Addressing Spectrum Congestion by Spectrally-Cooperative Radar Design

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Scope of Presentation

- Introduction
  - Motivation
  - Categories of current solutions/approaches
- Proposed Solution (2-step approach)
  - Spectrally Efficient Waveform Design
  - Results from this step
  - Sparse Spectrum Allocation using Information Theory
  - Results from this step
- Applications of results to Radar System Implementation
- Conclusion

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Motivation: Resolving Spectrum Congestion issues

- Rise in demand for Radio Frequency (RF) spectrum in recent years in wireless communications due to increase in demand in:
    - Mobile Telephony services such as FaceTime and Skype
    - Cable/Satellite TV streaming
    - 5th Generation Mobile Telecommunications Protocol
    - Internet of Things (IOT)

- This imposes a strain on current radar systems who maintains largest share of RF spectrum

Motivation: Resolving Spectrum Congestion issues

- Leads to Spectrum Congestion issues and rise of Mutual Interference among systems (e.g. radar versus cell phone) who need to coexist within finite spectrum allocation, i.e. Spectrum Sharing

- Main initiative to resolve these challenges rests within the radar community

- Question Posed: Does Radar needs all these spectrum to be fully filled or can it be just partially filled in an optimal manner ?

Current Approaches

- Presently, resolving the issues associated with spectrum sharing can be broadly classified into 3 categories:
  - Category 1: Design of Cognitive Radio to ensure radar's performance is not degraded
  - Category 2: Design of Cognitive Radar as main party responsible in interference mitigation
  - Category 3: Joint design of both Cognitive Radio/Radar spectrum allocation and waveforms

- These categories have also been given acronyms such as the "Three 'A's of Communications – Radar Spectrum Sharing : Avoid, Accept and Amalgamate"

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Dissertation Research

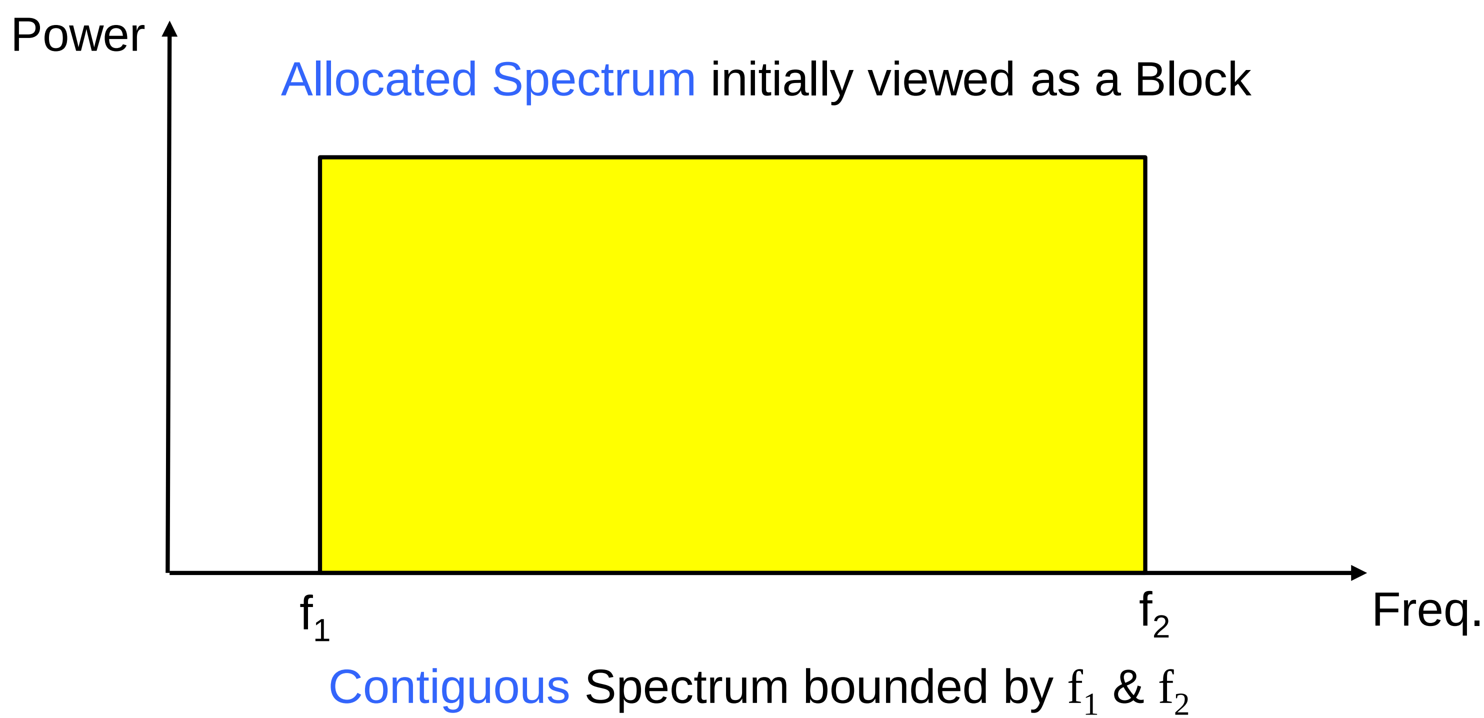
- Dissertation research is on developing a 2-step approach grouped under the 2nd category of Cognitive Radar

- Step 1 involves the design of a Spectrally Efficient Radar Transmit waveform so as to minimize mutual interference:
  - Built on the existing framework of Poly-phased Coded Frequency Modulated (PCFM) waveforms

- Step 2 involves the design of a Sparse Spectrum Allocation algorithm so as to reduce radar's spectrum usage while maintaining range resolution performance
  - An alternative approach to Sparse Frequency Waveform design

Overall Problem Formulation

- The next few slides provides some description of the Proposed Solution achieved via the 2-step approach



Overall Problem Formulation

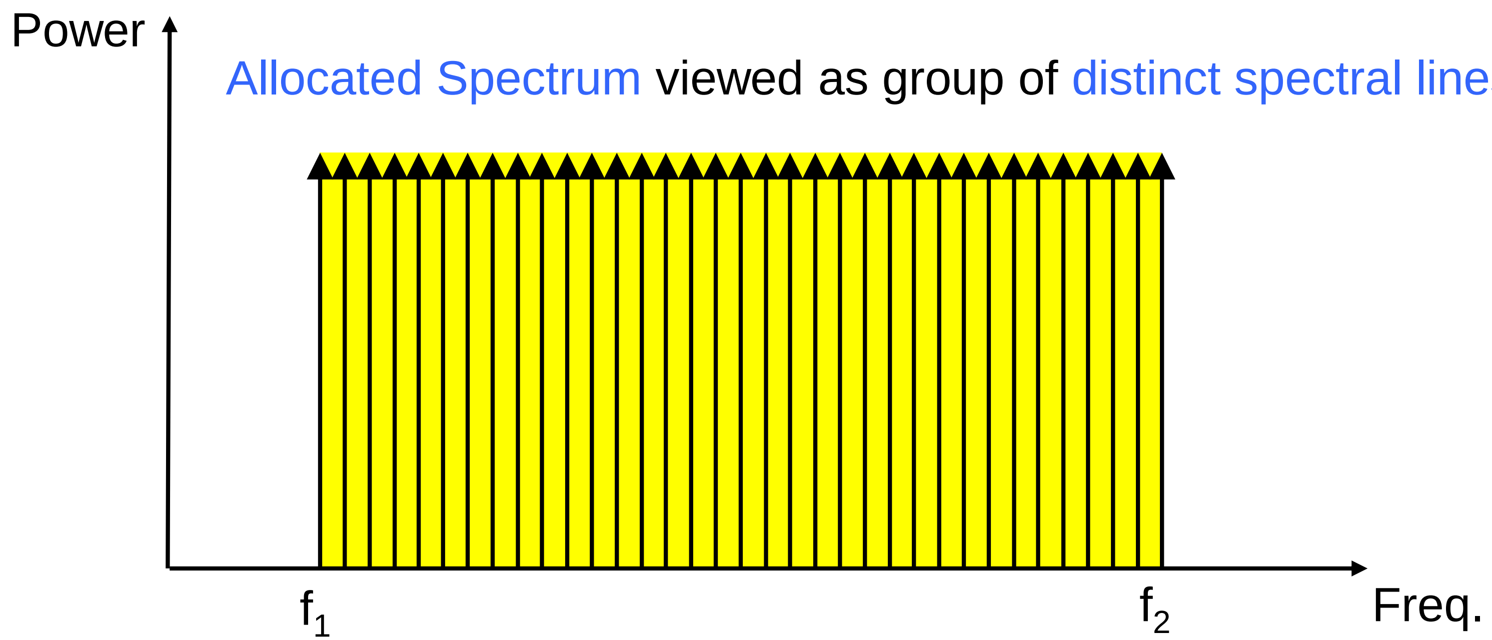
- Question: Can we build a radar transmit signal that does not fully utilize the allocated spectrum ?

- How do we evaluate its performance ?

- How do we process this type of sparse radar transmit signal ?

Overall Problem Formulation

- We are going to represent the spectrum by *N* number of spectral lines, for instance, CW tones or Pulsed Radar



Contiguous Spectrum bounded by f_1 & f_2

Solution Formulation

- Let this group of *N* spectral lines represent an array of *N* radarlets starting with no bandwidth (e.g. CW tones)

Group of distinct spectral lines viewed as array of radarlets

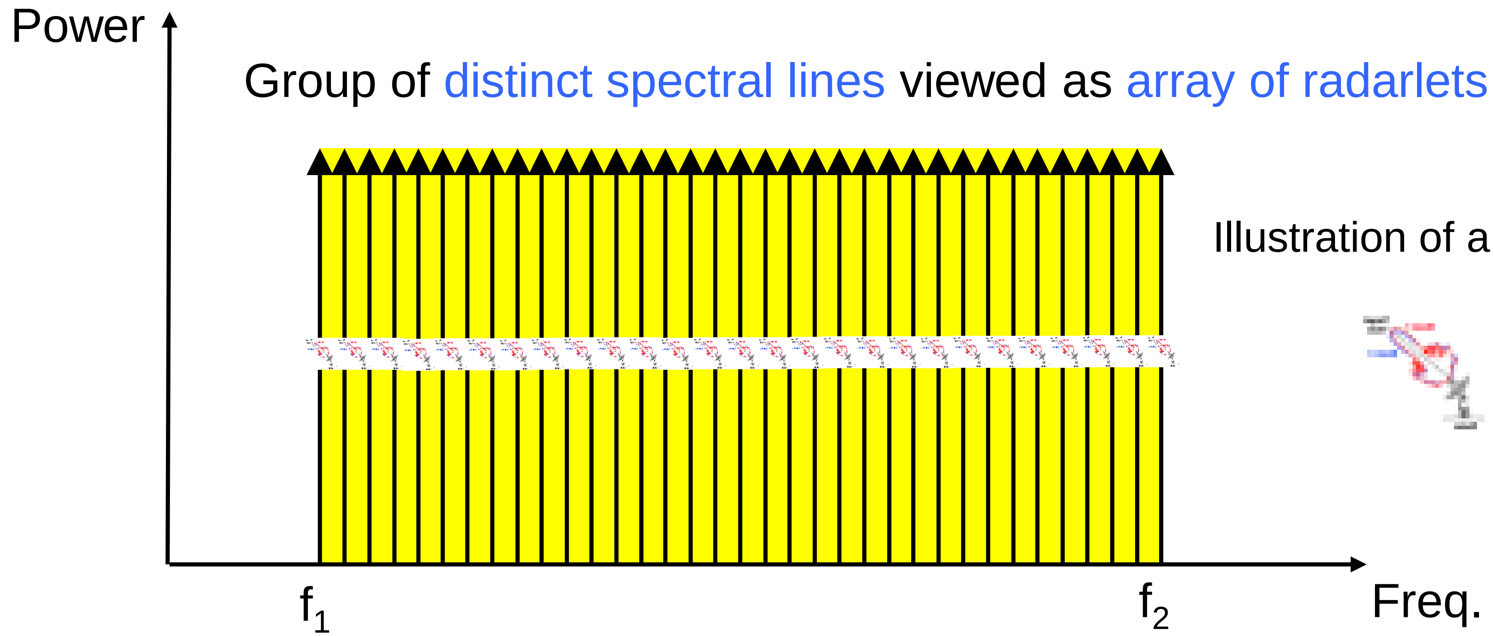
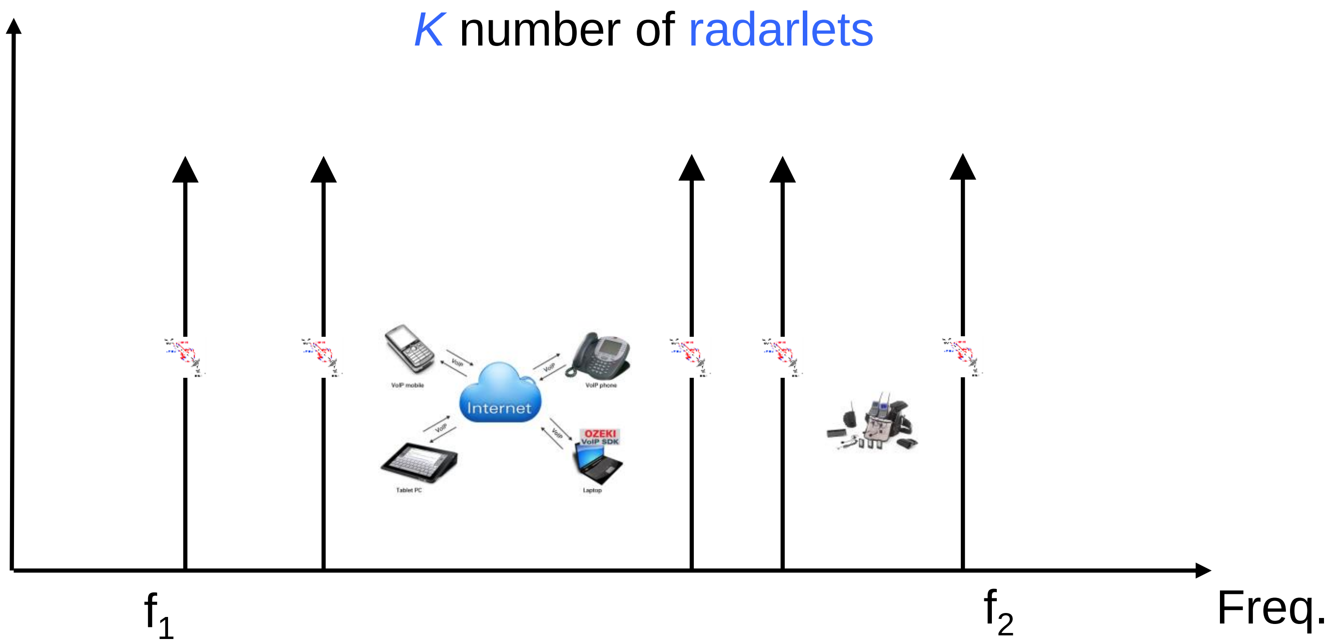


Illustration of a radarlet

Contiguous Spectrum represented by spectral lines

Formulation of Two-step approach

- We are going to thin the radarlets from *N* to *K* radarlets. The net spectrum usage will be the ratio (*K* / *N*)



Contiguous Spectrum replaced by Sparse Spectrum

Formulation of Two-step approach

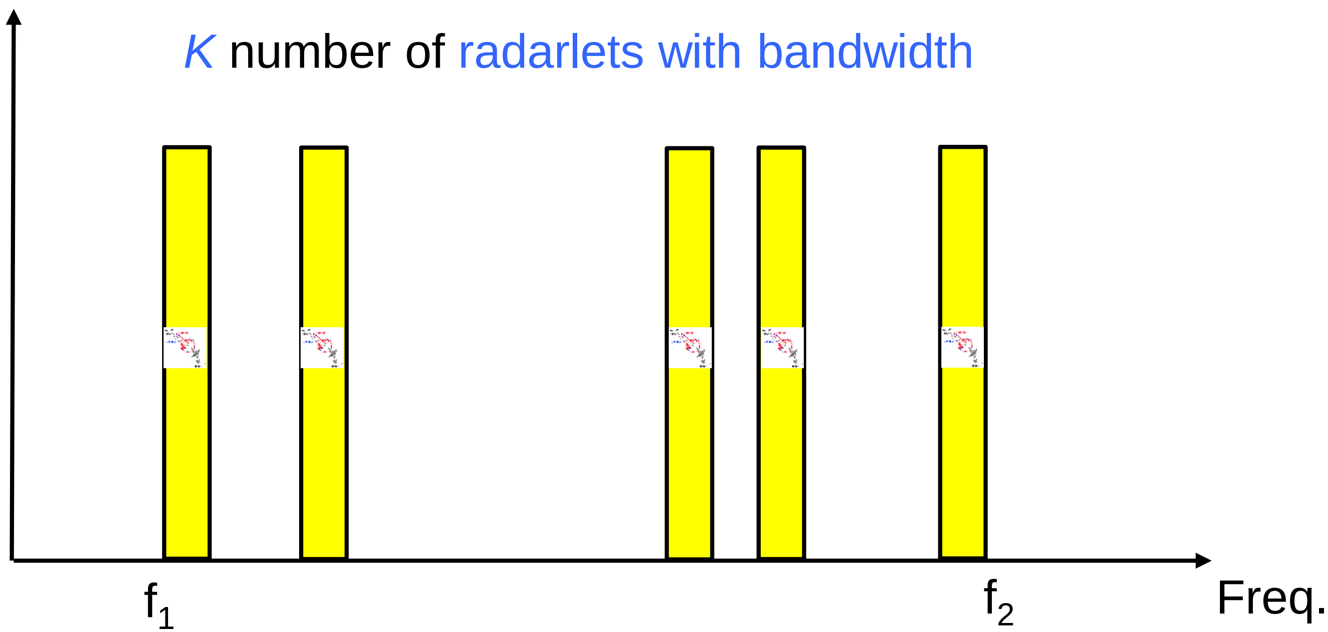
- The locations of the K resulting spectral lines are not confined to integer multiple of the Pulse Repetition Frequency so as to increase the degrees of freedom for the optimization process

- How do we design such a sparse radarlet array ?

- What optimality criteria do we use to determine the locations of this sparse radarlet array ?

Formulation of Two-step approach

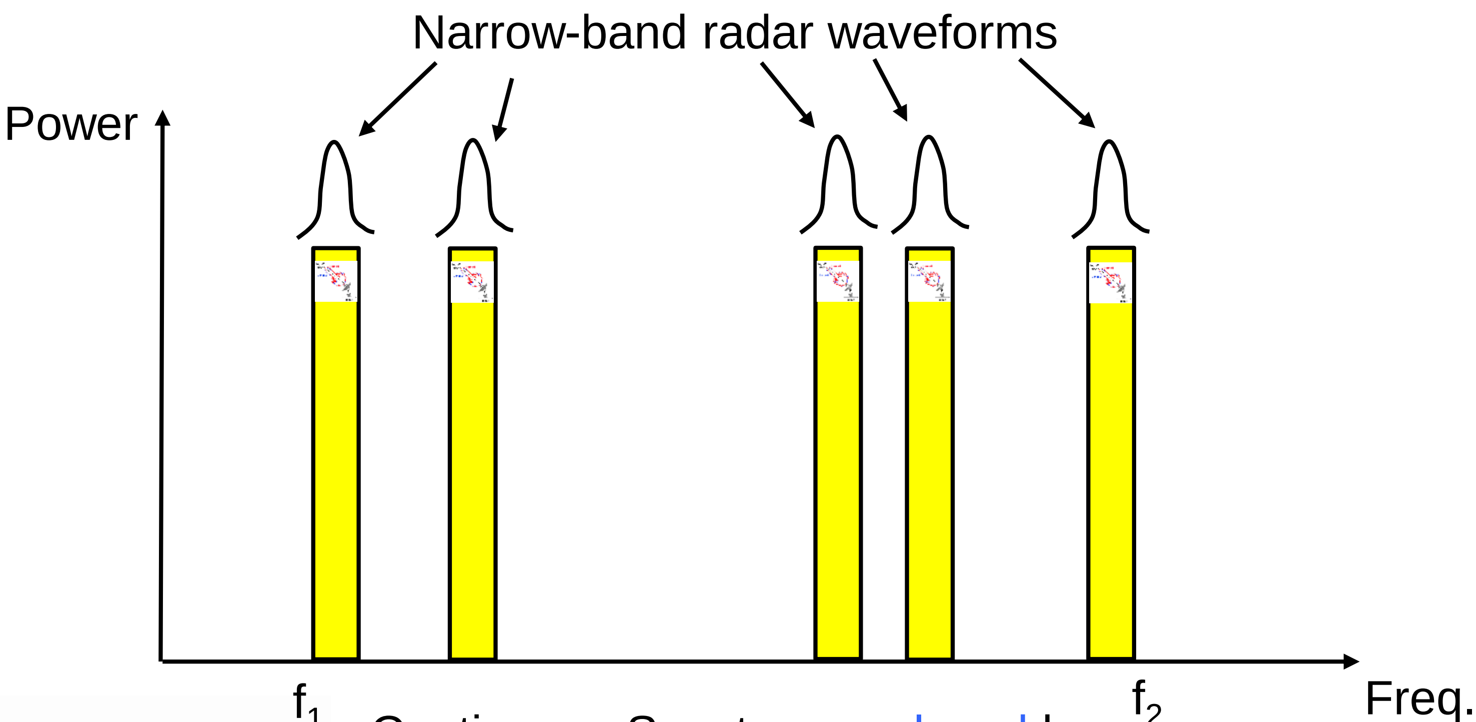
- We are going to modulate each of the K radarlet so that each radarlet will posses a finite bandwidth



Contiguous Spectrum replaced by Sparse Spectrum

Formulation of Two-step approach

- The modulated radar waveform for each radarlet should provide good spectral containment properties



Contiguous Spectrum replaced by Sparse Spectrum

Formulation of Two-step approach

- Thus, in addition to determining the locations of the sparse radarlet array, we also want to confine the spectral content of each radarlet

- This will ensure that the spectral content of each radarlet will not leak into the spectrum of other systems to become interference signals

- Need to select the type of radar waveform that is both spectrally well-contained as well as other properties like low side-lobe performance

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Higher-order PCFM

Problem Setup

Background

- Polyphase-coded Frequency Modulated (PCFM) radar waveforms are realized by a variant of Continuous Phase Modulation (CPM) signals from communications
  - Converts an arbitrary polyphase code into a physically-realizable FM waveform

- PCFM waveforms are:
  - Spectrally efficient – phase is continuous and differentiable thus providing good spectral containment
  - Power efficient – constant modulus
  - Able to achieve low autocorrelation sidelobes relative to time-bandwidth (*BT*) product where *B* is the 3 dB bandwidth

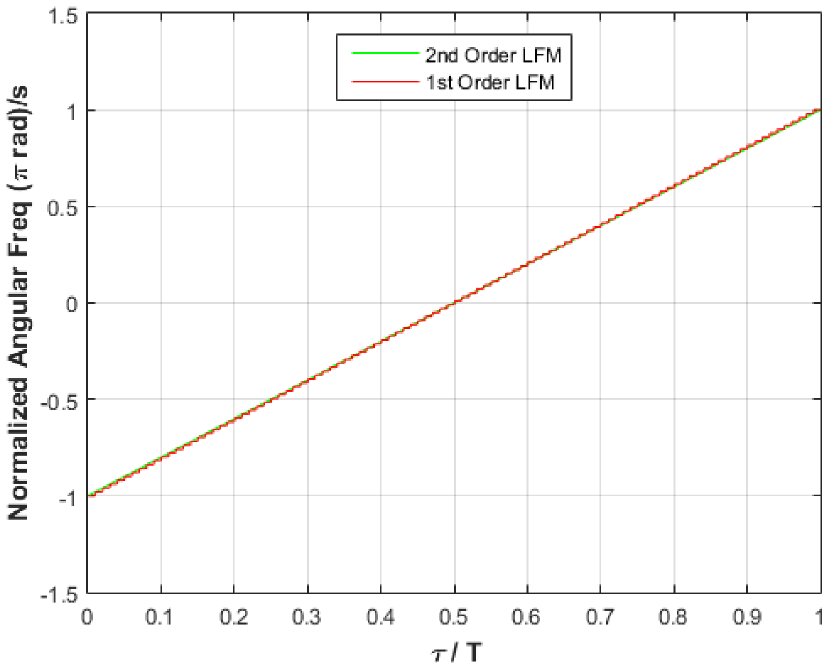
Higher-order PCFM Waveform

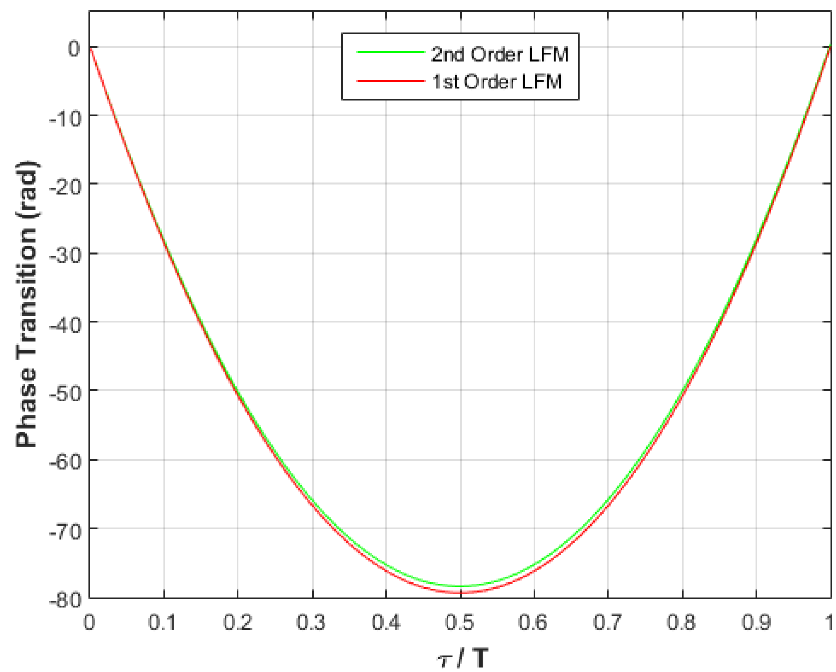
- Previous research demonstrated PCFM waveforms generated from Polyphase codes akin to first-order hold in phase (where traditional codes represent a zero-order hold)

- In my research, I have investigated the Higher-order PCFM waveform implementation as the prospective benefits are:
  - Offers additional degrees-of-freedom (DOF) in waveform design without any increase in the *BT* product
  - Higher-order terms produce smoother phase trajectory, maintaining good spectral containment
  - Allows for the possibility to combine multiple orders to obtain even lower autocorrelation sidelobes

Higher-order PCFM Waveform

- As an example, let’s examine the plots of instantaneous frequency and phase of a LFM signal generated using first-order PCFM waveform versus second-order PCFM waveform



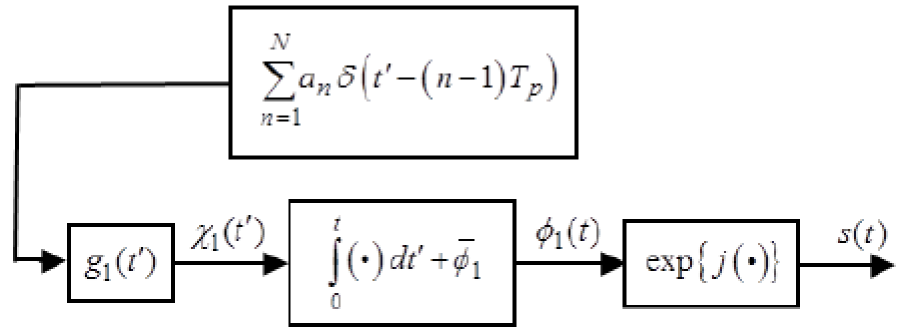


1st Order Implementation

- The 1st order PCFM implementation to realize phase function $\boldsymbol{\phi_1(t)}$:

$$\chi_1(t) = \sum_{n=1}^{N} a_n\, g_1\big(t-(n-1)T_p\big)$$

$$\phi_1(t) = \int_0^t \chi_1(t')\, dt' + \overline{\phi}_1$$



- $\chi_1(t)$ is the 1st order coded function produced by the *N* **“phase change” code** values a_n
- $g_1(t)$ is a shaping filter (e.g. rectangular)
- T_p is the duration of one phase change
- $\overline{\phi_1}$ is the initial phase value (arbitrary)

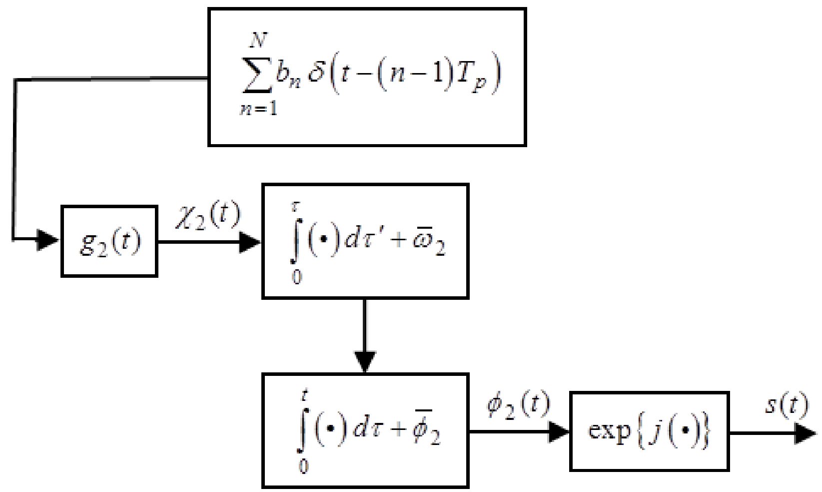
2^{nd} Order PCFM Implementation

- Generalize to 2^{nd} order PCFM implementation for phase function $\boldsymbol{\phi_2(t)}$:

$$\chi_2(t) = \sum_{n=1}^{N} b_n\, g_2\big(t-(n-1)T_p\big)$$

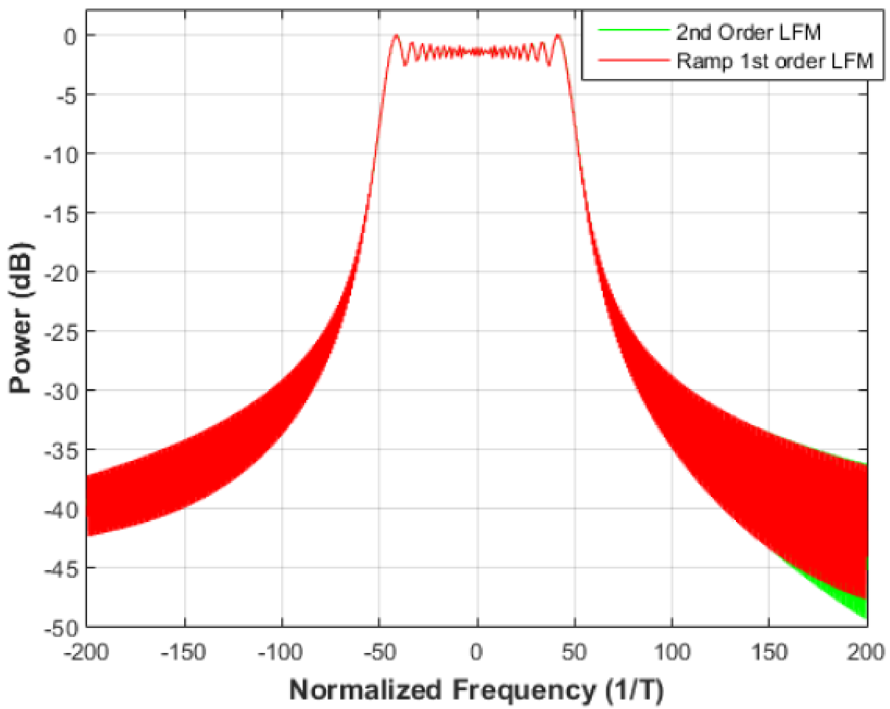
$$\phi_2(t) = \int_0^t \int_0^{t'} \chi_2(t'')\, dt''dt' + \int_0^t \bar{\omega}_2\, dt' + \bar{\phi}_2$$

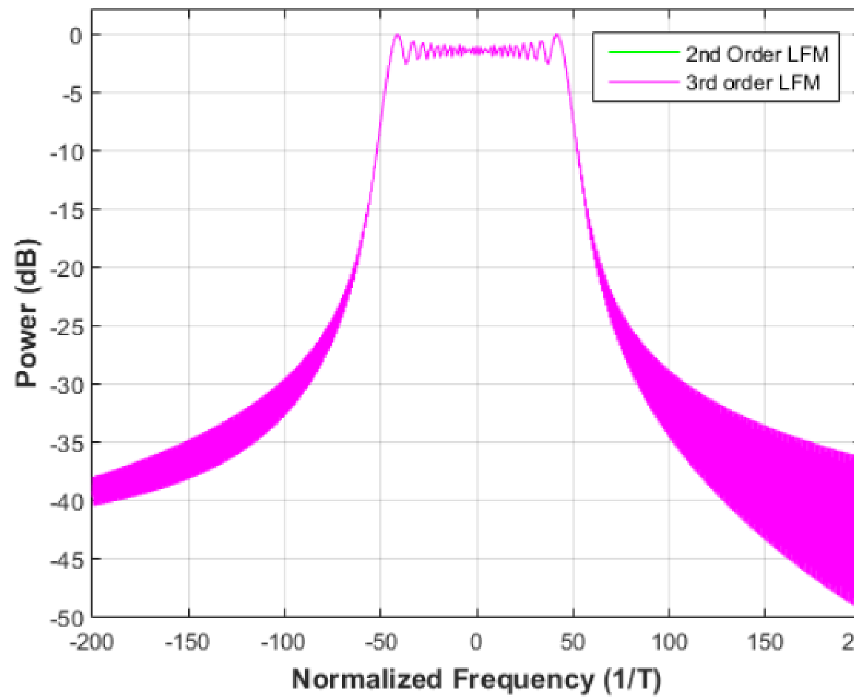
- $\chi_2(t)$ is the 2^{nd} order coded function produced by N **"frequency change" code** values b_n
- $g_2(t)$ is a shaping filter
- $\overline{w_2}$ & $\overline{\phi_2}$ are initial frequency & phase



Relationships between different orders of PCFM

- For instance, we can generate an exact LFM waveform of $BT = 100$ using either 1st, 2nd or 3rd order of implementations

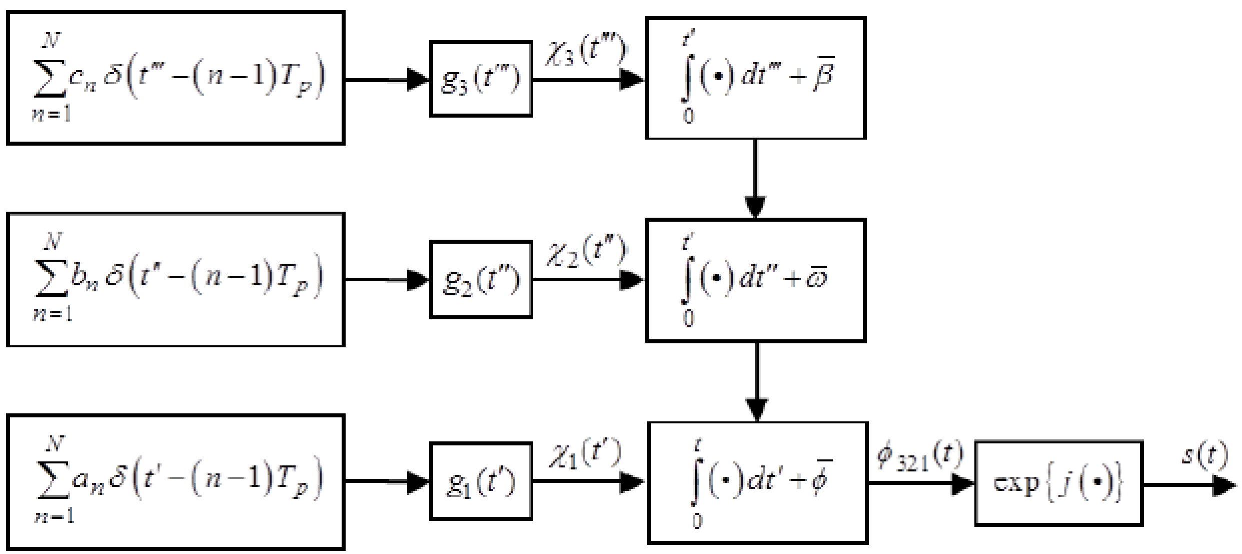




Multi-Order PCFM Implementation

- The 1st and higher orders of implementation can also be combined to become a multiple-order of implementation

$$\phi_{321}(t) = \int_0^t \chi_1(t')\,dt' + \int_0^t\int_0^{t'} \chi_2(t'')\,dt''dt' + \int_0^t\int_0^{t'}\int_0^{t''} \chi_3(t''')\,dt'''\,dt''dt'$$
$$+ \int_0^t\int_0^{t'} \bar{\beta}_{321}\,dt''dt' + \int_0^t \bar{\varpi}_{321}\,dt' + \bar{\phi}_{321}$$



Higher-Order Optimization Process

- When performing optimization for higher-order implementation such as second, third, fourth etc., the *Frequency Template Error (FTE)* metric is used to maintain spectral containment

- The greedy optimization approach denoted as "performance diversity" combining PSL, ISL & FTE metrics is used to optimize the higher-order PCFM codes
  - Multiple metrics help to avoid local minima via greedy search
  - Global optimality not guaranteed, but finds "good enough" local optimality

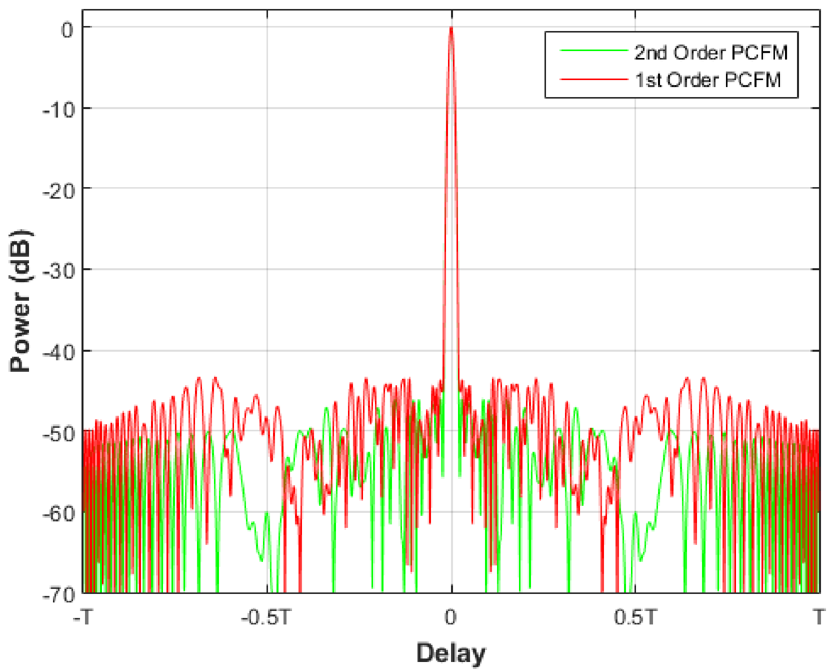
- When combining multiple orders, optimization may be performed jointly (i.e. simultaneously), or sequentially across the different orders

Higher-order PCFM

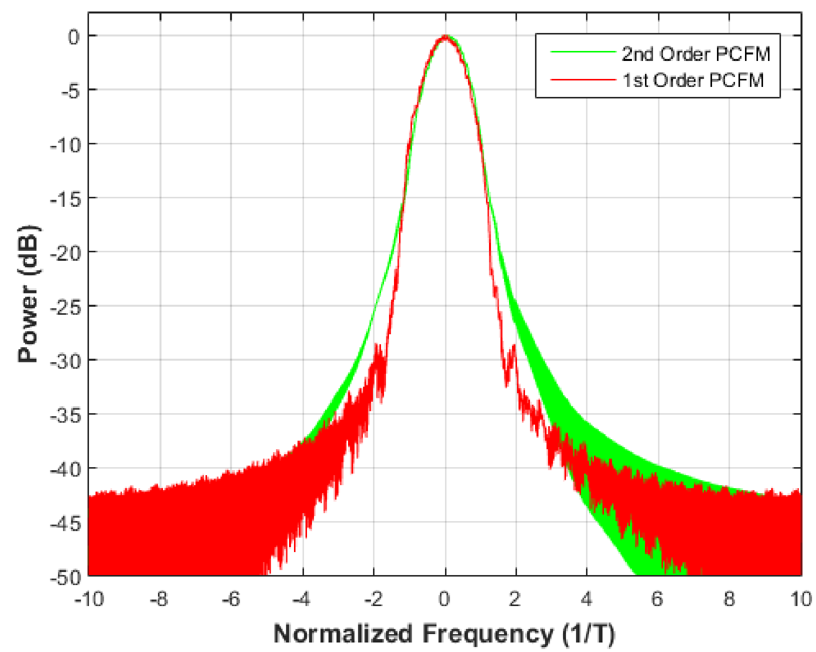
Simulation Results

2^{nd} order PCFM (Standalone)

- Let's examine the 1^{st} & 2^{nd} order PCFM implementations after optimization for $BT = 100$



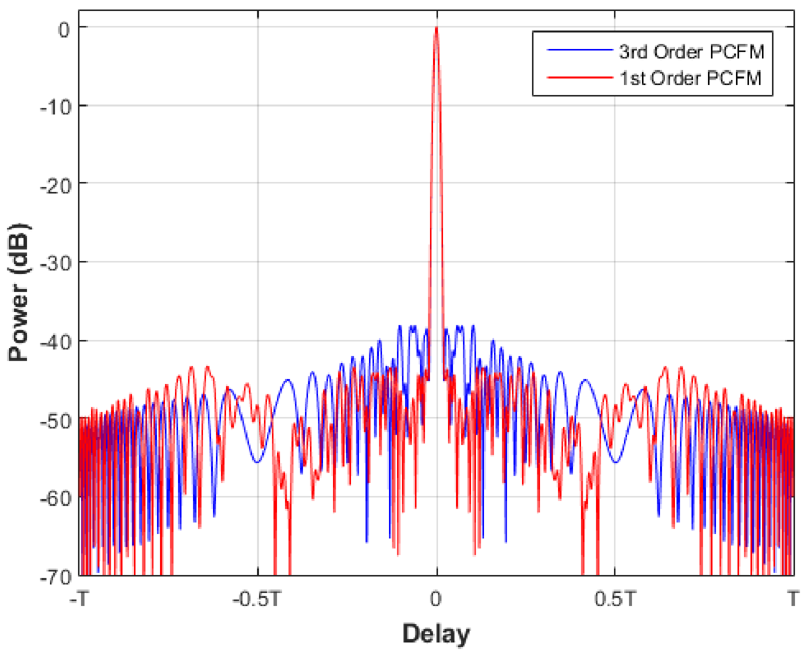
Autocorrelation of 1^{st} & 2^{nd} order optimized waveforms with $BT = 100$



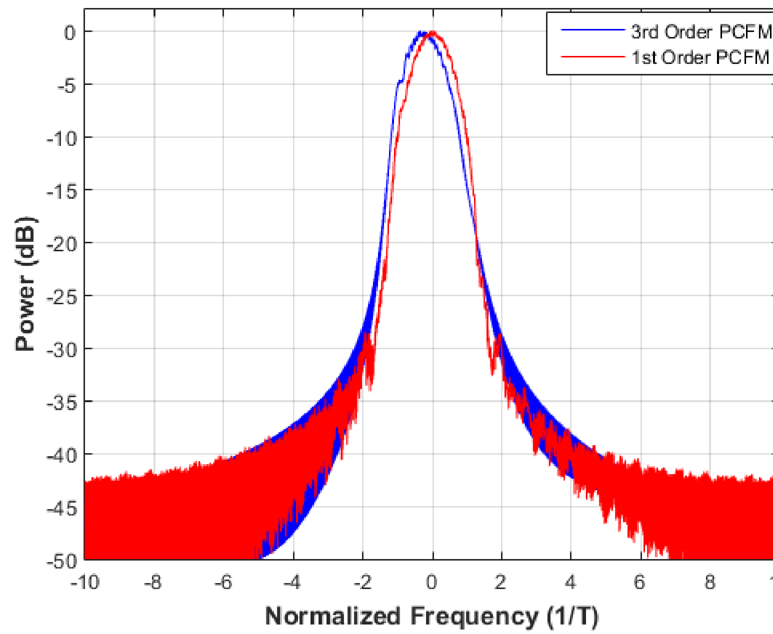
Spectral Content of 1^{st} & 2^{nd} order optimized waveforms with $BT = 100$

3rd order PCFM (Standalone)

- Likewise, we examine the 1st & 3rd order PCFM implementations after optimization



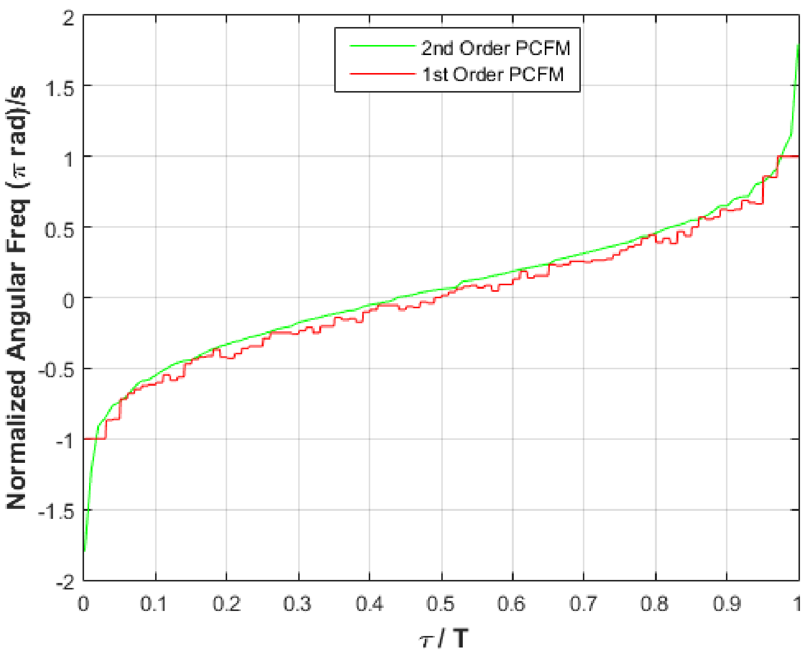
Autocorrelation of 1st & 3rd order optimized waveforms with BT = 100



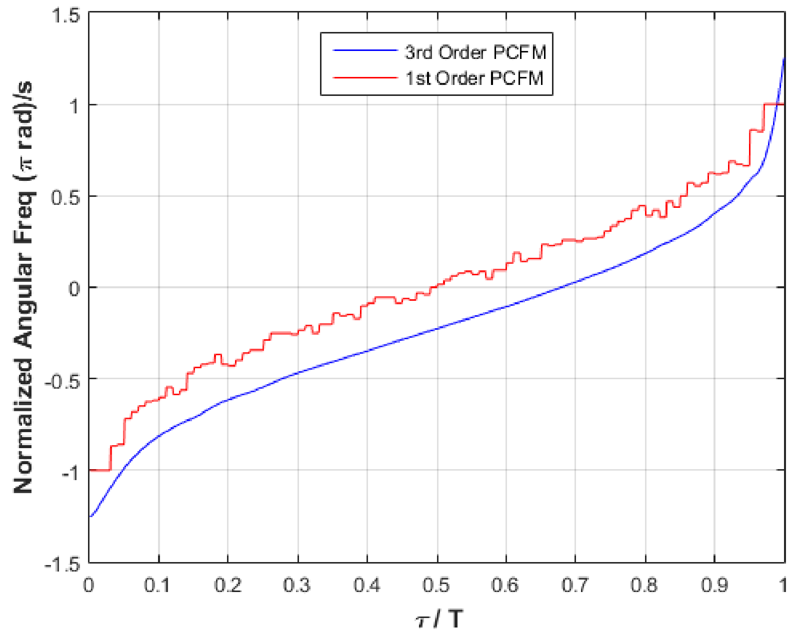
Spectral Content of 1st & 3rd order optimized waveforms with BT = 100

Instantaneous Freq. of Standalone PCFM

- Let's also examine the Instantaneous frequency of 1st , 2nd & 3rd order PCFM implementations after optimization



Instantaneous frequency of 1st & 2nd order optimized waveforms with *BT* = 100



Instantaneous frequency of 1st & 3rd order optimized waveforms with *BT* = 100

Summary of Individual Optimization Performance

- PSL and ISL values for optimizing BT = 100 waveforms for 1st, 2nd & 3rd order representations
  - Note: individually optimized (i.e. not combined with other orders)

PSL & ISL for 1st, 2nd & 3rd order optimized waveforms for *BT* = 100

	1st order	2nd order	3rd order	HFM bound
PSL (dB)	–43.4	–46.0	–38.1	–43.0
ISL (dB)	–59.5	–63.5	–57.4	

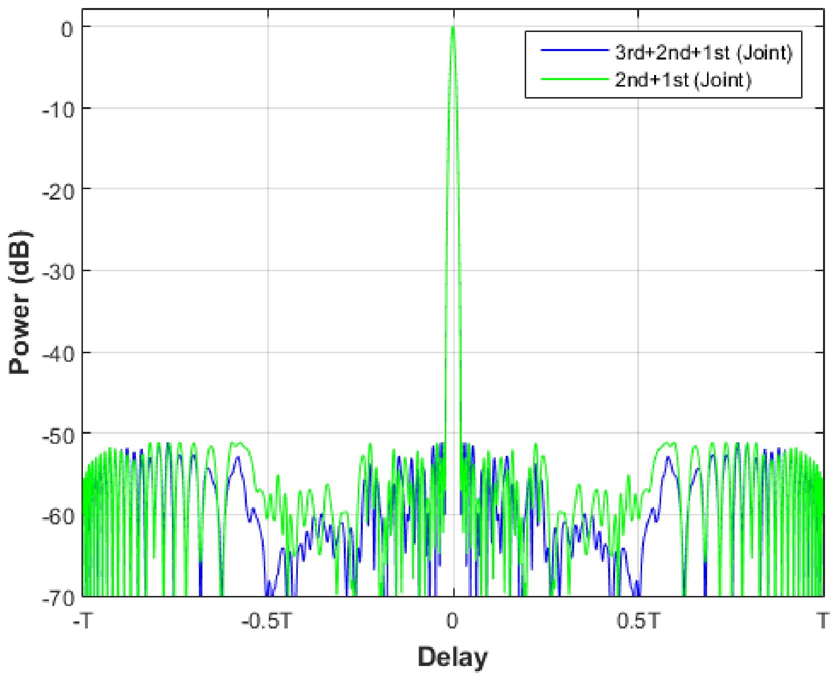
Original PCFM implementation

- Useful benchmark: hyperbolic FM (HFM) bound on PSL: $-20\log_{10}(BT) - 3$ dB

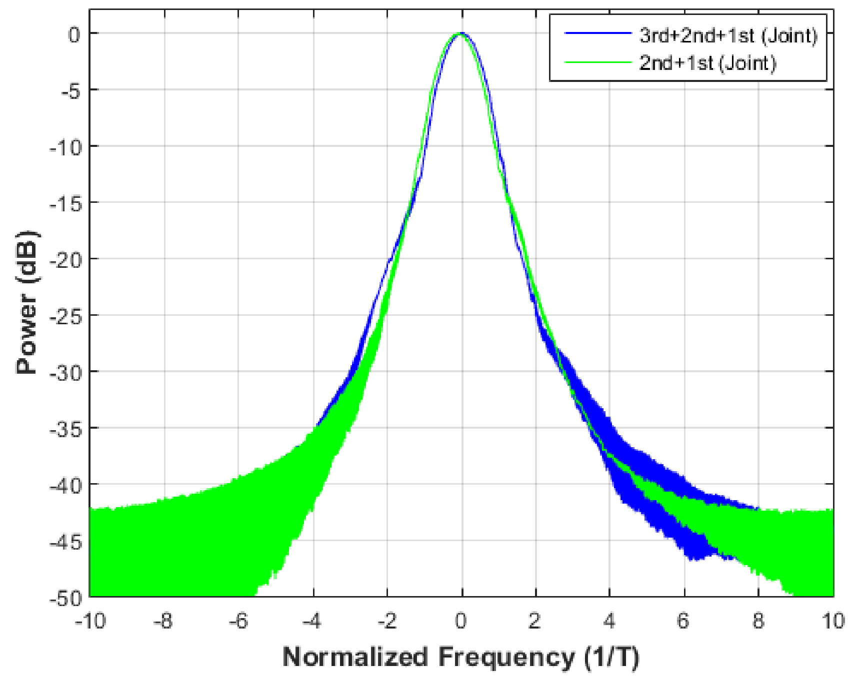
T. Collins & P. Atkins, "Nonlinear frequency modulation chips for active sonar" *IEEE Proc. Radar, Sonar & Navigation, Dec* 1999.

Multi-order PCFM (Combination)

- Let's examine the joint optimization of (3rd+2nd+1st) orders versus (2nd+1st) orders



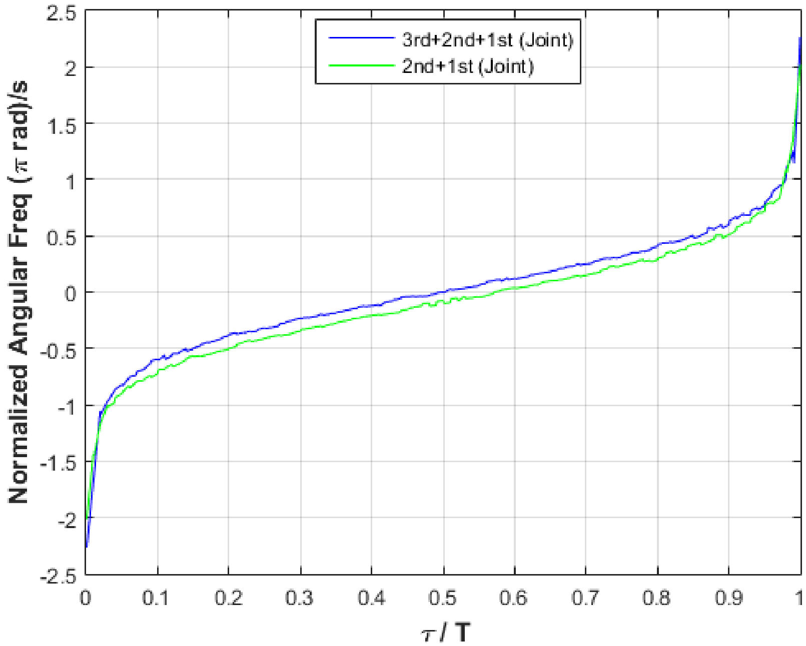
Autocorrelation of jointly optimized waveforms with $BT = 100$



Spectral Content of jointly optimized waveforms with $BT = 100$

Instantaneous Freq. of Multi-order PCFM

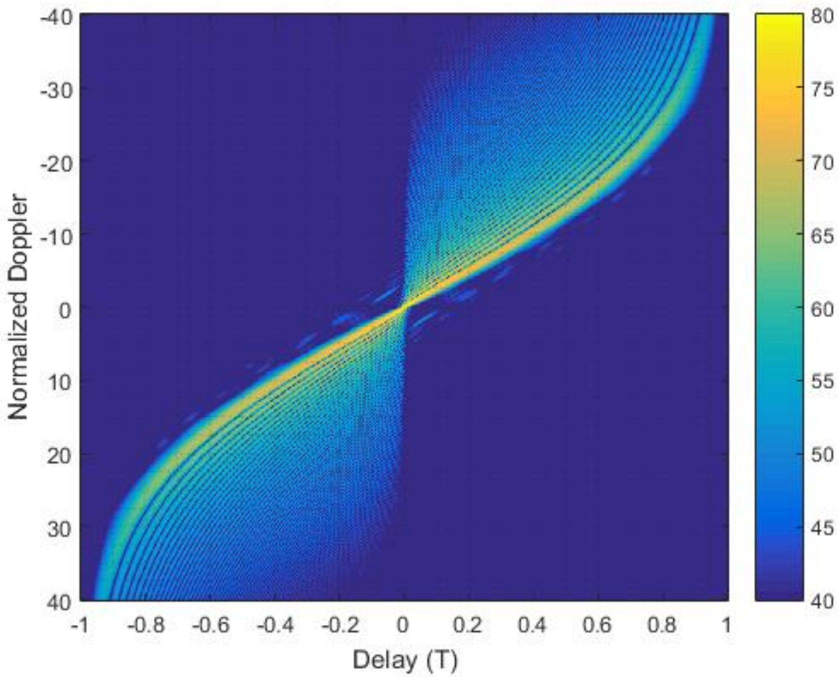
- Let's also examine the Instantaneous frequency of these multi-order PCFM implementations after optimization



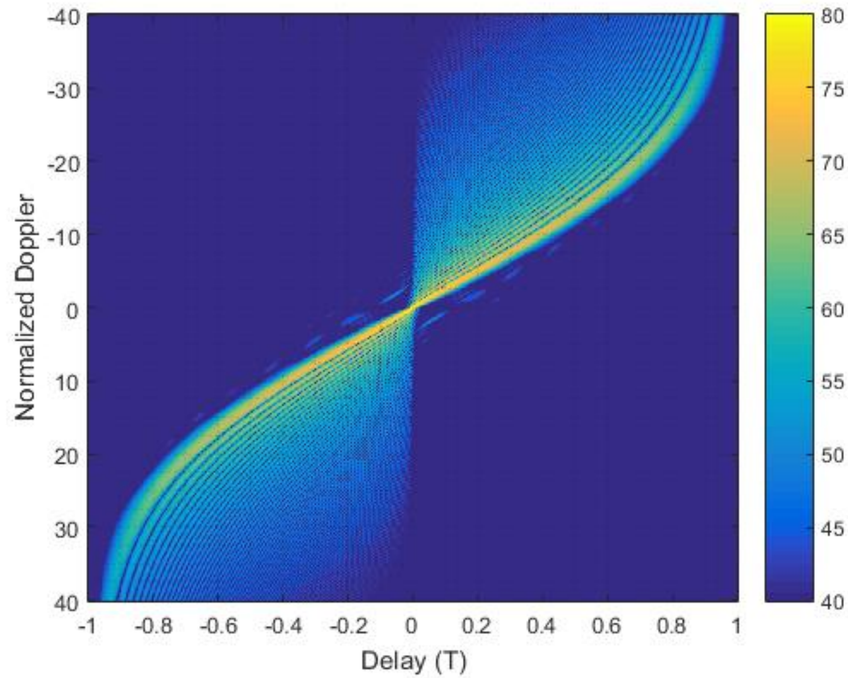
Instantaneous frequency of jointly 1st, 2nd & 3rd order versus of jointly 1st & 2nd order optimized waveforms

Multi-order PCFM (Combination)

- We also examine the ambiguity plots of these two multi-order PCFM waveforms



Ambiguity function of jointly 1st & 2nd order optimized waveforms with $BT = 100$



Ambiguity function of jointly 1st, 2nd & 3rd order optimized waveforms with $BT = 100$

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Summary of Multi-Order Performance

- The ordering of sequential optimization was based on the observation of how much each contributes to sidelobe reduction individually
- Based on these results, joint optimization appears marginally superior to sequential optimization for the multi-order PCFM implementations

PSL & ISL for sequential and joint optimization of multiple orders for *BT* = 100

	Joint 1st & 2nd orders	Joint 1st, 2nd & 3rd orders	Seq. 1st & 2nd orders	Seq. 1st, 2nd & 3rd orders
PSL (dB)	–51.1	–51.1	–50.7	–51.2
ISL (dB)	–66.4	–67.9	–66.0	–66.8

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Sparse Spectrum Allocation

Problem Setup

Problem Setup

- Let's view all the frequencies as measurements taken by the K radarlets in frequency domain

- The frequency measurements can be represented by the following measurement model:

$$\mathbf{v} = \mathbf{H}\boldsymbol{\gamma} + \mathbf{n}$$

$$\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, .., \mathbf{h}_i, \dots], i = 1 \dots . M$$

- $\mathbf{H}$ is the linear operator that relates the radar propagation to the resolution cell γ_i and back to the receiver

- This is the well known Linear model

Cramèr-Rao Bound (CRB)

- We want to perform estimation of the radar range profile $\boldsymbol{\gamma} = [\gamma_1, \gamma_2, \dots \dots, \gamma_m]$ from measurements made by the radarlet array
- Now, CRB provides a lower bound on estimation error variance for any unbiased estimator
- Also, CRB is equal to the inverse of the Fisher Information matrix **J** of the measurements:

$$\mathbf{J} = \left(\mathbf{H}'\mathbf{K}_{\mathbf{n}}^{-1}\mathbf{H} + \mathbf{K}_{\boldsymbol{\gamma}}^{-1}\right)$$

$\mathbf{K}_{\boldsymbol{\gamma}}$: A prior Covariance matrix of the vector $\boldsymbol{\gamma}$

$\mathbf{K}_{\boldsymbol{n}}$: Noise Covariance matrix due to to the measurements noise vector **n**

Cramèr-Rao Bound (CRB)

- For an efficient estimator such as the Minimum Mean Square Error estimator (MMSE), when applied to a Linear model, the error covariance $\mathbf{K}_{\boldsymbol{\varepsilon}}$ will be equal to the CRB

- Next, let's denote the Fisher Information matrix from *K* radar frequency measurements as $\mathbf{J}_K$

- Also, let's denote the Fisher Information matrix from (*K*-1) radar frequency measurements as $\mathbf{J}_{K-1}$

Marginal Fisher Information

- Therefore, for the k^{th} frequency measurement, the Marginal Fisher Information (MFI) matrix is defined as the nonnegative definite matrix $\Delta\mathbf{J}(K)$:

$$\Delta\mathbf{J}(K) = \mathbf{J}_{K-1}^{-1} - \mathbf{J}_{K}^{-1}$$

- From $\Delta\mathbf{J}(K)$, the MFI computed from the k^{th} frequency measurement is given as:

$$\begin{aligned} MFI &= \frac{1}{M} \times Tr\left(\Delta\mathbf{J}(K)\right) \\ &= \frac{1}{M} \times \left[Tr\left(\mathbf{J}_{K-1}^{-1}\right) - Tr\left(\mathbf{J}_{K}^{-1}\right)\right] \\ &= \frac{1}{M} \times \left[Tr\left(\mathbf{K}_{\varepsilon(K-1)}\right) - Tr\left(\mathbf{K}_{\varepsilon(K)}\right)\right] \end{aligned}$$

Marginal Fisher Information

- The MFI can be viewed as a measure of the unique or new information provided after adding the k^{th} measurement

- The new information will help to further reduce the uncertainty in estimating the radar range profile $\boldsymbol{\gamma}$

- In another words, the error variances within $\mathbf{K}_{\varepsilon}$ will be reduced with the new information

Sparse Spectrum Allocation

- Assuming that the contiguous spectrum consists of N radar frequencies & using the MFI as an optimization metric, a Sparse Spectrum Allocation algorithm can be developed for determining:
  - Locations of K out of N possible radarlet frequencies ($K < N$) that provides the least estimated error variances for that value of K
  - Optimization process (OP) is performed for one frequency at a time and will complete one iteration when all K radarlet frequencies are determined
  - OP can also be performed for one group of Q frequencies at a time ($K = P \times Q$) and will complete one iteration when all P groups of radarlet frequencies are determined

Sparse Spectrum Allocation

- The algorithm will continue in its iterations until no single frequency location or a group of frequency locations can be changed further to obtain additional MFI

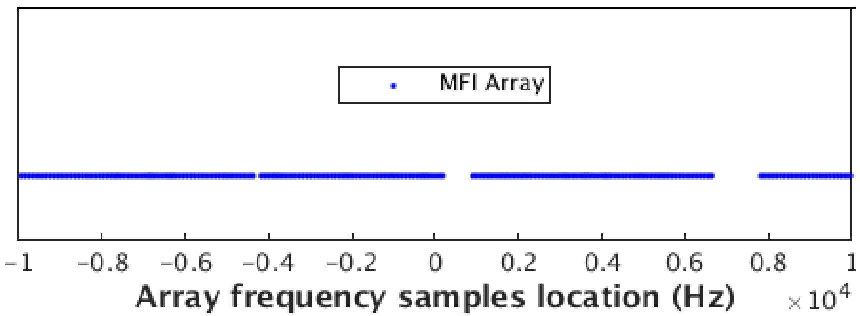
- The spectrum corresponding to the remaining $(N - K)$ radar frequencies can then be released for reuse

Sparse Spectrum Allocation

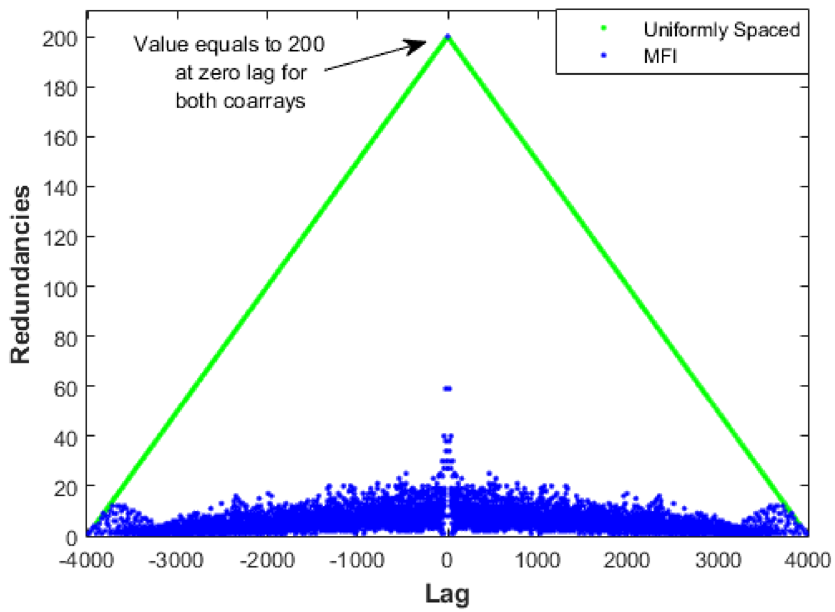
Simulation Results

Sparse Spectrum Allocation (SSA) Results

- Using the MFI measure as the metric of optimization, the sparse frequency array obtained for single frequency location insertion (1st approach) is as shown below for 50% spectrum usage



Frequency locations for 50% of spectrum usage



Coarrays from Sparse frequency array and Uniformly-spaced frequency array

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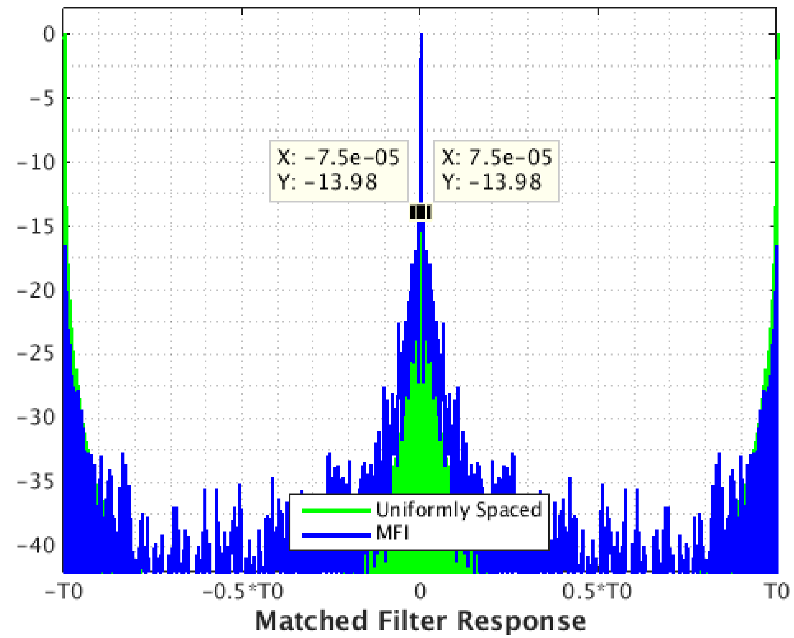
Matched Filter Response of SSA Results

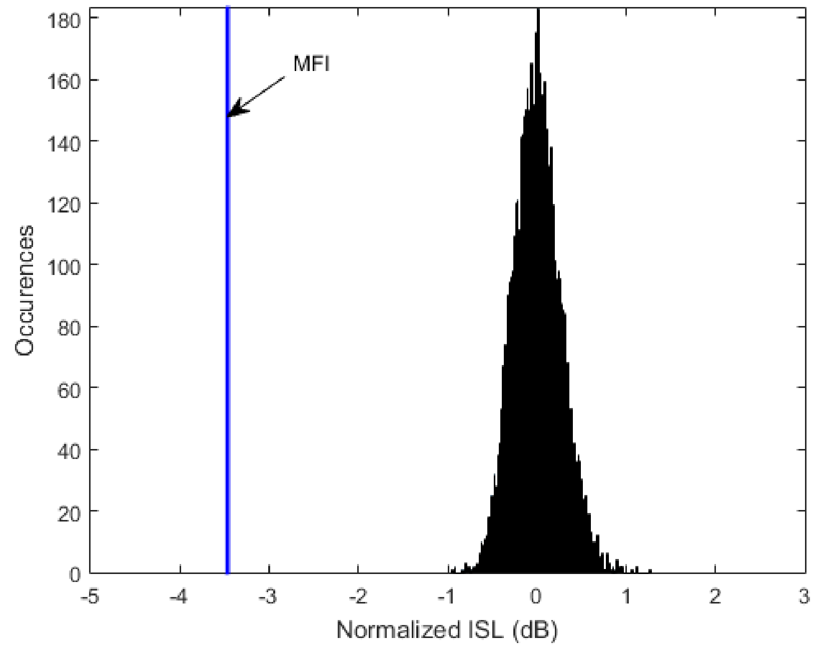
- To investigate the estimation error variances obtained using the previous sparse frequency array, we perform a Matched Filter operation
- Resulting plot is analogous to beam pattern obtained using Delay-Sum beamformer as weight vector
- Although it has higher side-lobes compared to uniformly-spaced sampling, but there are no grating lobes



Matched Filter response from Sparse frequency array versus Uniformly-spaced frequency array

ISL of SSA Results versus Randomly-spaced

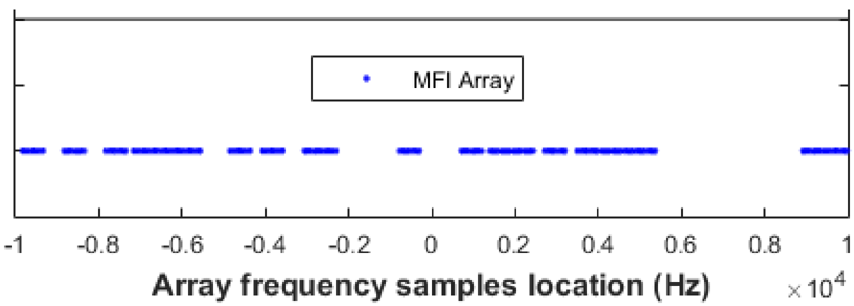
- Next, the Integrated Sidelobe level (ISL) obtained from the SSA is benchmarked against that obtained from a randomly-spaced frequency array
- Results obtained from 10000 trials of randomly-spaced frequency array are plotted using a histogram
- The ISL obtained from SSA is at least 13 σ away from the mean value of the randomly-spaced frequency array



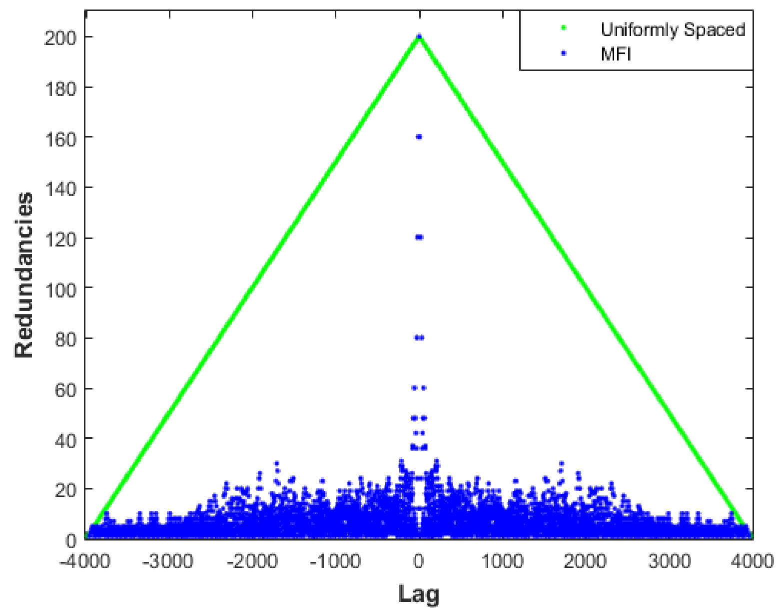
Error variances from Sparse frequency array versus randomly-spaced frequency array

SSA Results (Block implementation)

- To improve the utilization of the unused spectrum, the K radarlet frequencies is grouped into frequency blocks of equal sizes (2nd approach) and results are shown below for 50% spectrum usage



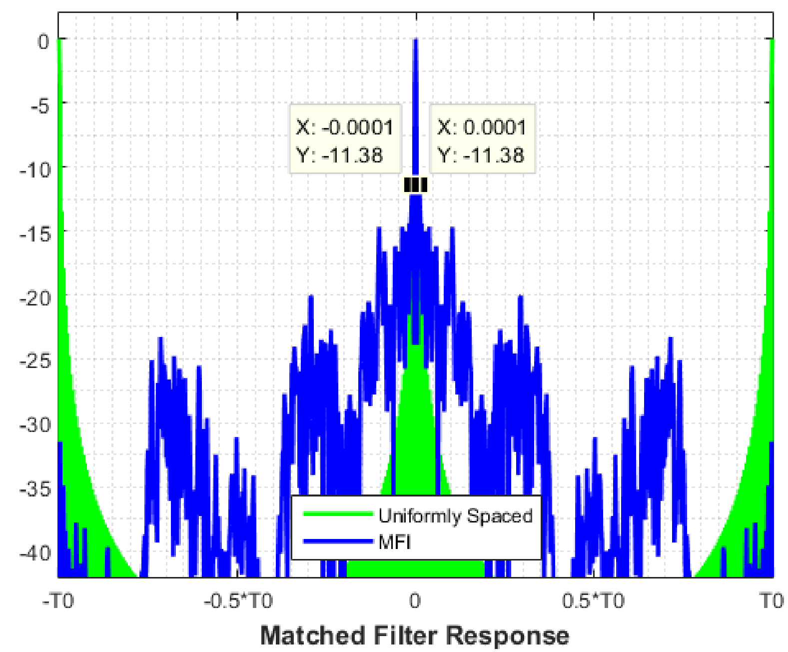
Block size of 1.25% each (50% of spectrum usage)



Coarrays from Sparse frequency array (block implementation) and Uniformly-spaced frequency array

SSA Results (Block implementation)

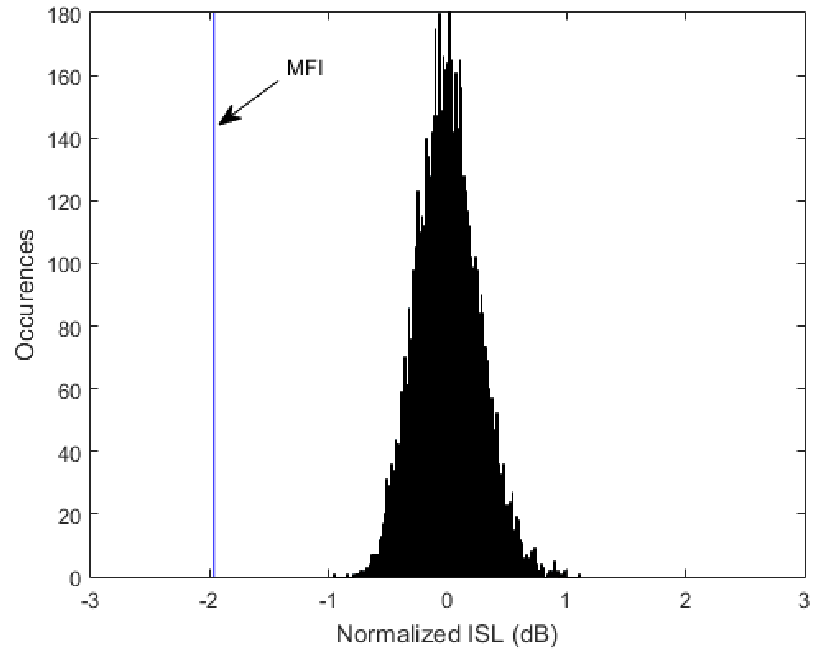
- Likewise, we perform the Matched Filter operation on the results obtained using the approach of frequency block implementation
- Compared to using single frequency insertion, the block implementation suffers from additional PSL and ISL degradation
- However, there are again no grating lobes as compared to uniformly-spaced frequency implementation



Matched Filter response of Sparse frequency array (block implementation) versus Uniformly-spaced frequency array

Sidelobe of SSA Results versus Randomly-spaced

- Again, the ISL from Block implementation of SSA is benchmarked against that from a randomly-spaced frequency array
- Results obtained from 10000 trials of randomly-spaced frequency array are again plotted using a histogram
- The ISL obtained from Block SSA implementation is still 7.66 σ away from the mean ISL value of randomly-spaced frequency array



Error variances from Block-based SSA results versus randomly-spaced frequency array

Summary of SSA Performance

- Results obtained from constructing the sparse frequency measurement array model using SSA algorithm indicates that this approach is viable as :
  - Range resolution is still preserved even when using 25.0% of the original spectrum at the expense of sidelobe degradation
  - Coarray derived has features of a low-redundancy linear array (LRLA)
  - The sidelobe performance obtained from both single-frequency location insertion and block-frequency location insertion approaches are much superior compared to that from random insertion of these frequency locations

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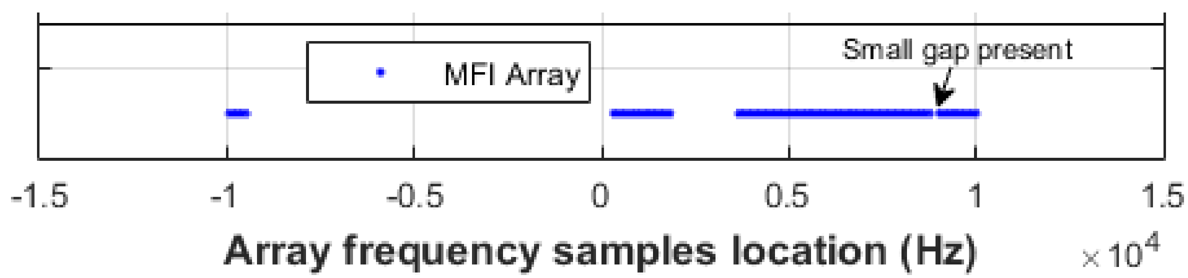
Conclusion

Application 1: Composite PCFM waveform

Waveform results

Applying SSA results to PCFM waveform design

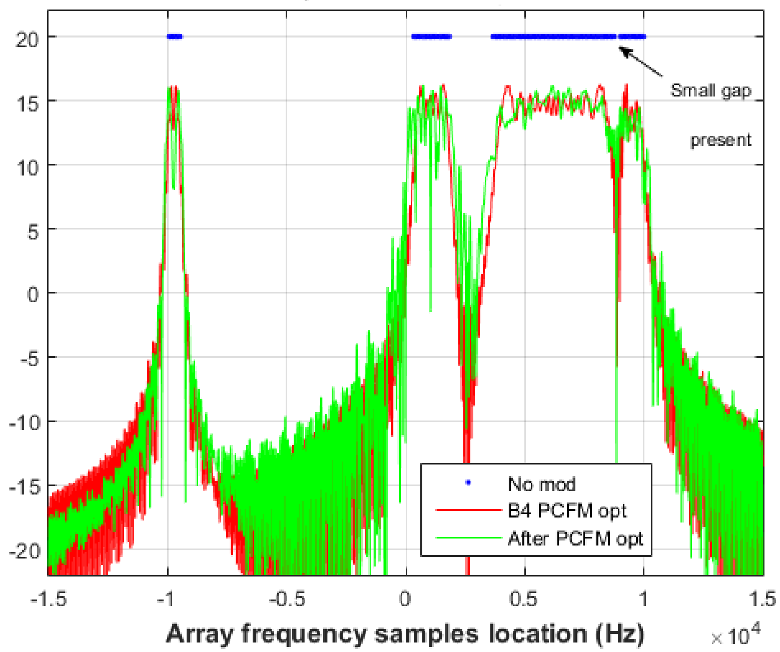
- In this example, SSA results for spectrum usage of 40% is used to generate the composite PCFM waveform
- From the SSA results shown below, it is seen that the spectral locations that are selected can be represented by 4 disjointed segments



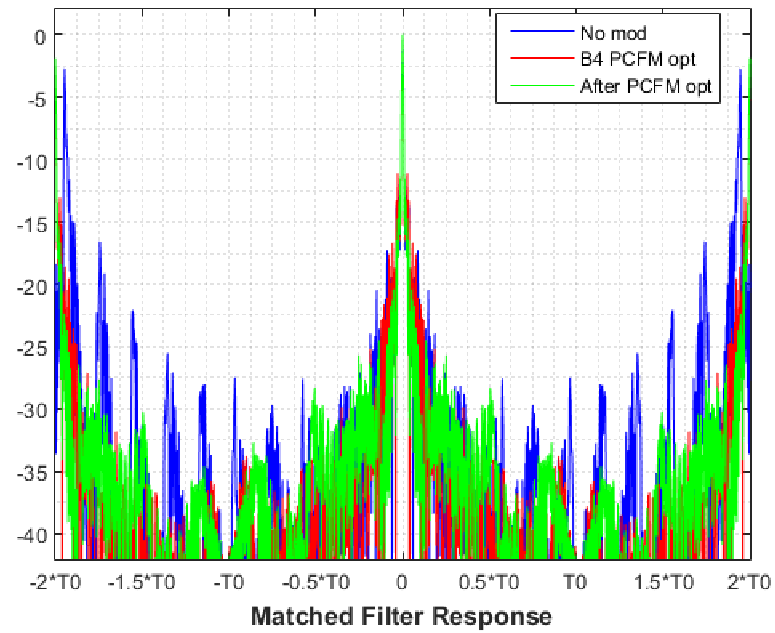
SSA results for spectrum usage of 40% and block size of 2.50%

Applying SSA results to PCFM waveform design

- Plots of Spectral Content and Autocorrelation function of the PCFM waveform before/after optimization are shown



Spectrum Content of composite PCFM waveform with *BT* = 200



Autocorrelation of composite PCFM waveform with *BT* = 200

Application 2: Radar Range Profile Estimation

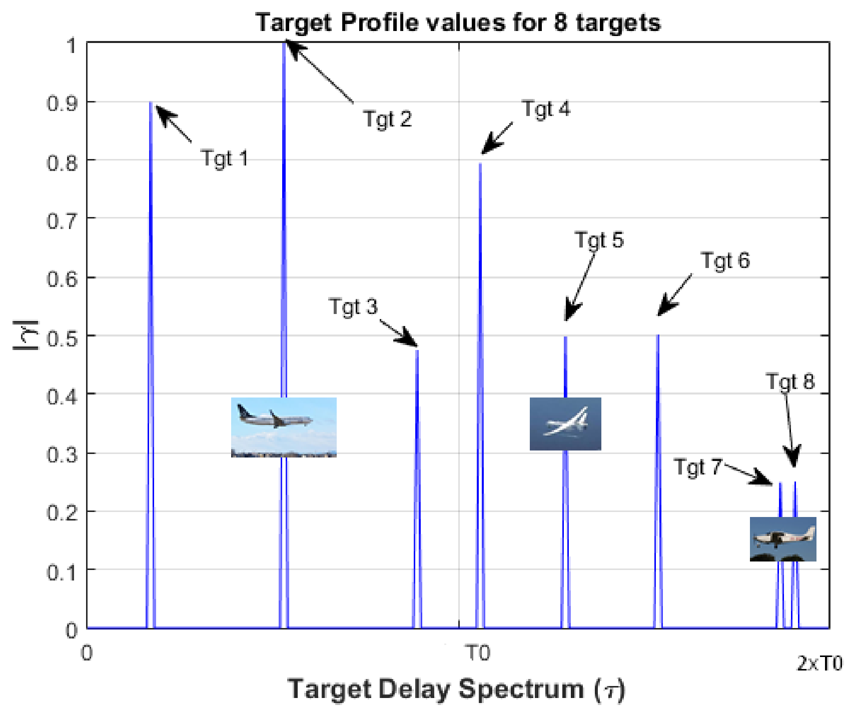
Simulation Results

Estimation of Radar Range Profile $\boldsymbol{\gamma}$

- Next, I will demonstrate the feasibility of using the SSA results for a radar range profile estimation application
- The problem setup is defined as low-density target distribution scenario (25 range cells containing complex target scattering coefficients out of M = 400 range cells)
- The remaining range cells are filled with very low-valued random Gaussian complex numbers
- Complex Gaussian noise is added to the measurements

Radar Range Profile

- Below is an example snapshot of the radar range profile before clutter and noise are added, i.e. low-density target distribution scenario



Iterative MMSE estimator

- For the radar range profile estimation application, an iterative MMSE estimator is developed for this application

- The equations for the MMSE estimator as well as computing the estimated range profile are as follows:

$$\mathbf{W}_{MMSE} = \mathbf{K}_\gamma \mathbf{H}' (\mathbf{H}\mathbf{K}_\gamma \mathbf{H}' + \mathbf{K}_n)^{-1}$$

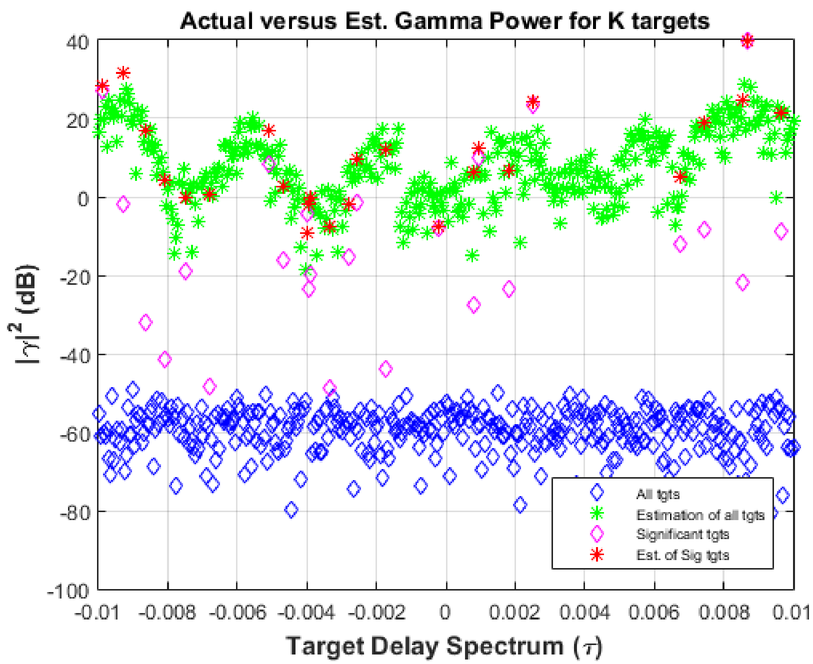
$$\tilde{\gamma} = \mathbf{W}_{MMSE} \mathbf{v}$$

Iterative MMSE estimator

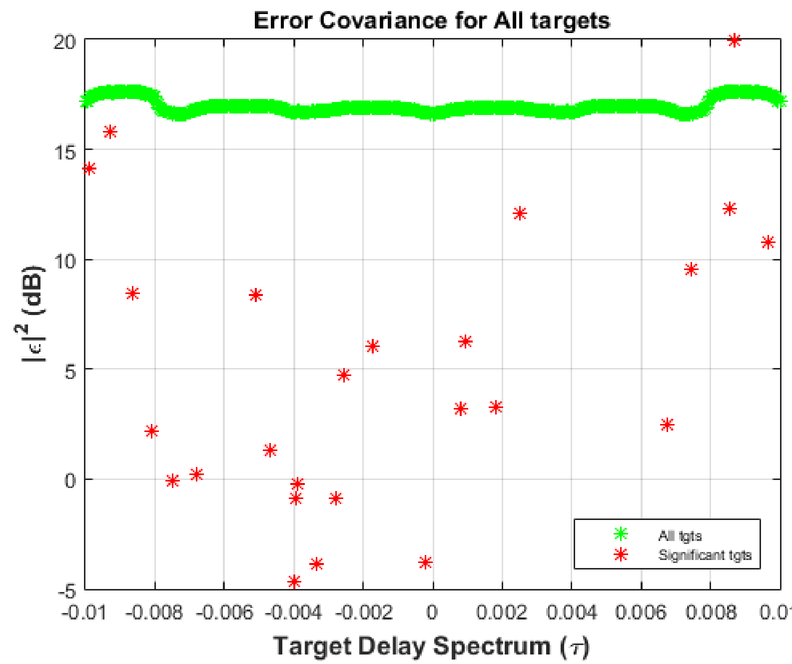
- In each P^{th} iteration, the $\hat{\gamma}_i$ from the range bins $i = 1,2, \dots \dots, M$ that contains the largest magnitude is identified and added to a set Θ containing range cells $[j_1, j_2, \dots \dots, j_{p-1}]$. Also, $i \notin \Theta$
- The $\hat{\gamma}_{j_q}$ for each element in this set Θ of range cells is assumed to be the true estimate of the scattering coefficient for that range cell
- Also, the a priori target covariance, $\mathbf{K}_\gamma$ for all locations is updated after each iteration

Initial MMSE estimation of γ using 50% spectrum

- At the 1st iteration, the results of the estimated $\hat{\gamma}$ is equivalent to performing a Matched Filter to each range cell within the range profile



Actual versus Estimated γ for 50% spectrum usage (1st iteration)

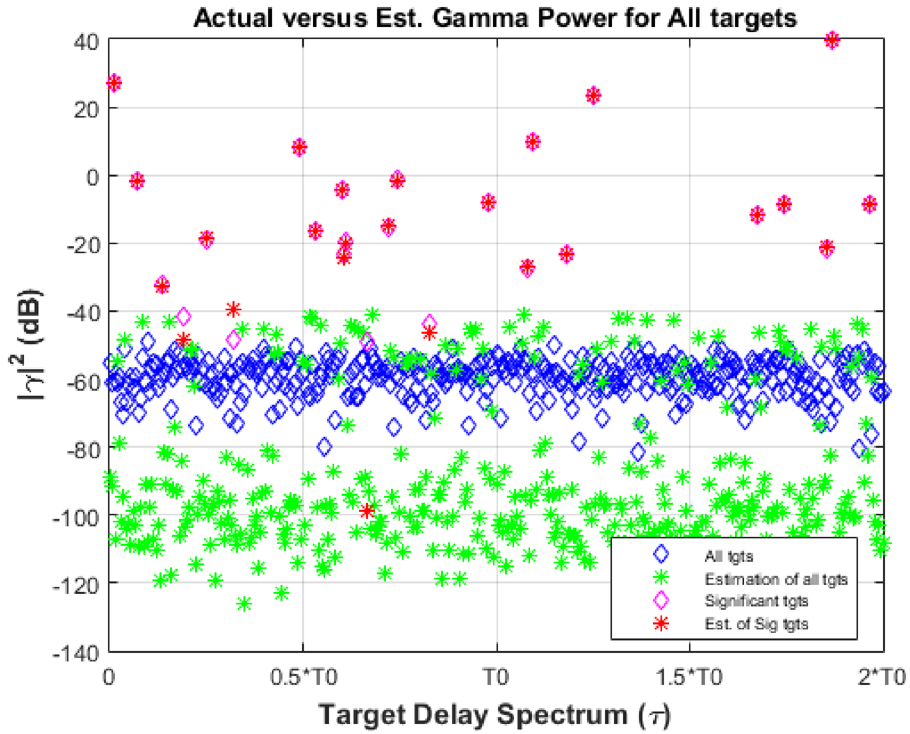


Error Covariance for all targets (1st iteration)

Final MMSE estimation of γ using 50% spectrum

- The Iterative MMSE filter is then reiteratively applied to obtain the final results of the estimated γ for all range cells in the unambiguous range

- Results demonstrates the viability of using the block implementation approach for this the SSA algorithm



Actual versus Estimated γ for 50% spectrum usage (block insertion implementation)

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Conclusions

- In this presentation, I have successfully illustrated a two-step approach to address the issues of both Spectrum Congestion and Spectrum Sharing between radar and communication systems

- The results obtained from this approach demonstrates that
  - 3-dB range resolution can be preserved while utilizing as low as 25.0% of the original spectrum represented as disjointed spectrum segments
  - The PCFM waveform implementation for these disjointed spectrum segments is able to prevent spectrum leakage to forbidden spectrum bands
  - It is viable to apply the frequency measurements obtained from such sparse spectrum usage to perform radar range profile estimation

Future Directions

- For Step 1 of the approach involving higher-order PCFM waveforms, the next step is to implement these waveforms in the lab using AWG and evaluate the measured output waveform's spectrum shape as well as performance in transmit-receive operations

- For Step 2 of the approach involving SSA algorithm and MFI, the next step is to apply this algorithm to a real-life system's spectrum usage so as to derive a sparse spectrum solution for this system

List of PhD. Publications

- S.D. Blunt, J. Jakabosky, P. McCormick, **Peng Seng Tan**, and J.G. Metcalf, "Holistic Radar Waveform Diversity," to appear in **Academic Press Library in Signal Processing Volume 7 (SIGP): Array, Radar and Communications Engineering**, eds. F. Gini, N.D. Sidiropoulos, M. Pesavento, and P.A. Naylor, Elsevier, 2017

- **Peng Seng Tan**, John Jakabosky, James M. Stiles and Shannon D. Blunt , "Higher-Order Representations of Polyphase-Coded FM Radar Waveforms: Relationships between various orders" to be submitted to **IET Radar, Sonar & Navigation** (after NRL release approval)

- **Peng Seng Tan**, James M. Stiles and Shannon D. Blunt, "Optimizing Sparse Allocation for Radar Spectrum Sharing," **2016 IEEE Radar Conference**, Philadelphia, Pennsylvania, May 02-06, 2016.

- **Peng Seng Tan**, John Jakabosky, James M. Stiles and Shannon D. Blunt, "On Higher-Order Representations of Polyphase-Coded FM Radar Waveforms," , **2015 IEEE International Radar Conference**, Arlington, Virginia, May 11-15, 2015.

- **Peng Seng Tan**, John Paden, Jilu Li, Jie-Bang Yan and Prasad Gogineni, "Robust Adaptive MVDR Beamforming for Processing Radar Depth Sounder Data," **2013 IEEE International Symposium on Phased Array Systems and Technology**, Waltham, MA, Oct 14-18, 2013

- Ulrik Nielsen, Theresa M. Stumpf, **Peng Seng Tan**, Prasad Gogineni, and Jorgen Dall, "Towards a Comprehensive Model of Ice Sheet Scattering Properties at VHF and P-band for Design and Optimization of Multichannel Ice Sounding Techniques," **2013 Progress in Electromagnetics Research Symposium (PIERS)**, Stockholm, Sweden, Aug 12-15, 2013

Thank you!

Questions?