A Novel Method for Measuring Polarization-Mode Dispersion Using Four-Wave Mixing

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***Abstract*— A method for measuring polarization-mode dispersion (PMD) on fiber links using four-wave mixing (FWM) generation is presented. This method uses a probe signal to analyze the signal polarization state via FWM generation. The FWM power transfer function is derived in terms of the Stokes parameters, and is validated using both simulated and experimental results. Based on this transfer function, PMD measurements are presented that agree well with the actual PMD values. Compared to the traditional frequency-domain methods, this new method does not require a motionless condition for the measurement apparatus.**

***Index Terms*— Four-wave mixing (FWM), nonlinear effects, polarization-mode dispersion (PMD) measurement.**

I. Introduction

POLARIZATION-MODE dispersion (PMD) is one of the major limiting factors of ultrahigh-bit-rate optical fiber communication systems. Currently, PMD is a big concern when upgrading legacy networks of installed fiber to 10 Gb/s (OC-192) rates and higher. At OC-192 bit rates, the maximum acceptable amount of PMD is about 10 ps [1].

Existing PMD measurement techniques fall mainly in two categories [2]. One involves time-domain measurements, and includes the interferometric and optical pulse methods. The other involves frequency-domain measurements, based on the evolution of states of polarization (SOP) as a function of frequency or wavelength. Included in this category are the fixed-analyzer method, the Jones-matrix method, and the Poincare-sphere method. A major drawback of the time-domain methods is that their results are degraded by polarization state fluctuations, caused by polarization mode coupling in the fiber [2]. On the other hand, a limitation of the frequency-domain methods is that any motion of the measurement apparatus, especially at the ends the fibers, can totally destroy the measured results [2], [3]. Maintaining a motionless condition is often difficult, especially with field measurements.

This paper presents a novel method for determining PMD in a fiber link by measuring four-wave mixing (FWM) products. In this method, a constant wavelength "probe" wave and a variable wavelength "signal" wave are launched into the test fiber of unknown PMD. The output signals are then input to a low-dispersion, low PMD "measurement fiber" where FWM products are generated that depend on the PMD of the test fiber. Average PMD can then be determined by measuring the power of the FWM products as the difference frequency of the two signals is varied, since the magnitude of FWM products depends upon the polarization states of the two waves. This technique is insensitive to mechanical vibrations and instabilities in the test equipment, since the measured PMD depends only on the relative SOP change between the probe and signal waves in the measurement fiber, not the position coordinates of fiber or the equipment.

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II. The Theory of the Nonlinear Method

A. Dependence of the FWM Transfer Function on the State of Polarization (SOP)

FWM is a nonlinear process induced by the Kerr effect in optical fibers. If three signals at frequencies f_i, f_j, and f_k copropagate through a single-mode fiber, the newly generated frequency through FWM will be $f_l = f_i + f_j - f_k$. For the partially degenerate case, $f_i = f_j$, the newly generated frequency is $f_l = 2f_i - f_k$. The generated FWM power depends not only on the signal frequency separation, the input signal powers, the fiber loss, and nonlinear characteristics, but also on the signal polarization states. The dependence of the FWM power on these parameters for the partially degenerate case can be expressed as [4]–[6]

$$P_{\mathrm{FWM}}(L) = F_1(\gamma, \alpha, P_1, P_2, L) F_2(D, \Delta\lambda, \alpha, P_1, P_2, L) \cdot F_3(\mathrm{SOP}_1, \mathrm{SOP}_2). \tag{1}$$

Here, the first term, F_1, is called the power term, which is a function of the fiber nonlinear coefficient γ, the fiber attenuation coefficient α, the fiber length L, and is the input signal powers P_i $(i = 1, 2)$. The second term, F_2, is called the FWM efficiency factor, which depends on the fiber dispersion D, the signal wavelength separation $\Delta\lambda$, the fiber loss α and length L, and the signal power [6]. The third term, F_3, is the FWM state of polarization (SOP) transfer function, where SOP_i $(i = 1, 2)$ are the states of polarization of signals 1 and 2, respectively. If $E_1 = [\cos(\phi_1), \sin(\phi_1) e^{j\Delta_1}]^T$ and $E_2 = [\cos(\phi_2), \sin(\phi_2) e^{j\Delta_2}]^T$ are the complex polarization vectors of signals 1 and 2, respectively, and the fiber length is much longer than the coupling length (10–100 m in most communication fiber), then F_3 can be written as [7]

$$F_3(\mathrm{SOP}_1, \mathrm{SOP}_2) = \cos^2(\phi_1)\cos^2(\phi_2) + \sin^2(\phi_1)\sin(\phi_2) + \frac{1}{2}\sin(2\phi_1)\sin(2\phi_2)[\cos(\Delta_1)\cos(\Delta_2) + \sin(\Delta_1)\sin(\Delta_2)]. \quad (2)$$

This equation is valid for fiber with small PMD, since PMD may significantly change the relative polarization states of the two signals along the fiber when the frequency separation between the two signals becomes large.

From the definitions of Stokes vectors [8], we can write the sine and cosine functions in (2) as

$$\cos(\phi_i) = \sqrt{\frac{1}{2}[s_0^{(i)} + s_1^{(i)}]} \quad (3a)$$

$$\sin(\phi_i) = \sqrt{\frac{1}{2}[s_0^{(i)} - s_1^{(i)}]} \quad (3b)$$

$$\cos(\Delta_i) = \frac{s_2^{(i)}}{\sqrt{(s_2^{(i)})^2 + (s_3^{(i)})^2}} \quad (3c)$$

$$\sin(\Delta_i) = \frac{s_3^{i}}{\sqrt{(s_2^{(i)})^2 + (s_3^{(i)})^2}}, \quad (3dc)$$

where s_0, s_1, s_2, and s_3 are the four normalized components of Stokes vector, with the superscript i standing for the signals 1 and 2, respectively. The normalized Stokes components satisfy the relation

$$s_0^i = \sqrt{(s_1^{(i)})^2 + (s_2^{(i)})^2 + (s_3^{(i)})^2} = 1 \qquad (i = 1, 2). \quad (4)$$

Substituting (3a) to (3d) into (2), we obtain

$$F_3(\mathrm{SOP}_1, \mathrm{SOP}_2) = \frac{1}{2}[1 + s_1^{(1)}s_1^{(2)} + s_2^{(1)}s_2^{(2)} + s_3^{(1)}s_3^{(2)}] = \frac{1}{2}[1 + \overline{s}_1 \cdot \overline{s}_2], \quad (5)$$

where $\overline{s}_1 = [s_1^{(1)} \; s_2^{(1)} \; s_3^{(1)}]^T$ and $\overline{s}_2 = [s_1^{(2)} \; s_2^{(2)} \; s_3^{(2)}]^T$ are the two vectors representing the polarization states of the two input signals on the Poincare sphere.

To see how FWM power generated in a low PMD, low dispersion measurement fiber can be used to measure the PMD in an arbitrary test fiber, we first note that, according to the fixed polarizer method [3], the first-order PMD of a fiber can be measured by launching a fixed SOP "signal" wave into the test fiber and then passing the output through a fixed polarizer. The output power from the polarizer is given by the expression,

$$T = \frac{1}{2}[1 + \overline{s}(\omega) \cdot \overline{P}] \quad (6)$$

where $\overline{s}(\omega)$ is the SOP of the light incident on the polarization analyzer and $\overline{p}$ is the unit vector specifying the transmission state (i.e., the pass axis of the polarization analyzer). First order PMD is then estimated using the formula [3]

$$\langle \Delta\tau \rangle = k\frac{\pi \langle N_e \rangle}{\Delta\omega} \quad (7)$$

where $\langle \Delta\tau \rangle$ is the mean PMD, $\langle N_e \rangle$ is the mean number of maxima and minima of the T curve in the frequency band $\Delta\omega$, and k is the polarization coupling factor (which equals 1.0 when the fiber under test is a PMD emulator).

Comparing (5) with (6), we see that they are the same function, except that the polarization state of the 2nd, fixed frequency "probe" signal in (5) replaces the polarizer transmission state $\overline{p}$ in (6). This suggests that an alternative to the fixed polarizer method would be to launch two, fixed SOP signals into the test fiber and pass the output through a short measurement fiber that has both low PMD and dispersion. According to (5), the FWM power generated in a low PMD measurement fiber will vary with frequency changes of the test signal exactly as would the output of the test signal alone passing through a fixed polarizer. This means that we can use the FWM transfer function (5) in place of the T function (6) when calculating PMD using calculating first order PMD using (7).

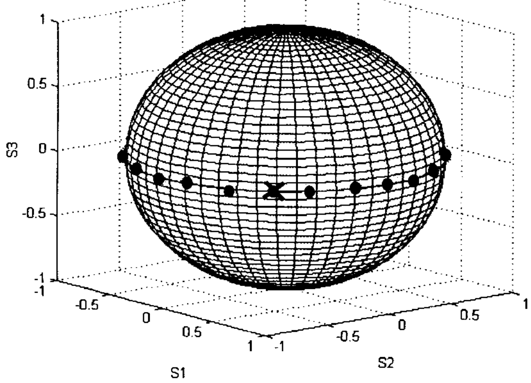
The advantage of calculating PMD using the FWM power produced in a separate, measurement fiber is that no special care need be taken to maintain a strict spatial orientation between the test fiber and the measurement equipment (such as a polarizer). This is because the probe wave follows the signal wave through both the test and measurement fibers and, therefore, automatically establishes the polarization reference in the measurement fiber.

B. Experimental Verification of the FWM Transfer Function

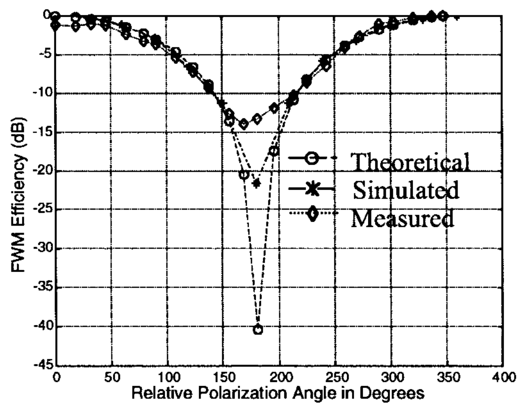
In order to verify (5), both experiments and numerical simulations were performed and compared with (5). A 17.5 km length of dispersion-shifted fiber (DSF) was used to produce FWM. The zero-dispersion wavelength of this fiber was 1551 nm. Two CW signals were input to this fiber, with wavelengths 1552.0 nm and 1552.8 nm, respectively. The polarization states of the two input signals were varied by polarization controllers, and a polarization analyzer was used to measure and record the input polarization states of the two signals on the Poincare sphere. The numerical calculations were performed using the Split-Step Fourier-Transform Method [9].

Fig. 1 shows the results for the case of when both signals were linearly polarized. Here, the SOP for one signal was fixed and the SOP for the other signal varied along the equator of the Poincare sphere. Fig. 1(a) shows the polarization states on the Poincare sphere. Fig. 1(b) shows the FWM efficiency in dB as calculated numerically, measured experimentally, and predicted by the FWM transfer function (5). For these plots, the FWM efficiency is defined as the FWM power, normalized by its maximum value. These results agree well except in the notch area, where the analytical curve from (5) goes to zero ($-\infty$ dB) when the two signals have orthogonal polarization states. The simulated results at this point do not approach zero due to the second-order FWM effects. The minimum measured FWM power is limited by the ASE noise level. Even so, the difference between measured maximum and minimum FWM efficiency is roughly 15 dB, which is more than enough to distinguish the minima and maxima needed for determining PMD.

Fig. 2 shows the variation of the FWM efficiency when the polarization state of one signal is a fixed, linear state, and the other signal's polarization state is varied from linear, to elliptical, to circular, and back to the original linear polarization

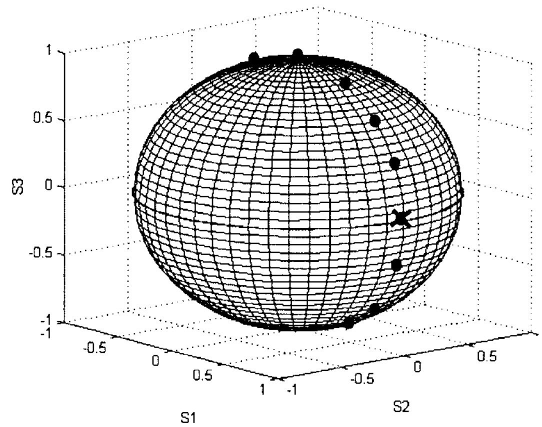


(a)

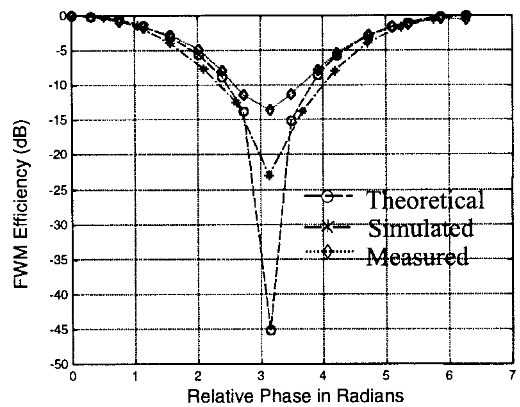


(b)

Fig. 1. The dependence of FWM on the input signal polarization states for linear polarization. × signal 1; • signal 2.



(a)



(b)

Fig. 2. The dependence of FWM on the input signal polarization states for linear, elliptical and circular polarization. × signal 1; • signal 2.

state, as shown in Fig. 2(a). Numerically calculated, measured, and analytical predictions (5) of FWM efficiency are shown in Fig. 2(b) and, again, show excellent agreement.

III. Measuring PMD Using FWM

An experimental setup for measuring PMD using FWM is shown in Fig. 3. Here, the device under test is a PMD emulator. The DSF measurement fiber is the same as that used above for verifying the FWM transfer function. An erbium-doped-fiber amplifier (EDFA) was used to boost the signal power to produce FWM in the DSF. The FWM power was measured with an optical spectrum analyzer. During the measurements, the wavelength for one input signal was fixed at 1554.0 nm and the other wavelength was varied over a range that depended on the expected PMD values. Fig. 4(a) and (b) shows the measured FWM power versus the signal wavelength separation for PMD values of 20 and 10 ps, respectively, where each is compared with the zero-PMD case. The PMD-induced variations of the FWM power are clearly observed as the signal wavelength changes, and look similar to the transmission curves obtained when using the fixed-polarizer PMD method. For the zero-PMD case, there are some fluctuations in FWM power, but the magnitudes of the variations are quite small and easily distinguished from PMD-caused FWM magnitude variations. Fig. 5 shows the measured PMD values for different settings of the PMD simulator. Here, it can be seen that the measured PMD values agree well with the actual PMD values.

IV. Conclusion and Discussion

We have demonstrated a method for determining PMD by measuring FWM generation in a section of DSF measurement fiber placed after a test fiber. Both numerical simulations and experiments were performed to verify the FWM power-transfer-function dependence on the polarization states of two waves launched into a fiber. PMD measurements were also performed, with good agreement with the given PMD values.

Like the well-known fixed polarization-analyzer method, this method uses a frequency-domain transfer function to determine SOP changes with frequency and, consequently, the PMD. The difference, however, is that this technique uses the FWM power generated in a separate measurement fiber to track the changes of polarization of a wave with frequency. This makes this technique relatively insensitive to mechanical vibrations and upset, since both the probe and signal waveforms are subjected to the same mechanical environments. Hence, the accuracy of this technique is limited only by the additional PMD and dispersion added by the measurement fiber itself, which is typically small in short lengths of DSF fiber. In addition, errors can be further reduced by first calibrating the measurement with the zero-PMD case

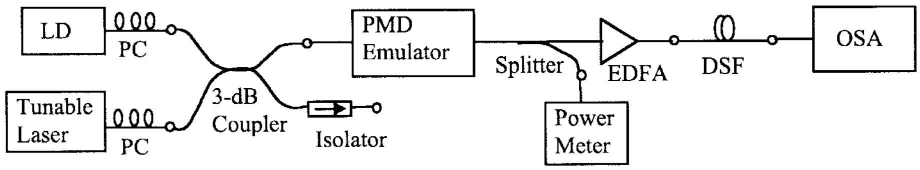
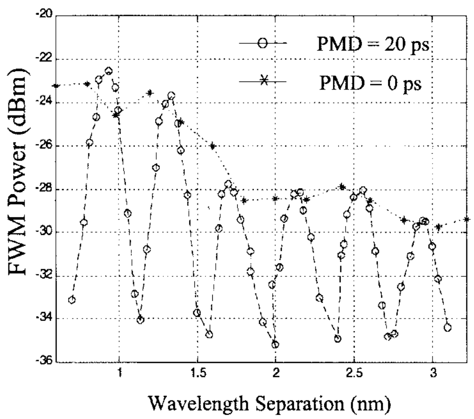
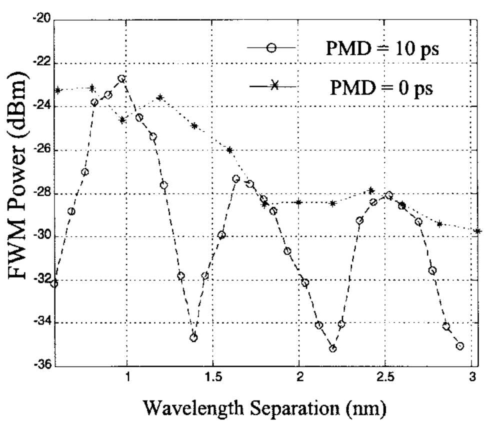


Fig. 3. Experiment setup for measuring PMD using FMD.



(a)



(b)

Fig. 4. Measured FWM power versus wavelength separation for different PMD values. DSF fiber length: 17.5 km, zero-dispersion wavelength: 1551 nm, Loss: 0.25 dB/km.

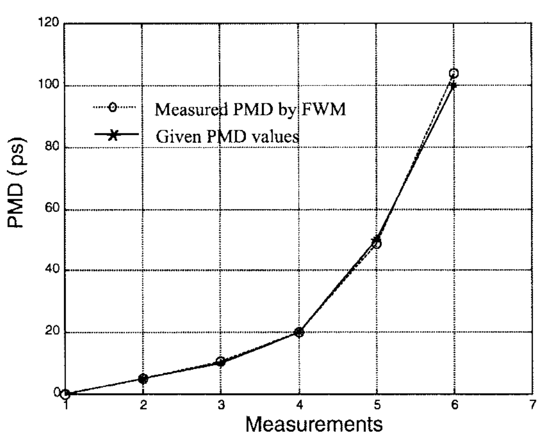


Fig. 5. Comparison of measured PMD using FWM and the given PMD.

(i.e., the test device or fiber under test removed), so that the frequency dependence of the FWM in the measurement fiber can be subtracted out.

An additional advantage of this technique is that it may be possible for it to provide in situ PMD measurement or monitoring on dense wavelength-division multiplexed (DWM), traffic-carrying links. If the polarization states of the transmitted signals are fixed, the FWM products generated throughout the bandwidth of the channels in a separate measurement fiber may provide an estimate of the PMD, either span by span or over several spans.

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