EECS 868 Mathematical Optimization with Applications
Spring Semester 2019
Assignment **#4** Due **19 March 2019**

Reading: Lecture Notes and Luenberger/Ye: Sections 8.6 - 8.11, 9.1 - 9.6 (just skim 8.9, 9.4, and 9.5)

1. Repeat the last problem of the last assignment, performing 3 iterations of Newton's Method starting from the point $x = 1$.
2. Consider the line search step for any algorithm that updates the selected points as $\mathbf{x}_{k+1} = \mathbf{x}_k + \alpha_k \mathbf{d}_k$, and let $\mathbf{g}_k$ be the gradient vector at point $\mathbf{x}_k$ in *column* vector form.
    a. When the objective function is a generalized quadratic function $f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T\mathbf{Q}\mathbf{x} - \mathbf{x}^T\mathbf{b}$ with $\mathbf{Q}$ being a symmetric positive definite matrix, show that the optimal step length α_k can be found explicitly as $\alpha_k = \dfrac{-\mathbf{g}_k^T\mathbf{d}_k}{\mathbf{d}_k^T\mathbf{Q}\mathbf{d}_k}$.
    b. Now, for the Steepest Descent algorithm applied to such a quadratic function, use part (a) to show that the optimal step length α_k can be found explicitly as $\alpha_k = \dfrac{\mathbf{g}_k^T\mathbf{g}_k}{\mathbf{g}_k^T\mathbf{Q}\mathbf{g}_k}$.
3. Luenberger, Problem 12, p. 258. Steepest Descent Convergence.
4. This problem will involve the so-called Rosenbrock function:

    $$f(\mathbf{x}) = 100\left(x_2 - x_1^2 \right)^2 + (1 - x_1)^2$$

    Some results of this problem will be re-used in Project 2.

    a. Compute the gradient and Hessian of the Rosenbrock function (above).
    b. If we wish to minimize this function with no constraints on x_1 and x_2, show that $\mathbf{x}^* = (1\ \ 1)^T$ is the only candidate relative minimum point of this function, and that this point is indeed a relative minimum point.
    c. Execute 3 iterations of the "pure" Newton's Method for this function starting at point $\mathbf{x}_0 = (0\ \ 2.5)^T$. You need not "program" Newton's Method (e.g., in Matlab), but you should certainly use Matlab (or other computational program) to calculate matrix inverses and do the matrix arithmetic. Show the "trajectory" of this method; that is, show the 4 points in E^2 ($\mathbf{x}_0$ through $\mathbf{x}_3$) and connect them in sequence by lines with arrows. Also show the location of $\mathbf{x}^*$. This can be done approximately, by hand; an exact plot is not necessary. *ALSO*, plot the objective function value as a function of iteration. Comment on these results.
5. Luenberger, Problem 5, p. 282. Showing that certain eigenvectors are conjugate. Hint: you may use results from appendix A.4.
6. Luenberger, Problem 1, p. 282. One method of generating a set of conjugate vectors.

