Laplacian pdf of DGD time derivative and application to predicting PMD-induced outage rates

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It is reported for the first time that the time derivative of differential-group delay (DGD) on buried fibres has a Laplacian pdf. Using this, the previously reported expression for predicting the polarisation-mode dispersion (PMD)-induced outage rate is simplified and it is shown that it is a function of the mean DGD and the fibre's Laplacian parameter only.

Introduction: Polarisation-mode dispersion (PMD) is a major impediment for network operators seeking to increase the per channel data rate to beyond 10 Gbit/s on long-haul fibre-optic links. To ensure the reliability of their fibre-optic network at higher bit rates, network engineers must be able to predict PMD-induced outage rates.

A PMD-induced outage is one which the instantaneous differential-group delay (DGD or $\Delta\tau$) exceeds a given threshold value, $\Delta\tau_{th}$, while the outage probability P_{out}, expressed in minutes/year, can be calculated using

$$P_{\text{out}} = P(\Delta\tau \geq \Delta\tau_{th}) = 1 - \int_0^{\Delta\tau_{th}} f_\tau(\Delta\tau)\, d\Delta\tau \qquad (1)$$

where $f_\tau(\cdot)$ is the Maxwellian probability distribution function (pdf) of DGD, P_{out} represents only the annualised outage probability and reveals nothing regarding outage rate or duration. Accurate estimation of the impact of PMD on network availability requires statistical analysis of DGD temporal variability. Caponi *et al.* [1] showed how the mean time between PMD-related outages for a given link could be estimated from its DGD temporal variations and the Maxwellian probability density function. They showed that the mean outage rate, R_{out} (defined as the mean number of outage events per unit time with units of events/year), is found using [1]

$$R_{\text{out}} = \frac{1}{2} f_\tau(\Delta\tau_{th}) \int_{-\infty}^{\infty} f_{\tau'}(\Delta\tau') |\Delta\tau'| d\Delta\tau' \qquad (2)$$

where $\Delta\tau'$ is the time derivative of DGD, and $f_{\tau'}(\cdot)$ is the pdf of $\Delta\tau'$. While P_{out} is the same for all random variables with a Maxwellian pdf, it has been reported that R_{out} is not the same since differences in cable and installation affect the DGD temporal characteristics [1, 2]. In this Letter, we first show that $\Delta\tau'$ has a Laplacian pdf by curve-fitting the histogram obtained from measured data. We then simplify the R_{out} expression by analytically reducing the integral in (2) using the Laplacian pdf of $\Delta\tau'$. Finally, remarkably good agreement is shown between the values of the PMD-induced outage rate obtained using the original and simplified expressions.

pdf of time derivative of DGD: Measurements were made of the instantaneous DGD on a 190 km direct buried, standard singlemode fibre-optic cable made available by Sprint. A polarisation analyser employing the Jones-Matrix-Eigenanalysis (JME) method provided instantaneous DGD data for wavelengths from 1535 to 1565 nm with a spectral resolution of 0.1 nm. These measurements were repeated approximately every 23 minutes and for about 18 days providing 339 000 measured DGD values. $\Delta\tau'$ data were obtained by numerically differentiating the measured DGD data.

A histogram of the $\Delta\tau'$ data is shown in Fig. 1. Through curve-fitting we found that this histogram closely resembles a Laplacian pdf (a two-sided, first-order exponential) of the form

$$f_{\tau'}(\Delta\tau') = \frac{\alpha}{2} e^{-\alpha|\Delta\tau'|} \qquad (3)$$

where $\alpha = \sqrt{2}/\sigma$ and is the Laplacian parameter with units of hours/picosecond and σ is the standard deviation of $\Delta\tau'$. This is the first time that the Laplacian nature of $\Delta\tau'$ is being reported. For comparison, a curve representing a Laplacian pdf with $\alpha = 0.6$ h/ps (obtained using the variance of $\Delta\tau'$ data) is also shown in Fig. 1.

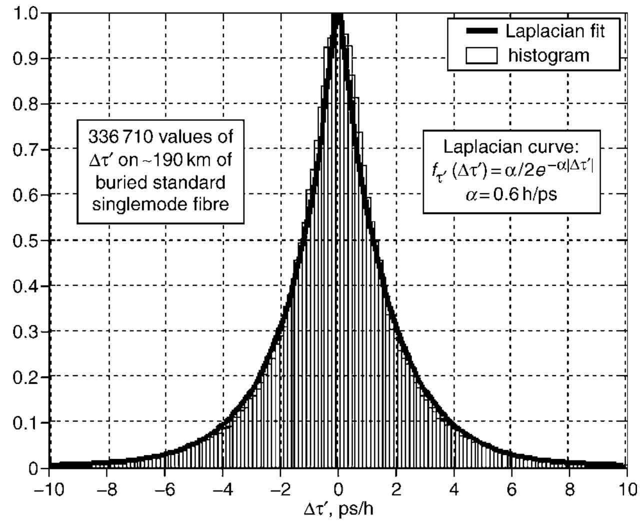

Fig. 1 *Histogram of $\Delta\tau'$ data and its Laplacian fit*

Similar agreement was seen between similar Laplacian fits and $\Delta\tau'$ histograms measured on other 95 and 190 km fibre spans. However, we have observed that α value decreases as the fibre length increases.

Closed-form expression for R_{out}: Using the Laplacian distribution as the $\Delta\tau'$ pdf, a closed form solution for the integral in (2), and hence R_{out}, can be obtained. Substituting (3) for the pdf of $\Delta\tau'$, the integral in (2) evaluates to $1/\alpha$. Then the expression for R_{out} in (2) reduces to

$$R_{\text{out}} = \frac{1}{2\alpha} f_\tau(\Delta\tau_{\text{th}}) \qquad (4)$$

The significance of (4) is that the mean outage rate due to PMD on any fibre route can be readily estimated given its mean DGD and Laplacian parameter α, greatly simplifying the route's PMD-induced outage analysis.

Whereas the fibre's mean DGD may be known from its PMD coefficient (ps/$\sqrt{\text{km}}$), the Laplacian parameter α must be estimated from a time series of DGD measurements made on each fibre. We have observed that estimation of α is observation time-independent. While the uncertainty in the α estimate decreased as the observation time increased, the estimated value for α was consistently close to 0.6 h/ps regardless of whether the observation time was one day or 18 days.

A comparison of R_{out} values for different normalised thresholds (threshold/mean DGD) obtained using numerically determined $\Delta\tau'$ pdf in (2) and the analytical expression of (4) is shown in Fig. 2. Excellent agreement between the values of the two cases is evident.

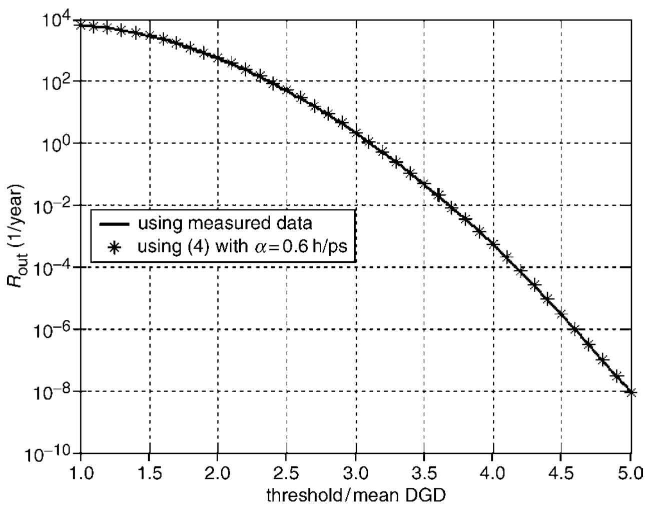

Fig. 2 *R_{out} values for different normalised thresholds using (a) numerically determined $\Delta\tau'$ pdf in (2) and (b) using (4)*

Conclusions: In this Letter, we have; (i) shown for the first time that the time derivative of DGD has a Laplacian pdf, the characteristics of which are determined exclusively by the Laplacian parameter α that has units of h/ps; (ii) simplified the expression for the mean PMD-induced outage rate reported earlier by Caponi *et al.* [1] using the above-mentioned finding, resulting in an expression for mean outage rate that depends only on the fibre's mean DGD and its Laplacian parameter α that represents its temporal characteristics; (iii) shown excellent agreement between the outage rates obtained using the original expression and the simplified closed-form expression; (iv) noted that α values decrease as the fibre length increases; (v) reported that estimation of the Laplacian parameter α from measurements is largely observation time-independent. These findings simplify the PMD-induced system outage analysis and will help network operators predict system downtime.

Acknowledgments: This work was funded by Sprint Corporation Company, L. P. and NSF grant ECS-0116213.

19 February 2004
Electronics Letters online no: 20040336
doi: 10.1049/el:20040336

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