all students will have the necessary background in random processes (part a) and circuit/communications theory (part c). However, it will be quite instructional for anyone with that necessary background.

   Consider the linear predictor problem outlined in Section I of Lecture 2.

   a. Derive the first order necessary conditions for a linear predictor of arbitrary order p, assuming that $\{x_k\}$ is a stationary random process.

   b. What are the FONC for the special case of $p = 1$?

   c. Using the results of part a, find the optimal predictor coefficients for a third order ($p = 3$) predictor when the input sequence $\{x_k\}$ is obtained as follows. Pass white noise through an RC lowpass filter with $RC = 10^{-3}$, then sample the filter output at an 8 kHz sampling rate to get $\{x_k\}$. Discuss your result.

   d. Defining the predictor gain as:

   $$G_p = \frac{E\left[x_k^2\right]}{E\left[e_k^2\right]}$$

   where e_k is the predictor error $x_k - \hat{x}_k$, what is the predictor gain in part c?

3. For Example 2 on p. 187, verify that the summation form of $f(\mathbf{a})$ is equivalent to the vector form at the bottom of the page. Example Application.
4. Let $\mathbf{a}$ be a given n-vector (column vector), and $\mathbf{A}$ be a given n x n symmetric matrix. Let $\mathbf{x}$ be a variable vector of dimension n (also a column vector). Compute the gradient and Hessian of $f_1(\mathbf{x}) = \mathbf{a}^T x$ and of $f_2(\mathbf{x}) = \mathbf{x}^T \mathbf{A}\mathbf{x}$.
5. For the following functions, there are no constraints on the variables. For each one, find all points that meet the first order necessary conditions for local minima of the function and determine which, if any, of these are in fact local minima.

   $$f_1(\mathbf{x}) = 2x_1^2 + x_2^2 - 3x_1 - x_1 x_2 \qquad\qquad f_2(\mathbf{x}) = 2x_2^3 + 3x_1^2 + 6x_1 x_2 + 12x_1$$

6. Are there any values of x, y and z that globally maximize:

   $$f(x, y, z) = \frac{xyz}{x + 2y + 2z^2}$$

   subject to $xy = 2$ and $x \geq 0$, $y \geq 0$, $z \geq 0$? If so, give the values of the variables and the value of the objective function. If not, justify your answer. Hint: minimizing $-f$ is not the only way to maximize f.

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