EECS 868 Mathematical Optimization with Applications
Spring Semester 2019
Assignment **#3** Due **19 February 2019**

Reading: Lecture Notes and Luenberger/Ye: Sections 7.4 - 7.9 (skip 7.6), Appendix A.3 - A.6, Appendix B.1, Sections 8.1 - 8.5

1. For the unconstrained function

   $$f(x,y,z) = 2x^2 + xy + y^2 + yz + z^2 - 6x - 7y - 8z + 9$$

   prove that the point x=1.2, y=1.2, z=3.4 is a *global* minimum point.

2. Luenberger, Problem 9, p. 214. Convexity.

3. On a nameless planet, an object was released vertically and then traveled unhindered. It was observed to have approximately the following heights h_j (in meters) at one-second intervals t_j (also approximate):

j	1	2	3	4	5	6	7	8	9	10	11
t_j	0	1	2	3	4	5	6	7	8	9	10
h_j	100	107	112	111	105	99	84	71	47	25	0

   According to the laws of physics on this planet, the height of the object at any time should be given by a formula of the following form:

   $$h_j = z - v \cdot t_j - 0.5a \cdot t_j^2$$

   where z is the initial height, v is the initial velocity (downward), and a is the acceleration (downward) due to gravity. But since the observations were not made exactly, there exists no choice of z, v, and a that will cause all of the data to fit this formula exactly. Instead, we wish to estimate these three values by choosing them so as to minimize the "sum of squares":

   $$\sum_{j=1}^{n} [h_j - (z - v \cdot t_j - 0.5a \cdot t_j^2)]^2$$

   where n is the number of observations. This expression is a measure of the error between the ideal formula and the observed behavior.

   a. Show that the objective function for this problem is convex. Note that you can make use of the results of Problems 3 and 4 of Assignment 2.

   b. Find optimal values for z, v, and a. Are these values unique? Justify your answer.

4. Does the sequence $x_k = 1/k!$ converge superlinearly? Does it converge with order 2.0?

5. Luenberger, Problem 10, p. 214. Closed Mappings. If your answer is yes, prove it. If your answer is no, prove it or give an example.

6. Refer for this problem to the lecture notes on the Fibonacci search. At step k, it was claimed that the point y_{k-1} was guaranteed to be at a distance $\left(\frac{F_{N-k+1}}{F_{N-k+2}}\right) \cdot w_{k-1}$ from one of the endpoints a_{k-1} or b_{k-1} of the $(k-1)^{th}$ uncertainty interval. This claim allows us to choose point x_k as the point

that is at distance $\left(\frac{F_{N-k+1}}{F_{N-k+2}}\right) \cdot w_{k-1}$ from the other endpoint of the $(k-1)^{th}$ uncertainty interval. This claim was obviously true for $k = 2$ by the choice of $x_1 = y_1$.

Use an induction argument to show that this claim is true for all k up to $k = N$. That is, assume that it is true at step $k - 1$ and then show that it must be true at step k. Since you have already shown that it is true for $k = 2$, this will show that it is true for all k up to $k = N$.

7. Tabulate and compare the ratio of the final uncertainty width to the initial uncertainty width for Fibonacci search and Golden Section search (stopped after the function f has been evaluated for N points) for the following values of N: 2, 3, 4, 5, 10, 15, 20, 30, 40, 50, 100.

8. Perform (a) Fibonacci search and (b) Golden Section search on the function $f(x) = e^{1.5x} - 9x$ in the interval [0.5,1.5] evaluating $N = 6$ points in each case. Be sure to give the endpoints of the uncertainty interval and the width of the uncertainty interval at each step. You may do the calculations by hand or write a program to do them; if you write a program, provide both the source code and the output. Compare the final uncertainty intervals and their widths; do they match your expectations?

