Synthetic Aperture Characterization of Radar Satellite Constellations

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Abstract- The concept of radar satellite constellations, or clusters, for SAR and other radar modes has been proposed and is currently under research. These systems are composed of multiple, formation-flying satellites with each satellite having its own, coherent receiver. Increased swathwidth compared to that of traditional SAR is attainable by processing the spatial data obtained from multiple satellites. The multi-channel system can also be scanned both forward and backward.

The size and orientation of a such a system's resolution cell can change dramatically, however, depending on the number of satellites in the constellation, the size of the constellation, the look geometry, and the subset of data that is coherently processed. In addition, any of these parameters can be varied on demand according to mission requirements. The constellation itself forms an array that is sparsely populated and irregularly spaced. Furthermore, if the constellation is of extremely wide extent, then the width of its array pattern determines resolution rather than system bandwidth and coherent integration length. The problem of predicting system resolution is further exacerbated by forward- and backward-looking scenarios.

In order to aid in the design, analysis, and signal processing of radar satellite constellations, we present a method of characterizing the resolution of such systems. We derive two *eigensensors* that can be interpreted as the dimensions of a two-dimensional synthetic aperture. Then, the synthetic aperture expression is used to derive resolution; simulations are presented to verify the theory.

I. Introduction

There is currently a strong push toward improving spaceborne radar technology. This is due to several advantages provided by spaceborne radar, the most important of which is the ability to achieve global coverage. However, spaceborne implementations are also problematic, especially concerning the size of the required antennas. The minimum size of a SAR antenna is restricted by the minimum SAR antenna area constraint, and low radial velocities observed by spaceborne platforms require high angular resolution in order to distinguish between moving and stationary targets. Therefore, spaceborne applications require large antennas that can be difficult to deploy and expensive to launch.

One proposed concept for space-based radar that decouples the antenna size requirements from the need to launch and deploy a single, monolithic antenna array is to place multiple transmitters and receivers into space, each on their own, small satellite [1]. These satellites, called *microsats*, would fly in a formation called a satellite *constellation*. Each satellite in the constellation would be able to coherently sample the signal transmitted from each of the transmitters in the constellation. In this way, the constellation would work as a single, virtual radar able to operate in multiple modes including interferometric, SAR, and MTI.

The constellation concept provides the ability to reconfigure the radar system according to mission requirements. In addition to possibly varying sensor parameters such as bandwidth and integration time, the constellation concept allows for reconfiguration of the antenna array by dynamically including different numbers of satellites in the constellation. The size of the array may also be able to be changed by adjusting the orbits of the microsats.

The disadvantages of the constellation concept stem from the fact that the resulting antenna array will be sparsely populated, non-uniform, and three-dimensional. It has been demonstrated that increased SAR swathwidth can be achieved by processing the sparse array properly [2], and research into MTI processing for the constellation concept is ongoing. In developing SAR and MTI algorithms for this type of system, however, it is important that we be able to assess typical radar performance parameters such as resolution and the radar ambiguity function.

This paper presents a method for analyzing radar resolution for sparse arrays. In addition, the technique can be used for both sidelooking and non-sidelooking scenarios. In Section II, we describe the radar model used in this paper. In Section III, we derive a 2D synthetic aperture that is a generalization of the 1D synthetic aperture interpretation of SAR. In Section IV, we use the synthetic aperture to express system resolution. Then, we compare the theoretical result with numerically generated radar ambiguity functions in Section V. We make our conclusions in Section VI.

II. Radar Model

Let the vector, $\mathbf{x}$, represent the location of a scatterer on the Earth's surface. The complex signal received by the radar system depends on the radiation pattern of the transmitter, $g(\mathbf{x})$, the scatterer's range, $R(\mathbf{x})$, the two-way propagation delay, τ, the scatterer's reflection coefficient, $\gamma(\mathbf{x})$, and the sensor's five measurement parameters: time, frequency, and 3D spatial location. Letting the sensor's five measurement parameters be represented by the vector, $\mathbf{s}$, the received signal, d, due to a single scatterer is then

$$d(\mathbf{x},\mathbf{s}) = \frac{\gamma(\mathbf{x})\, g(\mathbf{x})}{R(\mathbf{x})^2} w(\mathbf{s}) \exp(-j\omega\tau) \tag{1}$$

where $w(\mathbf{s})$ is a sensor weighting function that describes parameters such as the sensor's transmit signal spectrum and receiver frequency response. Then, noting that the propagation delay actually varies with time, sensor location, and scatterer location, we let the received phase be expressed as $\Psi(\mathbf{x},\mathbf{s}) = \omega\tau$, and (1) becomes

$$d(\mathbf{x},\mathbf{s}) = \frac{\gamma(\mathbf{x})g(\mathbf{x})}{R(\mathbf{x})^2} w(\mathbf{s}) \exp\left[-j\Psi(\mathbf{x},\mathbf{s})\right]. \tag{2}$$

III. Space-Time-Frequency Synthetic Aperture

Since the scatterers themselves vary only in position on the Earth's 2D surface, it is reasonable to assume that only two sensor dimensions are needed for representing the radar data. Therefore, we hypothesize that although the phase expression of (2) varies versus five sensor parameters, those sensor parameters can be projected into the coordinates of two independent *eigensensors*. The projection of a sensor's time, frequency, and spatial parameters onto these two eigensensors forms a 2D synthetic sensor that can be used to characterize SAR and MTI performance.

We derive the 2D synthetic sensor by performing two first-order Taylor expansions of the received phase. First, the phase is expanded around the radar sensor parameters, $\mathbf{s}$. The result is

$$\Psi(\mathbf{x},\mathbf{s}) \approx \Psi(\mathbf{x},\bar{\mathbf{s}}) + \left(\nabla_s \Psi\big|_{\mathbf{x},\bar{\mathbf{s}}}\right)^{\dagger} \Delta\mathbf{s} \tag{3}$$

where $\bar{\mathbf{s}}$ is the set of sensor parameters around which the expansion is performed, $\Delta\mathbf{s}$ is the deviation in sensor parameters from the expansion point, ∇_s is the 5D gradient operator that takes first-order derivatives of the received phase with respect to each of the sensor's five parameters, and $(\cdot)^{\dagger}$ denotes the matrix transpose operation. We ignore the first term on the right side of (3) since it does not vary with sensor parameters. Then, we expand the last term in (3) around the two dimensions of the scatterer location. Using similar notation for the scatterer location expansion point, $\bar{\mathbf{x}}$, the deviation from the expansion point, $\Delta\mathbf{x}$, and the 2D gradient operator, ∇_x, the received phase becomes

$$\begin{aligned}\Psi(\mathbf{x},\mathbf{s}) &\approx \left[\left(\nabla_s \Psi\big|_{\bar{\mathbf{x}},\bar{\mathbf{s}}}\right)^{\dagger} + \Delta\mathbf{x}^{\dagger}\left(\nabla_x \nabla_s^{\dagger} \Psi\big|_{\bar{\mathbf{x}},\bar{\mathbf{s}}}\right)\right]\Delta\mathbf{s} \\ &= \left[\mathbf{k}_0^{\dagger} + \Delta\mathbf{x}^{\dagger}\boldsymbol{\Lambda}_s\right]\Delta\mathbf{s}\end{aligned} \tag{4}$$

where $\boldsymbol{\Lambda}_s$ is coined the *sensor transformation matrix*.

This matrix has an interesting interpretation that can be seen by taking its singular value decomposition (SVD). There are two non-zero singular values, σ_1 and σ_2, in the decomposition; therefore, there are two eigenvectors for the columns of $\boldsymbol{\Lambda}_s$, $\mathbf{u}_1$ and $\mathbf{u}_2$, and two eigenvectors for the rows of $\boldsymbol{\Lambda}_s$, $\mathbf{v}_1$ and $\mathbf{v}_2$. Writing (4) in terms of the SVD, we have

$$\begin{aligned}\Psi(\mathbf{x},\mathbf{s}) &\approx \mathbf{k}_0^{\dagger}\Delta\mathbf{s} + \sigma_1\left(\Delta\mathbf{x}^{\dagger}\mathbf{u}_1\right)\left(\mathbf{v}_1^{\dagger}\Delta\mathbf{s}\right) + \sigma_2\left(\Delta\mathbf{x}^{\dagger}\mathbf{u}_2\right)\left(\mathbf{v}_2^{\dagger}\Delta\mathbf{s}\right) \\ &= \mathbf{k}_0^{\dagger}\Delta\mathbf{s} + k_\alpha\alpha + k_\beta\beta\end{aligned} \tag{5}$$

where $k_\alpha = \sigma_1\Delta\mathbf{x}^{\dagger}\mathbf{u}_1$, $k_\beta = \sigma_2\Delta\mathbf{x}^{\dagger}\mathbf{u}_2$, $\alpha = \mathbf{v}_1^{\dagger}\Delta\mathbf{s}$, and $\beta = \mathbf{v}_2^{\dagger}\Delta\mathbf{s}$. We can now see that $\boldsymbol{\Lambda}_s$ transforms the 5D sensor system into a synthetic 2D system. The axes of the 2D synthetic sensor are $\mathbf{v}_1$ and $\mathbf{v}_2$, and sensor coordinates in the 2D system are given by $\alpha = \mathbf{v}_1^{\dagger}\Delta\mathbf{s}$ and $\beta = \mathbf{v}_2^{\dagger}\Delta\mathbf{s}$. Likewise, the frequencies measured by the two axes of the synthetic sensor are obtained through $\mathbf{u}_1$ and $\mathbf{u}_2$. These frequencies are $k_\alpha = \sigma_1\Delta\mathbf{x}^{\dagger}\mathbf{u}_1$ and $k_\beta = \sigma_2\Delta\mathbf{x}^{\dagger}\mathbf{u}_2$. The two synthetic sensor coordinates retain all the information collected by the five dimensions of the actual sensor. In addition, orthogonality of the eigenvectors means that the two synthetic sensor dimensions are also orthogonal.

Last, we perform a similar transformation for the transmitting antenna. Expressing similar derivatives for the two scatterer locations and the three transmit antenna dimensions, the transmit radiation pattern is

$$g(\mathbf{x}) = \int_{S_A} w_l(\mathbf{l}) \exp\left(-j\Delta\mathbf{x}^{\dagger}\boldsymbol{\Lambda}_l\Delta\mathbf{l}\right) d\mathbf{l} \tag{6}$$

where $w_l(\mathbf{l})$ is the antenna amplitude taper, S_A is the surface of the antenna's conductor or aperture, the vector, $\mathbf{l}$, describes a point on the antenna surface, and $\boldsymbol{\Lambda}_l$ is called the *antenna transformation matrix*.

IV. Sensor Resolution

Radar resolution is traditionally considered in terms of the correlation between two adjacent targets. This correlation can be expressed in terms of a matched-filter response; therefore, we begin by determining the output of a matched filter in the presence of two targets. The filter is matched to the first signal, which is due to a target at $\mathbf{x} = \bar{\mathbf{x}}$. We assume a Gaussian amplitude taper on the transmit aperture with a width and orientation described by the matrix of second moments, $\mathbf{J}_l$. Therefore, the received signal due to a scatterer at $\bar{\mathbf{x}}$ with a scattering coefficient of γ_1 is

$$d(\bar{\mathbf{x}},\Delta\mathbf{s}) = \frac{\gamma_1}{R(\bar{\mathbf{x}})^2} w(\Delta\mathbf{s}) \exp\left[-j\mathbf{k}_0^{\dagger}\Delta\mathbf{s}\right] g_0, \tag{7}$$

and the received signal due to a scatterer at $\bar{\mathbf{x}} + \Delta\mathbf{x}$ with a scattering coefficient of γ_2 is

$$\begin{aligned}d(\bar{\mathbf{x}}+\Delta\mathbf{x},\Delta\mathbf{s}) &= \frac{\gamma_2}{R(\bar{\mathbf{x}}+\Delta\mathbf{x})^2} w(\Delta\mathbf{s}) \exp\left[-j\mathbf{k}_0^{\dagger}\Delta\mathbf{s}\right] \\ &\times \exp\left[-j\Delta\mathbf{x}^{\dagger}\boldsymbol{\Lambda}_s\Delta\mathbf{s}\right] g_0 \exp\left[-\frac{1}{2}\Delta\mathbf{x}^{\dagger}\boldsymbol{\Lambda}_l\mathbf{J}_l\boldsymbol{\Lambda}_l^{\dagger}\Delta\mathbf{x}\right].\end{aligned} \tag{8}$$

We let the sensor taper, $w(\Delta\mathbf{s})$, be 5D jointly Gaussian with widths and orientation described by the matrix of second moments, $\mathbf{J}_s$. Then, the output, ξ, of the matched filter is

$$\xi = \gamma_1 + \gamma_2 \exp\left[-\frac{1}{2}\Delta\mathbf{x}^\dagger \left(\mathbf{\Lambda}_s \mathbf{J}_s \mathbf{\Lambda}_s^\dagger + \mathbf{\Lambda}_l \mathbf{J}_l \mathbf{\Lambda}_l^\dagger\right)\Delta\mathbf{x}\right]. \tag{9}$$

The first term on the right side of (9) is the desired reflection coefficient of the first scatterer. The second term represents error due to leakage of the second target into the filter output. It depends on the exponential term, which we now recognize as the correlation between the two scatterers. If we say that two targets are resolved when their correlations drop to a specified level, κ_c, then the equation that defines resolution is

$$-2\ln\kappa_c = \Delta\mathbf{x}^\dagger \left(\mathbf{\Lambda}_s \mathbf{J}_s \mathbf{\Lambda}_s^\dagger + \mathbf{\Lambda}_l \mathbf{J}_l \mathbf{\Lambda}_l^\dagger\right)\Delta\mathbf{x}, \tag{10}$$

which is the equation for an ellipse.

V. Simulations

Using (10), we have predicted the resolution ellipses for two cases and compared them to ambiguity functions obtained numerically. In the first case, Gaussian tapers were used for the time and frequency sensor dimensions. A uniform taper, however, was applied across the physical array, with the spatial-dependent elements of $\mathbf{J}_s$ calculated according to the sampled second moments of the uniform, sparse array. Fig. 1 compares the theoretically predicted resolution ellipse and the numerically calculated ambiguity function. The amount of correlation chosen for the definition of resolution was $\kappa_c = 0.707$. In Fig. 1, the scenario is forward looking with a very large physical array of microsats. We see that the ellipse overlaid on the ambiguity function accurately predicts the size and orientation of the ambiguity function's mainlobe. The axes of the resolution ellipse do not align with the along- and cross-track directions because of the forward-looking scenario and the random orientation of the large, sparse array. Also, since the physical array is the dominant factor in determining the ambiguity function, and it is sparsely sampled, we see that sidelobes in Fig. 1 are very high.

Finally, we provide an example where each sensor parameter is uniformly tapered. The sampled second moments are used for each element of $\mathbf{J}_s$, and the result is seen in Fig. 2. We see that the Gaussian assumption used to derive (10) is not as restrictive as it may appear.

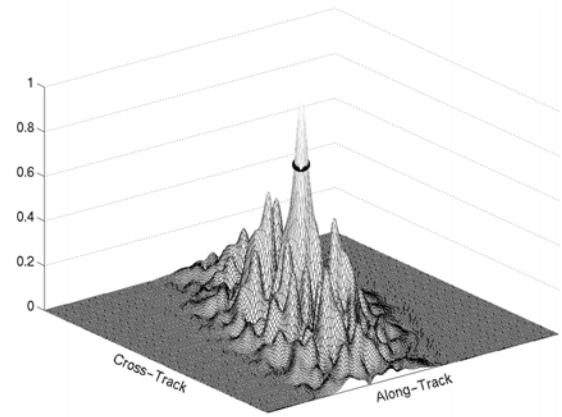

Figure 1. Theoretical vs. numerical resolution.

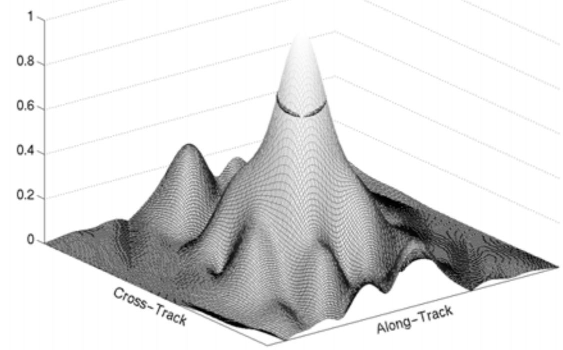

Figure 2. Theoretical vs. numerical resolution for uniform tapers.

VI. Conclusions

With the desire to implement radar on spaceborne platforms comes the desire to have one radar system perform in multiple radar modes. In addition, the performance parameters of those modes may need to be reconfigurable. As a result and considering the possibility of constellation-type implementations of spaceborne radar, more robust methods of characterizing radar performance parameters are needed. We have derived a method of characterizing radar sensor resolution for a wide range of radar configurations including look scenarios and sensor amplitude tapers. The characterization is based on projection of the sensor's five measurement parameters into an equivalent 2D synthetic aperture. The sensor transormation matrix also transforms the width of sensor parameters such as bandwidth, integration time, and array size into the width of the 2D synthetic aperture. Once this 2D width is determined, the size of the resolution ellipse on the ground can be readily determined. The effectiveness of our method was demonstrated with several simulations.

References

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[2] J.M. Stiles and N. Goodman, "Processing of multi-aperture SAR to produce fine-resolution images of arbitrarily large extent," *Proc. 2001 IEEE Radar Conf.*, Atlanta, GA, pp. 451-456, May, 2001.