EECS 868 Mathematical Optimization with Communications Applications
Spring Semester 2019
Assignment **#6** Due **16 April 2019**

Reading: Lecture Notes and Luenberger/Ye Sections 3.1 - 3.6

1. Luenberger/Ye, Problem 7, p. 71. Degenerate basic feasible solution.

2. An automobile manufacturer produces several kinds of cars. Each kind requires a certain amount of factory time per car to produce and yields a certain profit per car. A maximum amount of factory time has been scheduled for the next week. However, at least a certain number of each kind of car must be manufactured to meet dealer orders.

    a. Using the table below and a maximum factory time of 120 hours, formulate this as an LP problem in standard form, with the minimum number of variables (you can do it with only 4).

Car	Time	Profit	Orders
T	1	200	10
C	2	500	20
L	3	700	15

    b. Now solve this problem using the simplex method. Finding an initial basic (non-degenerate) feasible solution in this case is trivial. You should ultimately express your solution as the number of type T, C, and L cars produced and the total profit achieved. You should find that the solution specifies a non-integer amount of one of the cars. As a practical matter, how could you make use of this solution?

    c. Now add another constraint to the problem. Each kind of car achieves a certain fuel efficiency, and the manufacturer is required by law to maintain a certain "fleet average" efficiency. The fleet average is computed by multiplying the efficiency of each kind of car times the number of that car produced, and then dividing by the total cars produced. Extend your standard-form LP specification to include a minimum fleet average efficiency constraint of 35 miles/gallon, using 50, 30, and 20 miles/gallon as the fuel efficiencies of the T, C, and L cars, respectively.

    d. Solve this new problem using the simplex method. Finding an initial basic feasible solution is not trivial in this case, but can be found by simple row operations on the initial simplex tableau to get unit vectors corresponding to the basic variables. Again, you should ultimately express your solution as the number of type T, C, and L cars produced and the total profit achieved. Dealing with fractional amounts in the solution is not so easy in this case. What might you do?

3. Luenberger/Ye, Problem 13, p. 72. Alternate choice for entering basic variable. Explanatory note for the criterion: for each k we find the maximum quantity specified, then choose the k that has the smallest such maximum.

4. Luenberger/Ye, Problem 18, part *a* only, p. 73. Two-phase simplex procedure. Use the anti-cycling rule for your pivot choices.

5. This problem is a slightly re-written form of Problem 23, p. 74 in Luenberger/Ye. Consider the system of linear inequalities $\mathbf{Ax} \geq \mathbf{b}$, $\mathbf{x} \geq \mathbf{0}$ with $\mathbf{b} \geq \mathbf{0}$. This system can be transformed to standard form by the introduction of m surplus variables so that it becomes $\mathbf{Ax} - \mathbf{y} = \mathbf{b}$, $\mathbf{x} \geq \mathbf{0}$, $\mathbf{y} \geq \mathbf{0}$, and still with $\mathbf{b} \geq \mathbf{0}$. Unlike the case of *m slack* variables, the initial simplex tableau for this problem is *not* in canonical form, and these *m surplus* variables cannot be the basic variables for the initial basic feasible solution for the simplex method. However, we will need to add only one artificial variable to get the artificial problem for Phase I of the 2-phase simplex method, as you will show here.

   a. Why can't the *m surplus* variables in this problem be the basic variables for the initial basic feasible solution for the simplex method?

   b. Let b_k be the largest entry in $\mathbf{b}$, and consider the new system in standard form obtained by adding the k^{th} row to the negative of every other row. Show that the new system requires the addition of only a single artificial variable.

   c. Use this technique to find a basic feasible solution to the following system of constraints.

$$
\begin{aligned}
&x_1 + 2x_2 + x_3 \geq 4 \\
&2x_1 + x_2 + x_3 \geq 5 \\
&2x_1 + 3x_2 + 2x_3 \geq 6 \\
&x_i \geq 0, \quad i = 1, 2, 3
\end{aligned}
$$

