1. Implement the Golden Section line search along with the Strong Wolfe stopping conditions in a computer program. You may choose the language/tool to use (e.g., C, MATLAB, etc.). The implementation should be in the form of a function call or subroutine so that it can be used as part of a multi-dimensional iterative descent algorithm. That is, its inputs should be a point $x$, a direction $d$ (both vectors), constants $c_1$ and $c_2$ for the Wolfe conditions, and a limit $\alpha_{\text{max}}$ (a positive scalar). The minimum value of $\alpha$ should always be 0. The subroutine should find the minimum (approximately) of $f(x + \alpha d)$ for $0 < \alpha \leq \alpha_{\text{max}}$. Consider $\alpha_k$ to be the midpoint of the uncertainty interval after the $k^{\text{th}}$ iteration of the line search algorithm. For $k = 1$, there is no reduction from the initial uncertainty interval, so $\alpha_1$ will be $0 + \alpha_{\text{max}}/2$. The function $f$ to be minimized and $\phi'(\alpha)$ (the derivative of the line search function) should be subroutine or function calls, which will be dependent on the particular objective function being minimized. Provide a listing of the Golden Section subroutine and associated subroutines or function calls and a brief discussion of them.

2. Use this line search program to minimize objective function $f(x_1, x_2) = e^{x_2 - x_1} + (5/8)(x_1 - x_2)^2 + (5/8)(x_1 - x_2)$ along a line with starting point $x = (1, 3)^T$ and direction $d = (1/\sqrt{2}, -1/\sqrt{2})^T$, for $\alpha$ from 0 to $\alpha_{\text{max}} = 50.0$. Use Wolfe parameters of $c_1 = 0.01$ and $c_2 = 0.1$. For each iteration, indicate which (if either) of the Strong Wolfe conditions is satisfied. Illustrate the behavior of the algorithm with plots of $\alpha_k$, objective function value for $\alpha_k$, and $\phi'(\alpha_k)$ versus iteration $k$. Include the above values for $\alpha = 0$ on the plots associated with $k = 0$.

3. Investigate and report on the effects of the following. As part of this investigation, attempt to find values (in parts b and c) for which the line search fails to terminate. You need not include the information required in part 2 for each of these investigations; include only data that will help you describe your findings (which might be different from what was required in part 2). Note that this part is considerably more open-ended than the previous part.
   a. Using Wolfe vs. Strong Wolfe conditions (for the original $\alpha_{\text{max}}, c_1$, and $c_2$ values).
   b. Using different values of $\alpha_{\text{max}}$ (for Strong Wolfe conditions and the original $c_1$ and $c_2$ values).
   c. Using different values for $c_1$ and $c_2$, subject to $0 < c_1 < c_2 < 1$ (for Strong Wolfe and the original $\alpha_{\text{max}}$).