Model-Based Technique for Super Resolution and Enhanced Target Characterization Using a Step-Frequency Radar: A Simulation Study

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Abstract--In this investigation, the signal reflected by a target to the step-frequency radar is processed using both non-parametric and parametric approaches. An autoregressive moving average (ARMA) model of the reflected signal is first used to estimate the location of the targets. Then, an extended Prony's method is used to estimate the magnitude of the reflection coefficients of the targets. These model-based approaches are found to provide super resolution and improvement in target characterization as compared to a conventional non-parametric approach. Several simulations are made to compare the performances of these methods.

INTRODUCTION

In recent years, step-frequency radars [1,2] have been used frequently in estimating the location of the scattering targets, such as ice-layer, subsurface targets, etc. An important advantage in using step-frequency radar [1] is that the phase change rate of the signal received at the radar, for the preselected incremental steps of transmission frequency, is directly proportional to the distance of the scatterer from the radar. Frequently, non-parametric fast-Fourier-transform-(FFT) based procedures are used [1] to estimate the location of the scatterers from the complex reflected signal. It has been shown [2] that the non-parametric approaches suffer from inherent resolution constraints and a parametric modeling approach, popularly known as the MUSIC [3] algorithm, is found to provide super resolution in estimating target locations. In this investigation, we use a high performance ARMA [3,4] model to estimate the target locations with super resolution. On estimating the target locations, we use an extended frequency-domain Prony's method (EFDPM [5]) to estimate the reflection coefficient of the target. The backscattered signal received from targets is synthesized, and the performance of the model-based approach is compared with that of the conventional FFT-based approach.

DATA MODELING

Assume that the transmission frequency (f) of a step-frequency radar is incrememted in discrete steps (n) of a preselected frequency Δf. Then, the signal at the receiver can be expressed as [2]:

$$x(n) = \sum_{k=1}^{D} \Gamma_k(n).\exp(-j2\beta_n d_k) + N(n) \qquad (1)$$

where d_k represents the distance of the k^{th} target and D represents the number of targets seen by the radar. Also, n represents the n^{th} frequency of transmission and $\beta_n=2\pi.n.\Delta f/c$. $\Gamma_k(n)$ represents the reflection coefficient of the k^{th} target at the n^{th} frequency of transmission, and N(n) represents samples of white Gaussian noise. From the above expression it can be seen that x(n) represents samples of a single (for D=1) or multiple (for D>1) sinusoids. The rate of change of the phases of these sinusoids are proportional to the distance of the target from the radar. An inverse discrete Fourier transform of the sequence x(n) can be defined as

$$y(m) = \sum_{n=1}^{M} x(n).\exp(j\beta_n d_m) \qquad (2)$$

where M represents the total number of discrete frequencies under consideration. The samples of y(m) will have a peak (for D=1) or peaks (for D>1). From the location of a peak of |y(m)|, the corresponding location of a target can be estimated [1,2]. Once the location of the target is identified, the next task is to estimate its reflection coefficient. Since in (1), $\Gamma_k(n)$ is multiplied by a complex exponential term, the inverse Fourier transform of x(n) will be the inverse Fourier transform of $\Gamma_k(n)$, shifted by the target distance d_k. Thus, if the samples of the Fourier transform of $\Gamma_k(n)$ can be isolated for each target, a direct Fourier transform of these isolated sequences will provide estimated values of the reflection coefficient $\Gamma_k(n)$ of the target k.

In the first simulation, a single target is located at a distance of 4 m($=d_1$), which possesses a linear reflection coefficient shown in Fig.1. We assume that the radar operates over the frequency range of 2-18 Ghz, and uses Δf=10 MHz. (These radar parameters are identical to the step-frequency radar that will be used to acquire experimental data in the near future.) Samples of x(n) are computed using.(1) for infinite signal-to-noise ratios (SNR), and the samples of the |y(m)| are computed using an FFT. The location of the target is identified as 4.0031 m based on the peak location of |y(m)|. Next, a Hanning window is set up around this peak location and a total of five samples of y(m) are collected. The inverse transform of these samples of y(m) provided the estimated values of the reflection coefficient of the target. The r.m.s value of the estimation error, for infinite SNR, is shown in Table I and the estimated values of $\Gamma_k(n)$ are plotted in Fig.1.

It is seen from Fig.1 that the windowing effect introduces error in the estimated values of the reflection coefficient of the target. It is always preferable to use as few samples as possible around the peak location of y(m) since the error in the estimated values of y(m) increases as we move away from the peak location[3]. But, the fewer samples we use from y(m), the larger will be the deformation in the estimated values of Γ_k(n). In order to overcome these difficulties of non-parametric approaches, we use parametric model-based approaches for estimating the target locations and their reflection coefficients.

Since the samples of $|y(m)|$ basically provide the spectral information of the samples of x(n), a model-based spectrum estimation technique can be used to estimate the location of the targets with a super resolution. In this investigation, we use an ARMA-model-based approach [4] to estimate the location of the targets with high accuracy. According to this scheme, the samples of the random signal x(n) is modeled as the output of an ARMA filter of order (p,q), excited by a zero-mean white Gaussian noise sequence w(n) such that

$$x(n) = \sum_{k=1}^{p} a(k).x(n-k) + \sum_{k=1}^{q} b(k)w(n-k) \qquad (3)$$

where a(k) and b(k) represent the AR and MA coefficients of the ARMA filter. For a pure AR process (b(0)=1 and b(k)=0 for k>0), the AR parameters can be estimated using a high-performance approach[4], which estimates the order p by performing the singular value decomposition (SVD) of an extended order autocorrelation matrix (ACM). In this case, the order p actually correspond to the number of targets present in the field of view of the radar. The samples of ACM are extracted from the samples of x(n) [4]. Using these AR coefficients, the pole locations can be estimated as the roots of the polynomial equation of the z-domain [3,4]

$$\sum_{k=1}^{p} a(k)z^{-k} = 0 \qquad (4)$$

Since the poles of (4) define the sinusoidal components of x(n) [2,3], an accurate estimation of the poles will actually define the target locations accurately. Using this approach, the target location is estimated as 3.9998 m, which is more accurate than that obtained from the inverse-FFT- (IFFT-) based approach (see Table I).

Next, a total of five samples of y(m) are selected around the peak location estimated by the ARMA model. These samples are then used by EFDPM to model the transformation of the reflection coefficient of a target as a rational function model [5] of order (u,v-here u+v+1=5 and s(v)=1). That is,

$$y'(m) = \frac{\sum_{k=0}^{u} r(k).m^{k}}{\sum_{k=0}^{v} s(k).m^{k}} \qquad (5)$$

This rational function model is used to select the parameters r(k) and s(k) so that y'(m) can provide a suboptimal fit to the samples of y(m) over the entire range of m. A direct Fourier transform of this modeled data is used to estimate the samples of $\Gamma_k(n)$, and the result is shown in Fig.1. This scheme provided an r.m.s error of 0.0052, which is less than the 0.1005 provided by the FFT-based approach.

The aforementioned procedures are repeated for SNRs of 40, 30, 20, 10 and 0 dB, and the results are tabulated in Table I. It can be seen that as the SNR decreases, the r.m.s error in estimating the reflection coefficient of the target increases for both methods. The r.m.s error provided by the extended Prony's method stays consistently lower than that of the FFT-based approach.

SUPER RESOLUTION FOR MULTIPLE TARGETS

From Table I, it is seen that the target location estimated by the FFT-based approach did not change at all throughout the testing range of the SNRs. This is expected since we used 1600 samples for only one non-decaying sinusoid. However, when multiple and closely spaced targets are present in the radar's field of view, the situation is found to be very different.

In this section, we assume that the two targets are present at distances 4.0 and 4.05 meters from the radar. Then, we compare the performances of the ARMA-model-based approach versus the FFT-based approach in estimating the locations of the targets as the number samples (N) is decreased. The results are presented in Table II, which shows that the ARMA-model-based approach successfully resolves the two targets when the FFT-based approach cannot resolve a distance of 0.15 meters. Thus, the parametric modeling of the radar return is found to be useful in achieving super resolution as compared to the FFT-based approach.

ESTIMATION OF THE REFLECTION COEFFICIENTS FOR MULTIPLE TARGETS

From Table II we find that the ARMA model can estimate the target locations accurately, using only 100 samples of x(n). To estimate the reflection coefficient, however, we collected 1600 samples of x(n). This is done to make sure that the neighboring target is not undesirably influencing the five samples around each peak location of $|y(m)|$ [2]. Next, five samples are collected around the first peak location of the inverse transformed domain, and the reflection coefficient of the target is estimated using both the Hanning window and the extended Prony's method. The estimated values of the reflection coefficient are shown in Fig. 2, and the r.m.s errors associated the Hamming window and the extended Prony's method are found to be 0.1421 and 0.1143, respectively. In the presence of additive noise, with an SNR of 0 dB, the r.m.s errors are 0.1496 and 0.1293. Thus, the extended Prony's method provided superior performance in characterizing the target.

It is important to note that increasing window size actually decreases the performance of the conventional approach in estimating the samples $\Gamma_k(n)$ due to the undesirable influence from the neighboring target.

CONCLUDING REMARKS

In this investigation we show that an ARMA-model-based technique provides super resolution in estimating the locations of targets as compared to an FFT-based approach. By knowing the locations of these targets, we can collect more data to get enough samples inbetween the peaks of the inverse transformed domain so that the entended Prony's method can be used to estimate the reflection coefficient of the target. We find that the extended Prony's method estimates the values of the reflection coefficient with an improved accuracy as compared to the results obtained by the

FFT of the windowed data. Thus, a combination of the ARMA-model-based approach and the extended Prony's method might be used effectively in estimating the location of the targets and their reflection coefficients. We are in the process of applying the model-based techniques to the measured data for characterizing the ice type.

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Table I : Comparisons of the Estimated Values Between the FFT-based and the Model-based Approaches

SNR in dB	Estimated Target Location (Actual Location = 4.0 m)		R.M.S. Error in Estimating Reflection Coefficient (using 5 samples)	
	IFFT-based (m)	ARMA-model-based (m)	FFT-based	EFDPM-based
∞	4.0031	3.9998	0.1005	0.0052
40	4.0031	3.9998	0.1130	0.0098
30	4.0031	4.0001	0.1176	0.0376
20	4.0031	4.0006	0.1176	0.0847
10	4.0031	3.9992	0.1268	0.0991
0	4.0031	3.9989	0.1308	0.1122

Table II: Error in Estimating Target Locations Using ARMA model (Targets are located at 4.0 m and 4.05 m)

No. of Samples used (N)	Resolution From IFFT (m)	Distances Estimated by ARMA model (m)		Errors in Distance Estimation (%)	
		First Target	Second Target	First Target	Second Target
1600	0.0094	3.9997	4.0503	0.01	0.01
800	0.0187	3.9984	4.0514	0.04	0.03
400	0.0375	3.9950	4.0553	0.13	0.13
200	0.0750	3.9868	4.0675	0.33	0.43
100	0.1500	3.9856	4.1757	0.36	3.10

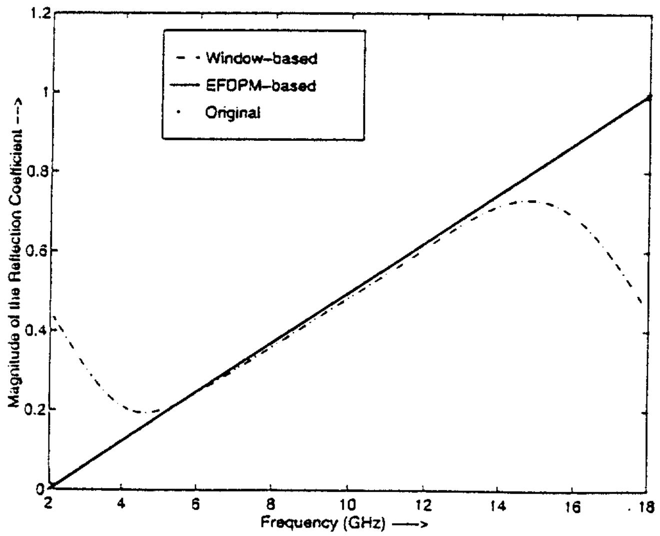

Fig.1. Original and estimated reflection coefficient of a target.

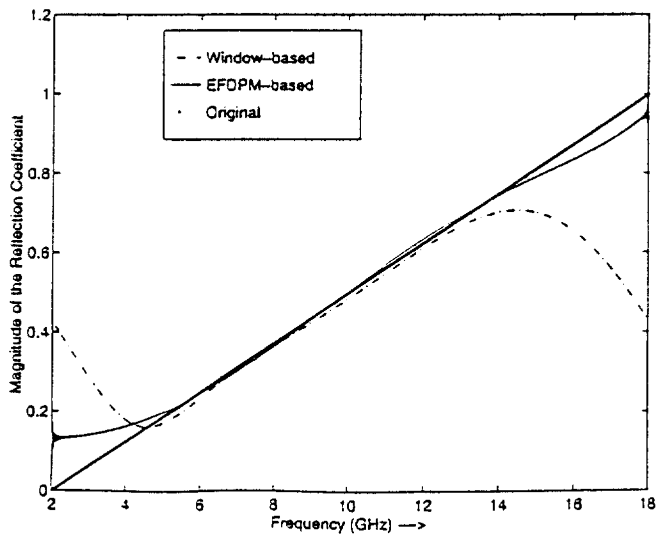

Fig.2. Original and estimated reflection coefficient of the first target.