Numerically reversing the nonlinear wave propagation in single-mode optical fiber

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Nonlinear wave propagation in single-mode fiber has been studied extensively in the last decade using analytical approximations and forward-propagating numerical simulations [1-4]. Little attention, however, has been directed towards the inverse wave propagation, whereby an input wave is determined from a desired output wave. In the studies of traditional linear systems, inverse operations are quite common and usually easy to do. However, when nonlinear effects are added in the system, such as the nonlinear effects in single-mode fiber, the inverse process is more difficult. This paper demonstrates a variation of the split-step Fourier method (SSF) [4] that performs inverse operations on both the linear and nonlinear effects in single-mode fiber, thus allowing the calculation of transmitted waveforms that produce a desired output result.

The algorithm used to perform the inverse operation to nonlinear wave propagation is essentially the reverse of the split-step Fourier Transform (SSF) method [4]. The general expression to solve the nonlinear Schrodinger equation (NLS) is

\[
A(z-h,T) = \exp\left(-\frac{h}{2}D_z\right)\exp\left[-hN_s(z-h/2)\right]\exp\left(-\frac{h}{2}D_z\right)A_z,T
\]

where \(A(z, T)\) is the slowly-varying envelope, \(z\) is the distance along the fiber, \(T\) is the normalized time, and \(h\) is the step length of the numerical operation. Also, \(D_z\) and \(N_s\) represent the linear effects and nonlinear effects, respectively. Since the operation performed by (1) is inverse to the operation performed by SSF, we call it ISSF.

First, we consider a inverse calculation when just fiber loss, chromatic dispersion and self-phase modulation (SPM) are present. Fig. 1 shows the envelope evolution of a 17.6-ps fundamental-soliton pulse in 42-km of standard single-mode fiber (SMF), followed by 8-km dispersion-compensation fiber (DCF). The dispersion and loss for SMF and DCF are 16 ps/km-nm, 0.25 dB/km and ~80 ps/km-nm, 0.35 dB/km, respectively. The soliton peak-power is 3 mW. Fig. 1(a) shows the forward evolution of the pulse envelope using SSF, which simulates the forward propagation of the pulse in the fiber. Fig. 1(b) shows the inverse evolution of the same pulse envelope, using the received signal from SSF as the input to ISSF. As can be seen, the envelopes are identical.

![Fig. 1 Evolution of a soliton in 42-km SMF followed by 8-km DCF](image)

Next, we consider a 4-channel WDM system with five (5) 100 km spans of standard single-mode fiber. The fiber dispersion is 0.5 ps/nm-nm at 1550 nm, the launched power for each channel is 1 mW, and the channel spacing is 100 GHz. Fig. 2(a) and Fig. 2(b) show the transmitted and received waveforms of channel 2, as calculated by SSF simulation, where the
effects of SPM, cross-phase modulation (XPM) and four wave mixing (FWM) are included. Fig. 2(c) shows the transmitted waveform, calculated using ISSF, using the waveform Fig. 2(b) as the input. Comparing Fig. 2(a) and Fig. 2(b), they are nearly identical, except for some small numerical distortion. This amount of distortion will not influence simulation results (eye-closure penalty or Q-factor) in most real applications.

![Fig. 2 Pulse sequences demonstrating inverse operation, including the effects of SPM, XPM and FWM](image)

Stimulated Raman scattering (SRS) can be accounted for in ISSF using the power-coupled model [2]. Figure 3 shows an example of how this inverse method can be used to pre-emphasize the transmitted signal power in DWDM systems to compensate the gain tilt caused by the stimulated Raman effect. Here, a 45-channel DWDM, 2-fiber span system with flat-gain optical amplifiers is modeled. The bit rate is 10 Gb/s per channel, the channel spacing is 1.6 nm and 0.8 nm, the received power is 0 dBm/ch., the fiber length per span is 100 km, and dispersion is 3.5 ps/km-nm, and the loss is 0.25 dB/km. Fig. 3(a) shows the power spectral density (psd) of the transmitted signals obtained by using ISSF, assuming that goal is to obtain a flat power spectral density at the receiver. Fig. 3(b) shows the received power spectral density as calculated using SSF. Outside of sampling error, this is the desired flat spectrum.

![Fig. 4 Transmitted and received optical spectral intensity](image)

In conclusion, we have demonstrated the inverse operation of nonlinear light wave propagation in single-mode optical fiber by using the inverse split-step Fourier method. Most effects are reversible, with some numerical distortion depending on system parameters. The idea of performing inverse operation to wave propagation in single-mode fiber unveiled a new aspect of studying nonlinear effects in single-mode fiber and may provide new approaches to tackle some nonlinear problems in optical fiber communications.

References