# Study of variation of the Laplacian parameter of DGD time derivative with fiber length using measured DGD data

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# Abstract

Temporal and spectral DGD measurements were made on different combinations of three 95-km fibers within a slotted-core, direct-buried, standard single-mode fiber-optic cable over long time periods to characterize DGD variability. From this data we observed that DGD varies slowly over time but rapidly over wavelength. We also observed that the DGD variation with time becomes more rapid for longer length fibers. The DGD time derivative data obtained from the measured DGD data showed good agreement with a Laplacian distribution. We observed that the Laplacian parameter, which dictates the Laplacian pdf of DGD time derivative, seems to converge to a lower value as the fiber length is increased. This observation, which needs further verification, if true, would greatly simplify the system outage analysis due to first-order PMD on long-haul optical fiber links.

### Introduction

In spite of the recent telecom bubble, statistics show that the net traffic growth (combined Internet, data and voice traffic) remains at the same level as it was three years ago and network capacity is being exhausted at the same rate as it was during the prebubble time [2]. To cope up with the demand for network capacity, major carriers are looking at increasing the transmission speeds. Polarization-mode dispersion (PMD) may be a major impediment for network operators seeking to increase the per channel data rate on long-haul fiber-optic links. While there are PMD challenges facing carriers at 10 Gb/s, these challenges are not as severe as originally feared. A marked improvement in the PMD tolerance of 10 Gb/s long-reach receivers will likely satisfy most length demands, obviating the need for PMD mitigation. However, transmission speeds of 40 Gb/s and beyond will most likely require some form of PMD mitigation in long-haul applications. To ensure signal quality on their fiber at such higher rates, network engineers must anticipate the impact of PMD on various fiber routes. A solid understanding of PMD-induced system outages will help engineers and researchers to develop new and cost-efficient mitigation alternatives to maintain high network reliability.

The availability of PMD data measured over long time periods on installed, buried fibers is limited. In this paper we present differential group delay (DGD) data measured over long time periods and simplified first-order PMD outage analysis based on measured data for buried, standard single-mode fiber.

# Measurement setup

Measurements of instantaneous DGD were made on different combinations of three 95-km fibers (1, 2, and 3) within a slotted-core, direct-buried, standard single-mode fiber-optic cable made available by Sprint. The combinations of fibers used are three individual fibers (1, 2, 3, each ~95 km), three 2-fiber concatenations (1-2, 2-3, 1-3, each ~190 km) and one 3fiber concatenation (1-2-3, ~285 km). A polarization analyzer employing the Jones-Matrix-Eigenanalysis (JME) method was used for measurements. The specifications of measurements on the three individual fibers and some results from the analysis of measured data were reported in our previous paper [3]. For measurements on 2-fiber concatenations two EDFAs were used along the link; one post-amplifier and one inline amplifier in between the two fibers. For measurements on 3-fiber concatenation five EDFAs were used along the link; one post-amplifier and four in-line amplifiers after every ~47.5 km of fiber except for the last 95 km. For all the concatenated fiber combinations measurements were made at wavelengths from 1535 nm to 1565 nm with a spectral resolution of 0.1 nm (about 12.5 GHz) and were repeated every 23 minutes.

Measurements were carried on for 18 days on 1-2 concatenation (Aug. 22, 2002 - Sept. 9, 2002), for 21 days on 2-3 concatenation (Aug. 1, 2002 - Aug. 22, 2002), for 16 days on 1-3 concatenation (Sept. 27, 2002 - Oct. 13, 2002) and for 34 days on 1-2-3 concatenation (April 13, 2004 - May 17, 2004). Over the 34 days 2,127 measurements were made on 1-2-3 concatenation across 300 discrete wavelengths representing 638,100 measured DGD values. The corresponding number of measured DGD values for 2fiber concatenations are 339,000 for 1-2, 394,200 for 2-3, and 306,000 for 1-3.

# Plots of DGD vs. wavelength and time

Figures 1, 2, 3, and 4 show in a color-coded format normalized DGD data (i.e., DGD/mean DGD) measured on the three 2-fiber concatenations and the 3fiber concatenation, respectively. From the plots it is evident that the DGD varies significantly with wavelength and relatively high-DGD events are spectrally localized. A comparison of the abovementioned plots with the corresponding plots for individual fibers reported in [3] shows that the variation of DGD with time is more rapid on concatenated fibers than on individual fibers.



Figure 1. Measured, normalized DGD vs. wavelength and time for 1-2 fiber concatenation (18 days of data).



Figure 2. Measured, normalized DGD vs. wavelength and time for 2-3 fiber concatenation (21 days of data).



Figure 3. Measured, normalized DGD vs. wavelength and time for 1-3 fiber concatenation (16 days of data).



Figure 4. Measured, normalized DGD vs. wavelength and time for 1-2-3 fiber concatenation (34 days of data).



Figure 5. Histogram of measured, normalized DGD data for 1-2-3 fiber concatenation (34 days of data).

A histogram of the measured, normalized DGD data on 1-2-3 fiber concatenation, shown in figure 5, is seen to have shape consistent with a Maxwellian distribution as expected. A curve representing a Maxwellian distribution for a 1-ps mean DGD is also plotted for comparison. Also, we observed that the histograms of measured DGD data on the other combinations of the three fibers were in good agreement with the Maxwellian pdf (plots not shown here).

### The pdf of DGD time derivative

Caponi et al. [1] defined a first-order PMD outage as an event in which a given threshold value of DGD ( $\Delta \tau_{th}$ ) is exceeded. They showed that the mean outage rate,  $R_{out}$  (defined as the mean number of outage events per unit time with units of events/year), is found using [1]

$$\mathbf{R}_{\text{out}} = \frac{1}{2} \mathbf{f}_{\tau} \left( \Delta \tau_{\text{th}} \right) \int_{-\infty}^{\infty} \mathbf{f}_{\tau'} \left( \Delta \tau' \right) \left| \Delta \tau' \right| d\Delta \tau' \qquad (1)$$

where  $\Delta \tau'$  is the time derivative of DGD,  $f_{\tau}(\cdot)$  is the pdf of  $\Delta \tau'$  and  $f_{\tau}(\cdot)$  is the Maxwellian pdf of DGD. In [1] and [3], where predictions of  $R_{out}$  based on measurements on different buried links were reported, the integral in (1) was evaluated numerically from measured data. However, we observed [4] that  $\Delta \tau'$  has a Laplacian pdf of the form

$$f_{\tau'}(\Delta \tau') = \frac{\alpha}{2} e^{-\alpha \left| \Delta \tau' \right|}$$
(2)

where  $\alpha = \frac{\sqrt{2}}{\sigma}$  and is the Laplacian parameter with units of hours/picosecond and  $\sigma$  is the standard deviation of  $\Delta \tau'$ . Using the Laplacian pdf of  $\Delta \tau'$ , we reported [4] the following closed-form expression for  $R_{out}$ , which depends only on mean DGD and  $\alpha$ .

$$R_{out} = \frac{1}{2\alpha} f_{\tau} (\Delta \tau_{th})$$
(3)

To verify the Laplacian nature of  $\Delta \tau'$ , histograms of  $\Delta \tau'$ were obtained from measured DGD data on all fiber combinations mentioned earlier. Figures 6, 7, and 8 show the  $\Delta \tau'$  histograms obtained using measured DGD data on fiber 1, 2-3 fiber concatenation and 1-2-3 fiber concatenation respectively. For comparison, curves representing Laplacian distribution with different values for  $\alpha$ , are also shown in the figures. It can observed from the figures that the  $\Delta \tau'$  histograms are in good agreement with the Laplacian distribution. Similar histograms were obtained for the data on the other fiber combinations (plots not shown here) and they also showed good agreement with the Laplacian distribution.



Figure 6. Histogram of measured ∆τ' data from fiber 1 and its Laplacian fit.



Figure 7. Histogram of measured Δτ' data from 2-3 fiber concatenation and its Laplacian fit.



Figure 8. Histogram of measured ∆τ' data from 1-2-3 fiber concatenation and its Laplacian fit.



Figure 9. Comparison of R<sub>out</sub> values on 1-2-3 fiber concatenation calculated using (1) and (3).

A comparison of  $R_{out}$  values calculated using (1) and (3) for 1-2-3 concatenation is shown figure 9. Excellent agreement between both sets of  $R_{out}$  values is evident from the figure. Similar agreement between the values calculated using (1) and those calculated using (3) was observed for other fiber combinations also (plots not shown here).

# Variation of the Laplacian parameter $(\alpha)$ with fiber length

The Laplacian parameter  $(\alpha)$  values for all of the fiber combinations mentioned earlier are shown in table 1. From the values in table 1, it can be observed that the Laplacian parameter decreased by an order of magnitude for concatenated fibers (longer lengths) compared to individual fibers (shorter lengths). But the decrease in the Laplacian parameter from the 2-fiber concatenations to the 3-fiber concatenation is less significant. This appears to indicate that the Laplacian parameter is converging to a value as the fiber length is increased. However, the validity of this observation must be further verified, possibly using modeling. If such a behavior is in fact true, then this would be extremely useful for network engineers in predicting the first-order PMD outage rates on long-haul optical fiber links.

# Conclusions

We have measured DGD over long time periods on different combinations of three 95-km fibers within a slotted-core, direct-buried, standard singlemode fiber-optic cable. From these measurements we observed that DGD varies slowly over time but rapidly over wavelength or frequency. However, we also observed that the temporal variation becomes more rapid for longer length fibers.

Using the measured data we were able to verify the Laplacian nature of DGD time derivative and the closed-form expression for predicting first-order PMD outage rates that we reported in [4]. We observed that the Laplacian parameter, which dictates the Laplacian pdf of DGD time derivative, seems to converge as the fiber length is increased. This observation, which needs further verification, if true, would greatly simplify the system outage analysis due to first-order PMD on long-haul optical fiber links.

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	Individual fibers (~95 km)			2-fiber concatenations (~190 km)			3-fiber concatenation (~285 km)
Fiber combination	1	2	3	1-2	2-3	1-3	1-2-3
Laplacian parameter α (hr/ps)	7.5	4.25	10.9	0.6	0.95	0.7	0.38

### Table 1. Laplacian parameter $(\alpha)$ values for different fiber combinations studied