EECS 868: Mathematical Optimization with Applications

Spring Semester 2019

Assignment **#9** Due **13 May 2019 at 12 noon in the EECS office** (This is the Monday of Finals week)

Reading: Lecture Notes and Luenberger/Ye Sections 11.1 - 11.10, 12.1 - 12.5 (skim 12.2)

1. A rectangular cardboard box is to be manufactured with "width" x, "length" y and "height" z. The specifications call for double weight (that is, two pieces of cardboard) for the top (area xy), bottom (also area xy) and front face (area xz). The problem is to find the dimensions of such a box that maximize the volume for a given amount of cardboard, specified as 72 sq. ft.
   a. What are the first-order necessary conditions (FONC)?
   b. Find x, y, and z that satisfy the FONC.
   c. Verify the second-order conditions for the set(s) of dimensions that meet the first order conditions (found in part b).
   d. Suppose that the area constraint is relaxed to 72.1 sq. ft. How much additional volume would this allow in the optimal box? (Do *not* solve the problem again).
2. Consider a random variable X that can take on the following three values: $x_1 = -5$, $x_2 = 0$, $x_3 = 5$.
   a. What is the maximum entropy probability density function for this random variable, subject to the constraint that the mean of X must be 0?
   b. Re-work the problem for a mean of 2.5.
3. This problem will explore some consequences when points being considered are not *regular* points. In all cases, the objective function (in E^2) to be minimized is $f(\mathbf{x}) = x_1^2 + (x_2 + 1)^2$. Note that, on an x_1, x_2 axis, contours of constant f value are circles with center $x_1 = 0$ and $x_2 = -1$.
   a. First consider only the constraint $h_1(\mathbf{x}) = x_2 = 0$. Using the first order necessary conditions, find a possible relative extremum point. Is this point a *regular* point? Do the second-order necessary and/or sufficiency conditions hold for this point?
   b. Now add a second constraint $h_2(\mathbf{x}) = x_2 + \cos x_1 - 1 = 0$. Sketch the feasible region on an x_1, x_2 plane, identifying the points at which both constraints are met. Are these points regular points? What happens when you try to use the first-order necessary conditions in this case? Specifically, what values of x_1, x_2, λ_1, and λ_2 do the first-order necessary conditions yield? Do the second-order necessary and/or sufficiency conditions hold for this point (or these points)?
4. Luenberger/Ye, Problem 5, parts a and b only, p. 355. Constrained Hessian Eigenvalues.
5. Luenberger/Ye, Problem 8, *chapter 12,* p. 398.