

1

Optimal Space-Time Transmit Signal Design For Multi-Static Radar



Atulya Teja Deekonda Master's Thesis Defense January 31, 2005

COMMITTEE:

Prof. James Stiles (Chair) Prof. Glenn Prescott Prof. Muhammad Dawood



Outline



- Motivation
- Approach
- Algorithm-1(Collective Projection Algorithm)
- Algorithm-2 (Individual Projection Algorithm)
- Numerical Results
- Radar Model
- Results using the Radar Model
- Conclusions and Future Work



Objective



To develop Mathematical Algorithms that produce optimal space-time transmit signal, which would result in the responses from dissimilar targets to be as orthogonal as possible.





Motivation



- Why do we need the responses from dissimilar targets to be orthogonal to each other?
- REASON:
- \succ The response signal of a radar in vector-matrix form is given as,

$$\mathbf{r} = \sum_{i} \gamma_{i} \mathbf{H}_{i} \mathbf{s} + \mathbf{n} = \sum_{i} \gamma_{i} \boldsymbol{\rho}_{i} + \mathbf{n} = \mathbf{P} \boldsymbol{\gamma} + \mathbf{n}$$

Where, \mathbf{r} is the Measurement vector.

 \mathbf{H}_i is the Propagation matrix.

- γ_i is the scattering coefficient of target-i.
- s is the transmit signal.
- $\mathbf{\rho}_i$ is the normalized response vector.

$$\mathbf{\rho}_i = \mathbf{H}_i \mathbf{s}$$



KUZ

Estimators



• The response signal needs to be processed in order to estimate the scattering coefficients.

• <u>Matched Filter</u>

Most common and simple estimator.

>The Scattering Estimate of a given target pixel using the Matched filter is given by,

$$\hat{\gamma}_{i} = \gamma_{i} + \sum_{i \neq j} \gamma_{i} \frac{\boldsymbol{\rho}'_{i} \boldsymbol{\rho}_{j}}{\left|\boldsymbol{\rho}_{i}\right|^{2}} + \frac{\boldsymbol{\rho}'_{i} \mathbf{n}}{\left|\boldsymbol{\rho}_{i}\right|^{2}}$$

 \succ First term represents the desired estimate, second term is error due to clutter and the third term is error due to noise.

Maximizes Signal to Noise Ratio (SNR), does nothing to suppress error due to clutter.

 \triangleright A good estimate can be achieved if the responses from dissimilar target pixels are orthogonal to each other.

Hence in order to use the Matched filter and still get a good estimate we need,

$$\mathbf{\rho}'_i \mathbf{\rho}_j = 0$$
 where $i \neq j$





Approach



• <u>AIM</u> : To come up with a space-time transmit signal that produces the response signals from dissimilar targets to be as orthogonal as possible.

• <u>MINIMAX SOLUTION</u> :

- Minimize the maximum correlation between two dissimilar targets.
- > Try to find the "worst code" corresponding to the highest correlation.
- ➢ Find an orthogonal code to the worst code.
- \succ This code reduces the maximum correlation.

• Based on this Minimax solution, an Optimization criteria (\mathcal{X}) is designed that helps in selecting the worst transmit codes.





Optimization Criteria \mathcal{X}



• A criteria is needed to help us select the worst codes and to come up with the desired transmit signal.

• The Optimization Criteria,

$$\chi = \frac{(\boldsymbol{\rho}'_{i} \boldsymbol{\rho}_{i} + \boldsymbol{\rho}'_{j} \boldsymbol{\rho}_{j} + \boldsymbol{\rho}'_{i} \boldsymbol{\rho}_{j} + \boldsymbol{\rho}'_{j} \boldsymbol{\rho}_{i})}{(\boldsymbol{\rho}'_{i} \boldsymbol{\rho}_{i} + \boldsymbol{\rho}'_{j} \boldsymbol{\rho}_{j})}$$

• Range of χ values is given by, $0 \le \chi \le 2$

 $\mathcal{X} = 0 \text{ or } 2 \rightarrow \mathbf{\rho}_i \text{ and } \mathbf{\rho}_j \text{ perfectly correlated.}$

 $\mathcal{X} = 1 \rightarrow \mathbf{\rho}_i$ and $\mathbf{\rho}_j$ perfectly orthogonal to each other.





Analysis of \mathcal{X}



• The optimization criteria in terms of the propagation matrices (**H**) is given as,

$$\chi = \frac{\mathbf{s}'(\mathbf{H}'_i \mathbf{H}_i + \mathbf{H}'_j \mathbf{H}_j + \mathbf{H}'_i \mathbf{H}_j + \mathbf{H}'_j \mathbf{H}_i)\mathbf{s}}{\mathbf{s}'(\mathbf{H}'_i \mathbf{H}_i + \mathbf{H}'_j \mathbf{H}_j)\mathbf{s}}$$

• Introducing two matrices A and B as,

$$\mathbf{A} = \mathbf{H'}_i \mathbf{H}_i + \mathbf{H'}_j \mathbf{H}_j + \mathbf{H'}_i \mathbf{H}_j + \mathbf{H'}_j \mathbf{H}_i$$
$$\mathbf{B} = \mathbf{H'}_i \mathbf{H}_i + \mathbf{H'}_j \mathbf{H}_j$$

• The optimization criteria can then be written as,

$$\chi = \frac{\mathbf{s'} \mathbf{A} \mathbf{s}}{\mathbf{s'} \mathbf{B} \mathbf{s}}$$





The C matrix



• Defining another matrix C, we can write χ as,

$$\chi = \frac{\widetilde{\mathbf{s}}' \mathbf{C} \widetilde{\mathbf{s}}}{\widetilde{\mathbf{s}}' \widetilde{\mathbf{s}}}$$

Where,
$$\mathbf{C} = \mathbf{B}^{-1/2} \mathbf{A} \mathbf{B}^{-1/2}$$
 and $\mathbf{\tilde{s}} = \mathbf{B}^{1/2} \mathbf{s}$

• The C matrix is a Positive-definite matrix, so Eigen analysis can be used.

• Representing C in its eigen values and eigen vectors we have,

$$\mathbf{C} = \sum_{n=1}^{\mathbf{N}} \lambda_n \hat{v}_n \hat{v}_n'$$

Where λ_n are the eigen values of the **C** matrix.

 \hat{v}_n are the eigen vectors of the **C** matrix.

$$\chi = \lambda_n = \frac{\hat{v}_n'(\mathbf{H}'_i \mathbf{H}_i + \mathbf{H}'_j \mathbf{H}_j + \mathbf{H}'_i \mathbf{H}_j + \mathbf{H}'_j \mathbf{H}_i)\hat{v}_n}{\hat{v}_n'(\mathbf{H}'_i \mathbf{H}_i + \mathbf{H}'_j \mathbf{H}_j)\hat{v}_n} = \frac{\hat{v}_n'\mathbf{C}\hat{v}_n}{\hat{v}_n'\hat{v}_n}$$





Correlation Coefficient (ξ)



• The Correlation Coefficient gives us the measure of similarity between two response signals.

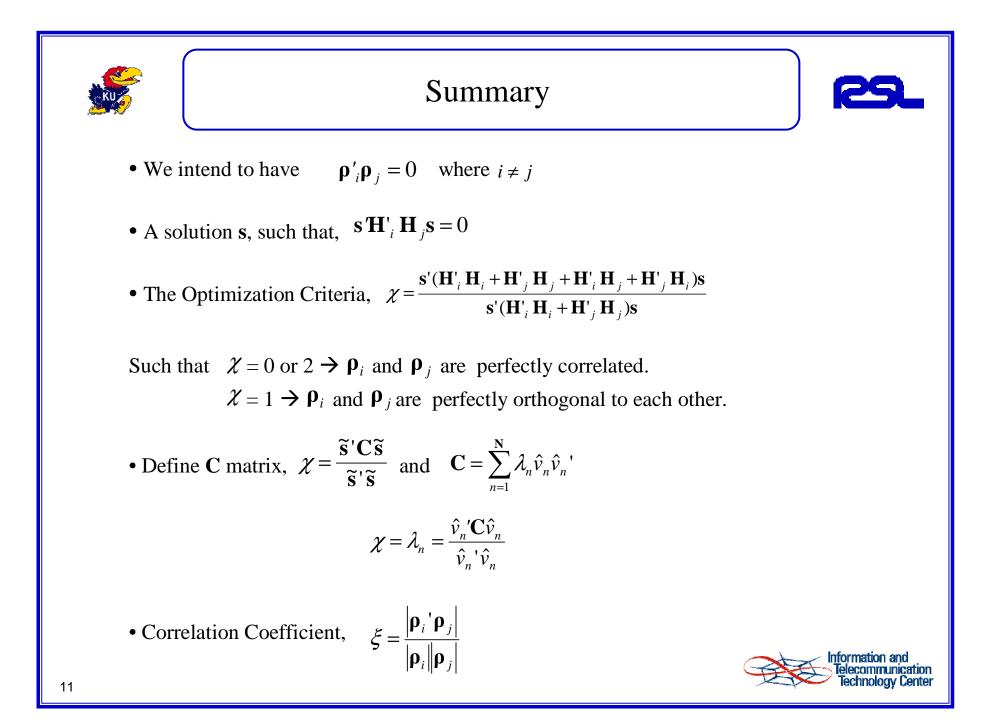
$$\boldsymbol{\xi} = \frac{\left| \boldsymbol{\rho}_{i} \cdot \boldsymbol{\rho}_{j} \right|}{\left| \boldsymbol{\rho}_{i} \right| \left| \boldsymbol{\rho}_{j} \right|}$$

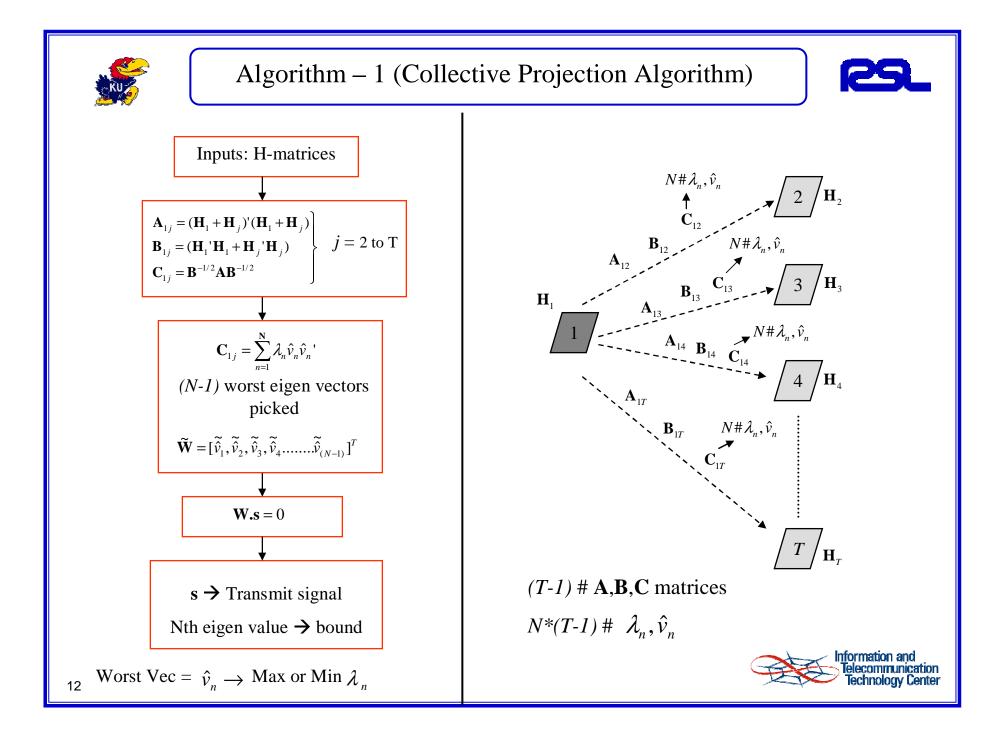
Minimize $Max{\{\xi\}} \rightarrow 0$

• Relation between
$$\chi$$
 and ξ

$$\chi = 1 - \operatorname{Re} \left\{ \frac{\boldsymbol{\rho}_i \cdot \boldsymbol{\rho}_j}{|\boldsymbol{\rho}_i| |\boldsymbol{\rho}_j|} \right\}$$
$$\chi = 1 - \boldsymbol{\xi} \to \operatorname{Im} \left\{ \boldsymbol{\rho}_i \cdot \boldsymbol{\rho}_j \right\} = 0$$
$$\chi \to 1; \ \boldsymbol{\xi} \to 0$$









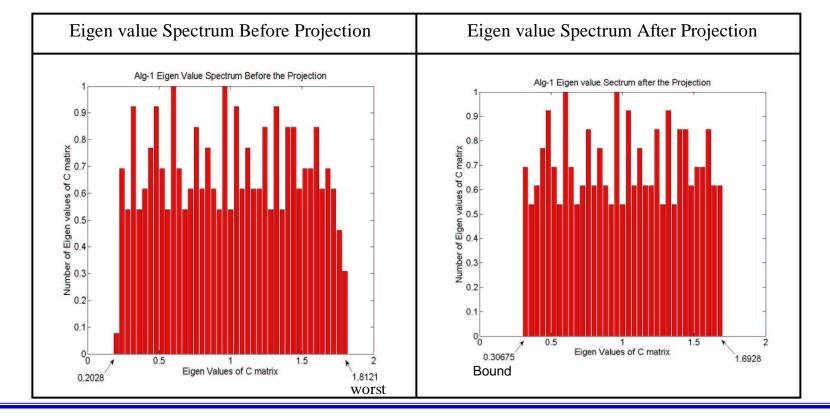
13

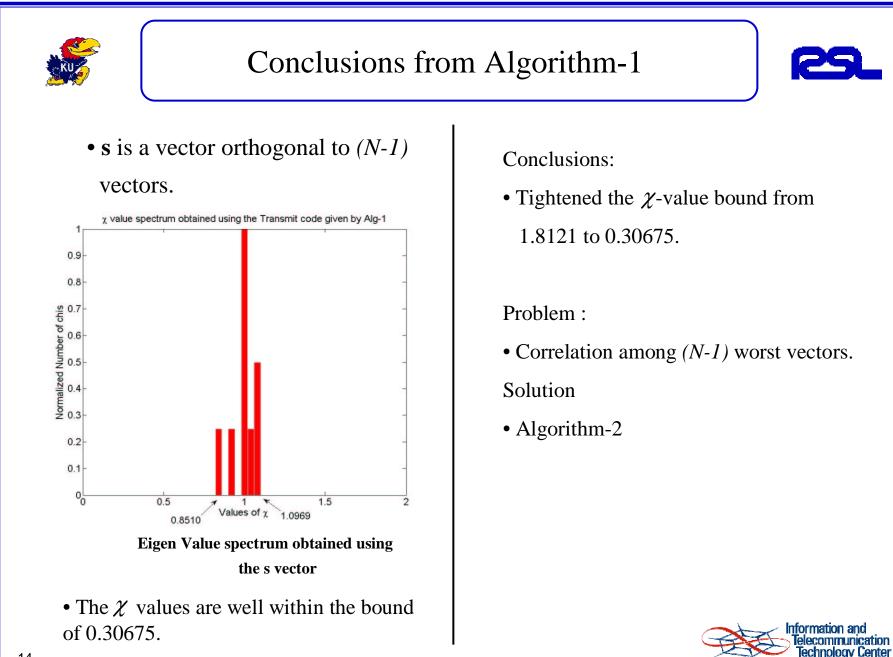
Numerical Analysis of Algorithm-1

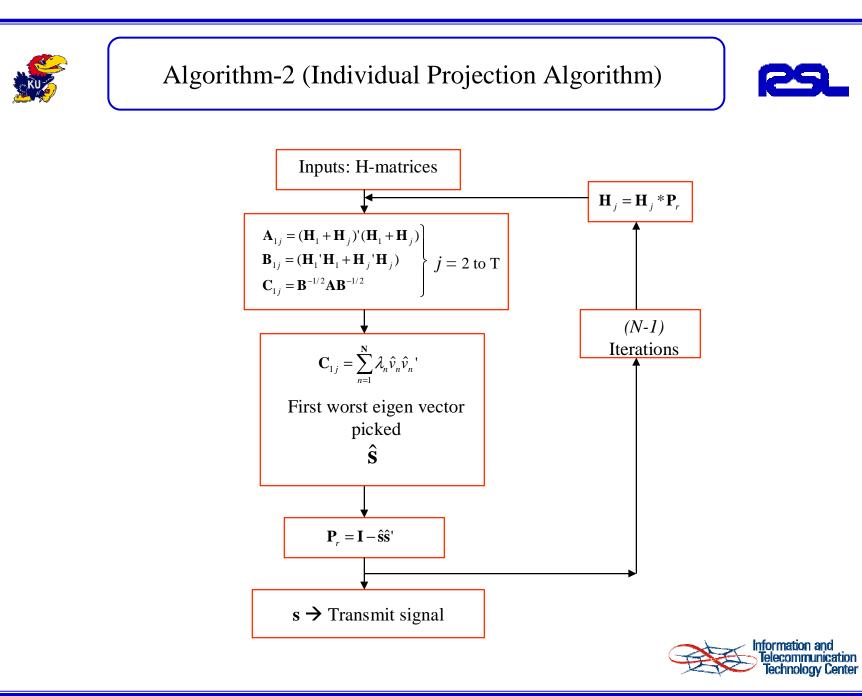


• The Propagation matrices (H-matrices) generated Randomly from a Gaussian Distribution of 0 mean and variance 1.

- Total Number of Measurements (M) = 100
- Total Dimensions of the Tx signal (N) = 40
- Total Number of Targets (T) = 10
- $\bullet \mathbf{H} \xrightarrow{} M \, x \, N \, x \, T$







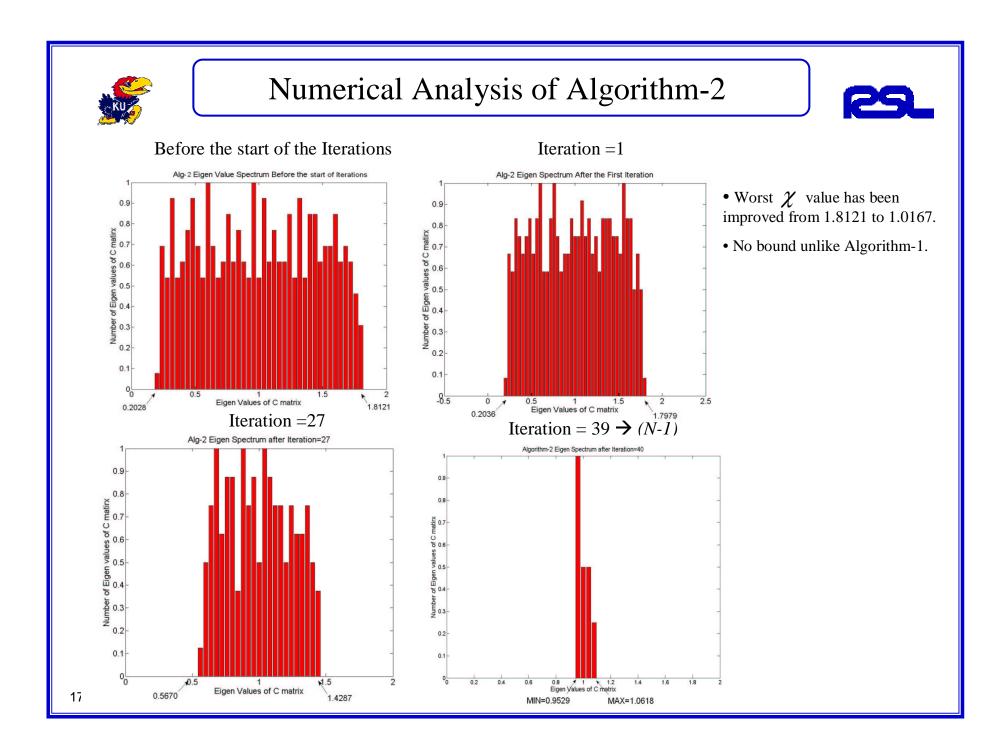


Numerical Analysis of Algorithm-2



• The Propagation matrices (**H**-matrices) generated Randomly from a Gaussian Distribution of 0 mean and variance 1.

- Total Number of Measurements (M) = 100
- Total Dimensions of the Tx signal (N) = 40
- Total Number of Targets (T) = 10
- The propagation matrices are given as inputs to the Algorithm.





Comparison of Alg-1 and Alg-2



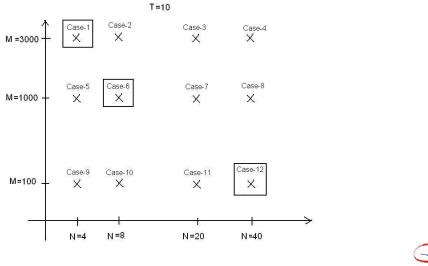
• Performances of Algorithm-1 (Collective Projection) and Algorithm-2 (Individual Projection) are compared.

• The Propagation Matrices (H-matrices) are generated randomly from 0 mean and variance 1 Gaussian Distribution.

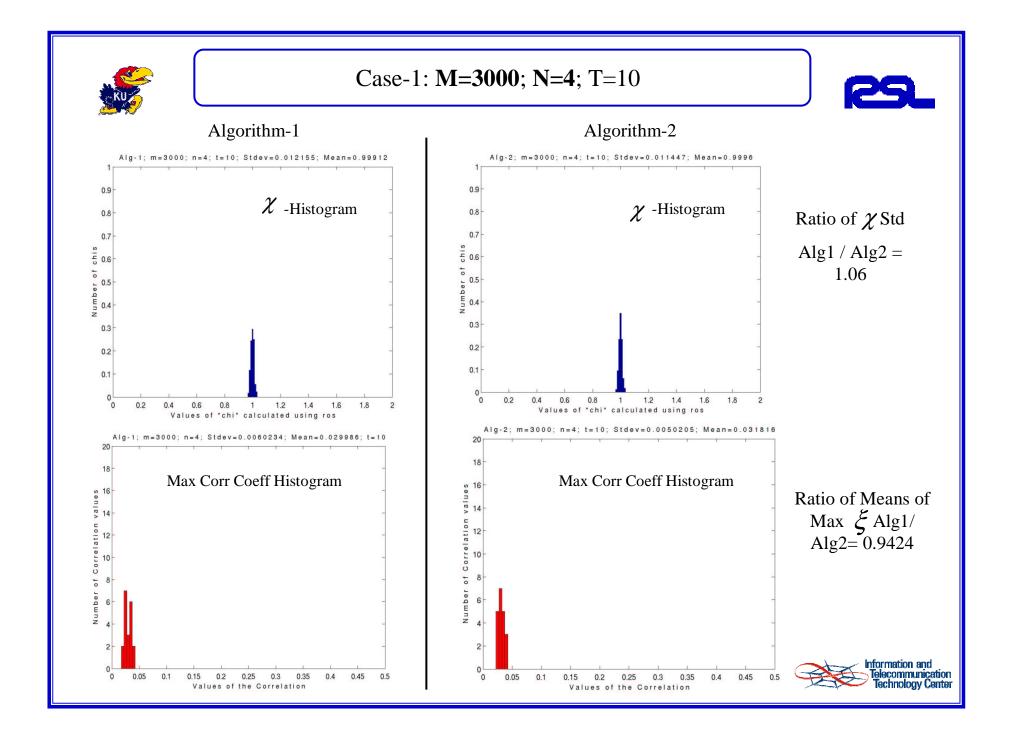
- Monte Carlo Loop = 20
- The histograms of the χ values and Maximum Correlation Coefficient values are compared

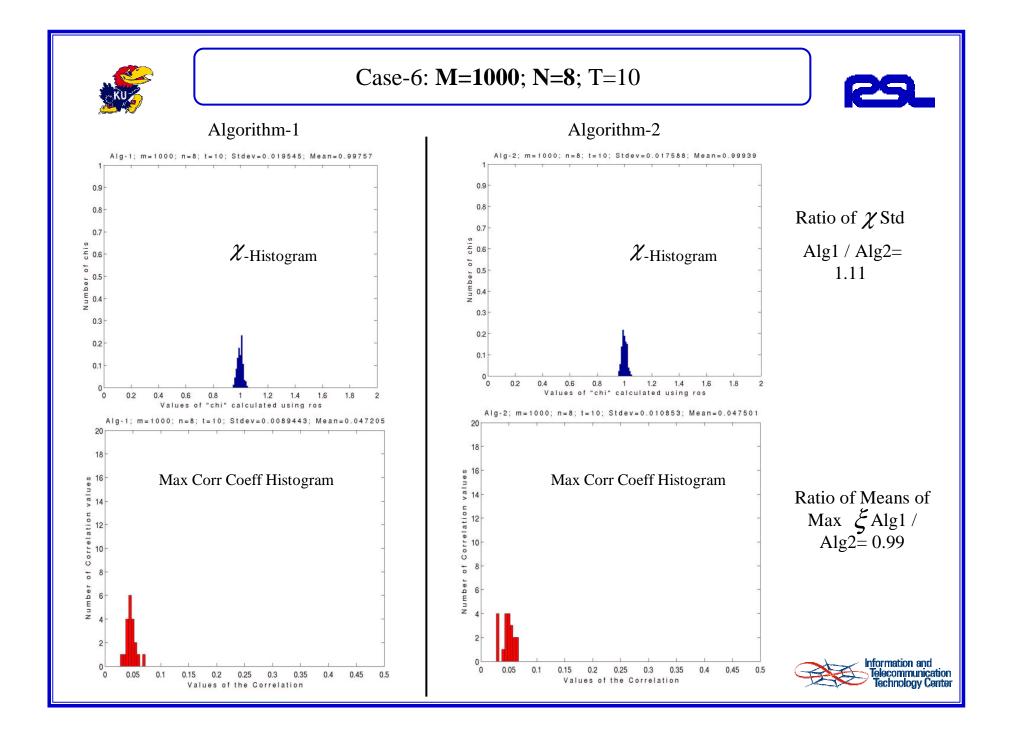
$$\chi = \frac{\widetilde{\mathbf{s}}' \mathbf{C} \widetilde{\mathbf{s}}}{\widetilde{\mathbf{s}}' \widetilde{\mathbf{s}}} \qquad \xi = \frac{|\boldsymbol{\rho}_i' \boldsymbol{\rho}_t|}{|\boldsymbol{\rho}_i| |\boldsymbol{\rho}_t|}$$

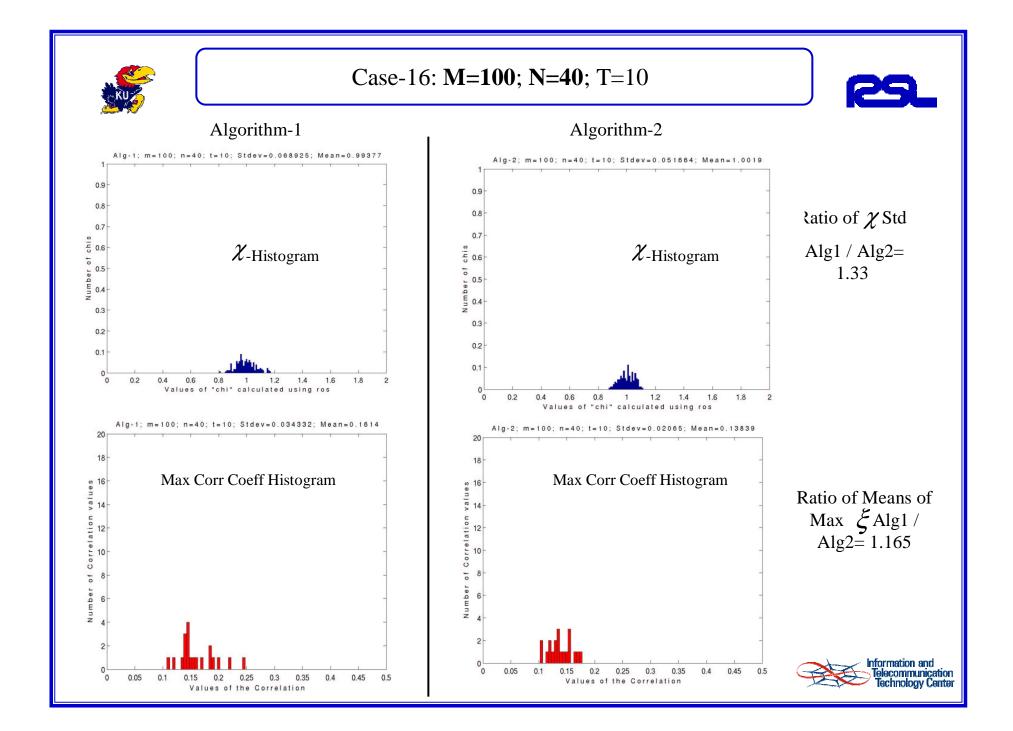
Different Cases for which the Algorithms have been compared

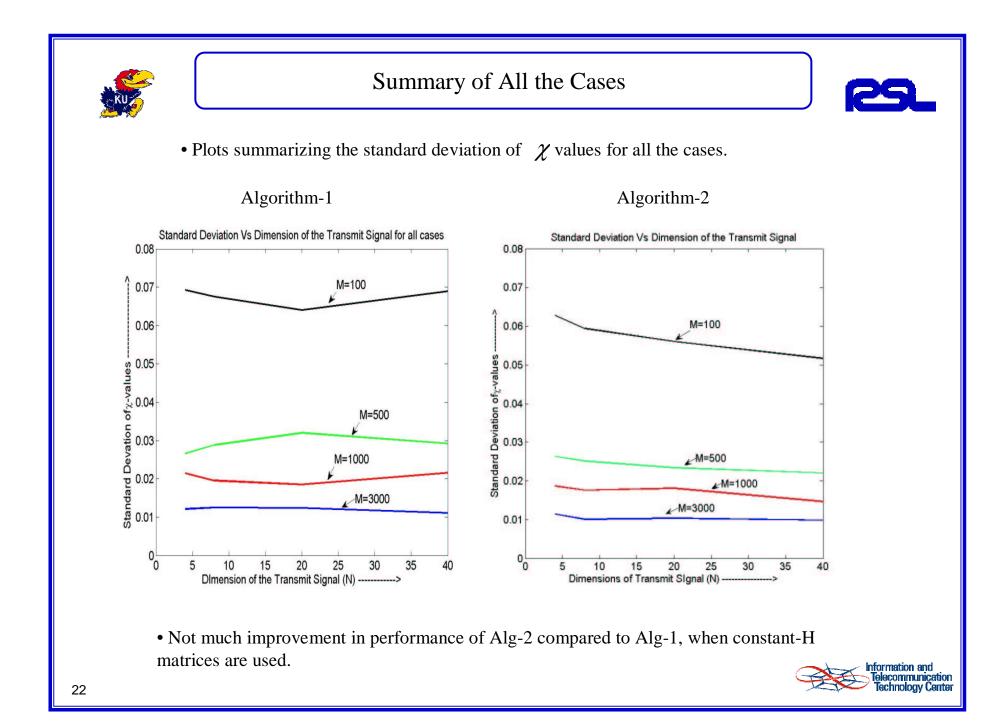


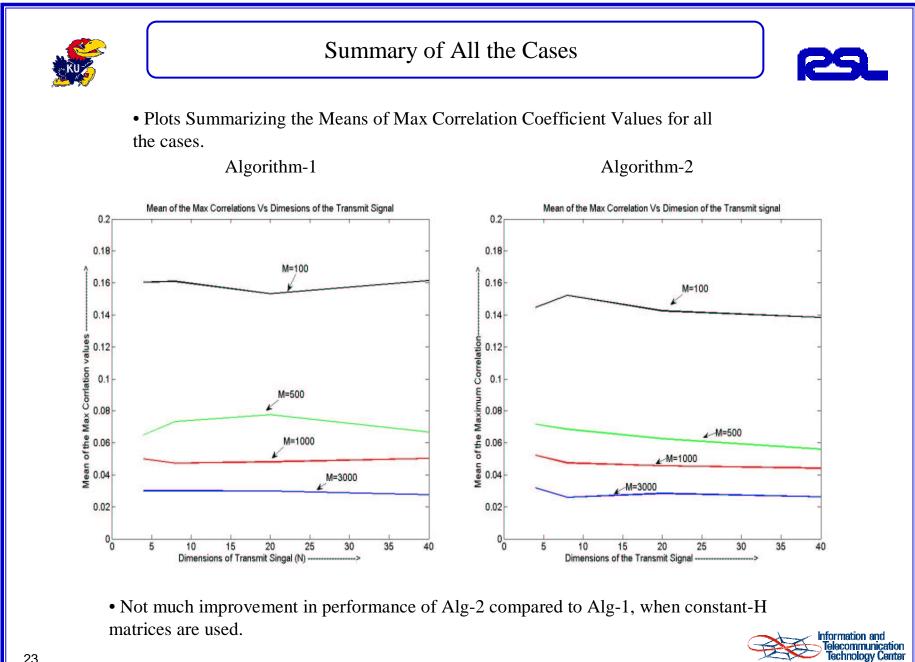


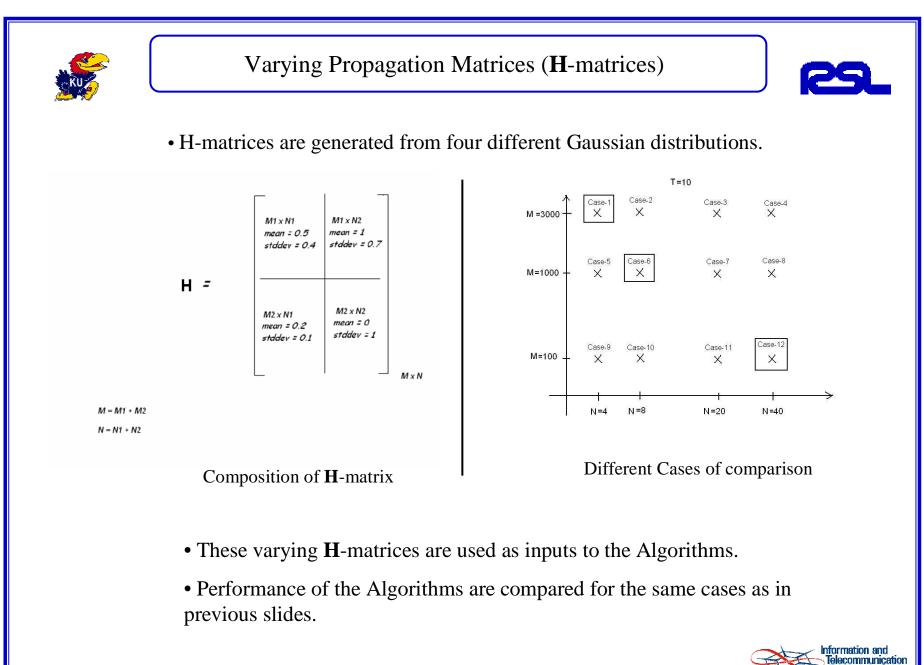




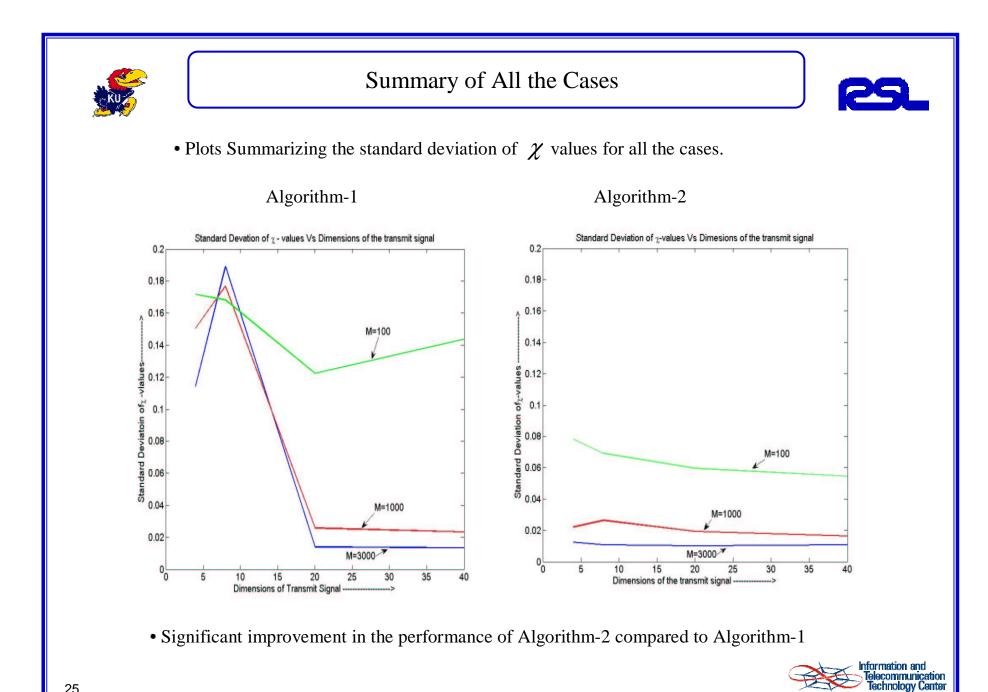


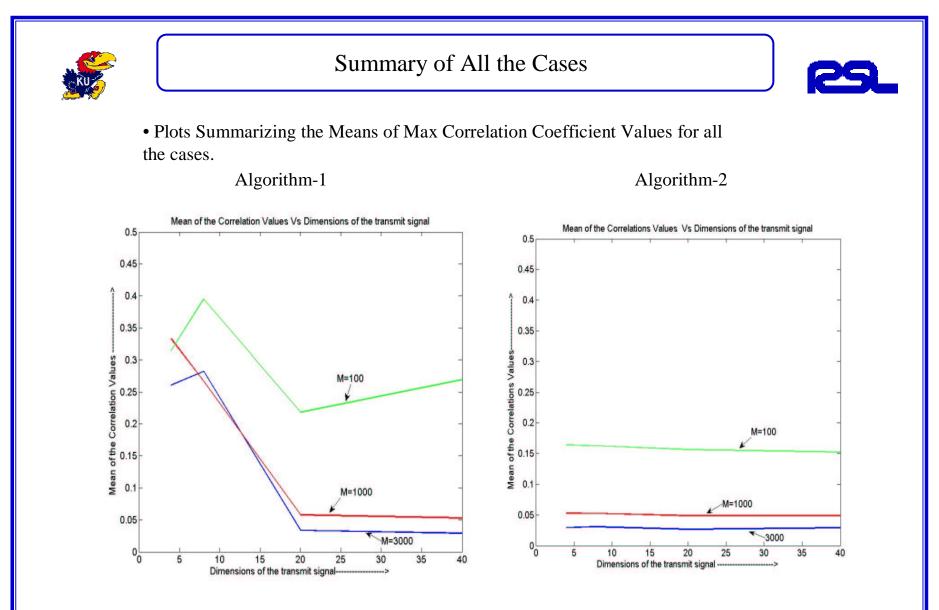






1010av Centei





• Significant improvement in the performance of Algorithm-2 compared to Algorithm-1







• Higher the total number of measurements (M), and higher the total number of dimensions (N), better is the performance of both Algorithms.

• The performance of Algorithm-2 largely depends on the structure of **H**-matrices.

• Hence, the performance of Algorithm-2 is either same or better than Algorithm-1.







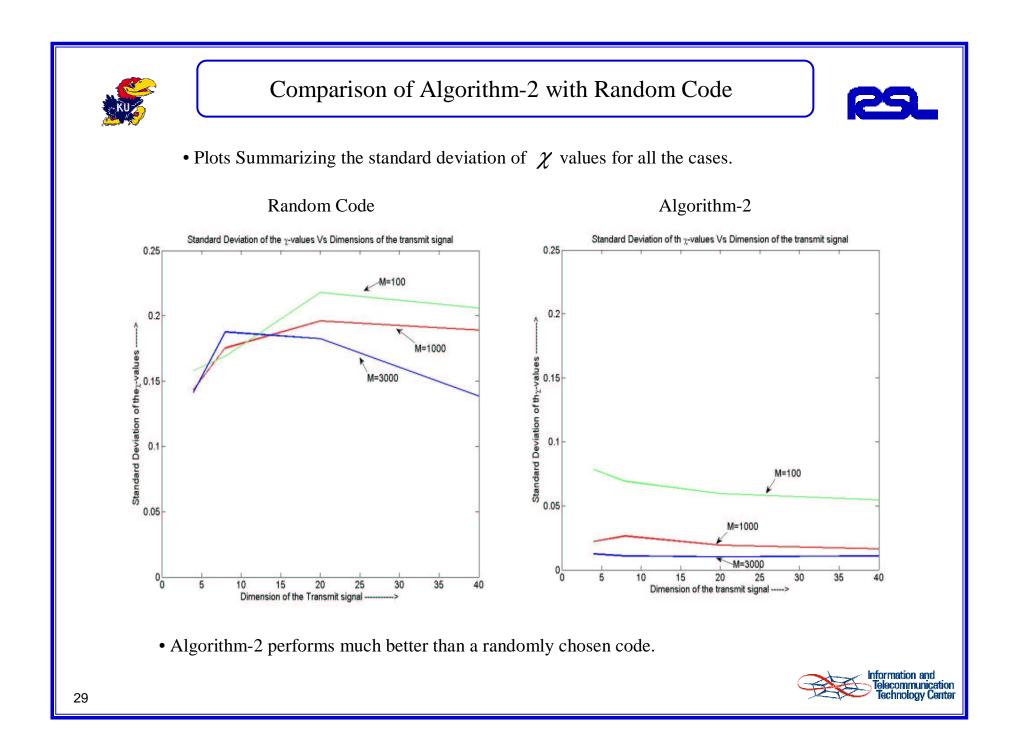
Question : How good is the result given by Algorithm-2 in general ??

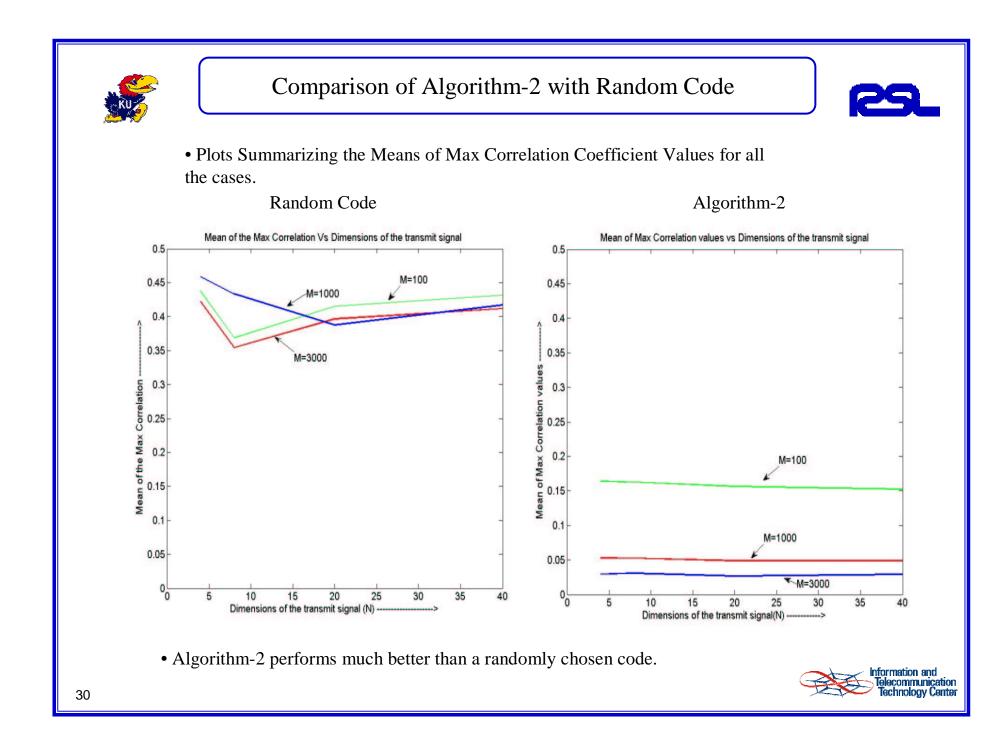
Or

Is there a yardstick to measure the performance of the Algorithm?

Answer : Compare the performance with *Random code* and code given by *Genetic Algorithm*











- Genetic Algorithm (Master's Project by Fernando Soto)
- ➤ A group of possible codes is considered.
- \succ A new group of fit solutions is selected out of the old group and are combined.
- \succ This process is continued till there is no improvement in the criteria.
- Varying-**H** matrices were used with M=100; N=8; T=10
- Criteria given to the Genetic Algorithm
- **s** solution that minimizes the largest β_t value.

$$\beta_{t} = \frac{|\rho_{1}'\rho_{t}|^{2}}{|\rho_{1}|^{2}|\rho_{t}|^{2}} = \frac{|\mathbf{s}'\mathbf{H}'_{1}\mathbf{H}_{t}\mathbf{s}|^{2}}{|\mathbf{H}_{1}\mathbf{s}|^{2}|\mathbf{H}_{t}\mathbf{s}|^{2}} \qquad \text{Where,} \quad t \in \{2, 3, 4, \dots, T\}$$





Comparison with Genetic Algorithm



Genetic Algorithm

Т	$oldsymbol{eta}_t$		
Target-2	0.0001	Target-6	0.0002
Target-3 0.0	0.0000	Target-7	0.0002
		Target-8	0.0002
Target-4	0.0002	Target-9	0.0000
Target-5	0.0011	Target-10	0.0008

Т	$oldsymbol{eta}_t$		
Target-2	0.0162	Target-6	0.0003
Target-3	0.0021	Target-7	0.0175
	0.0002	Target-8	0.0019
Target-4	0.0092	Target-9	0.0057
Target-5	0.0041	Target-10	0.0124

Algorithm-2

Max $\beta_t = 0.0011 = -30$ dB

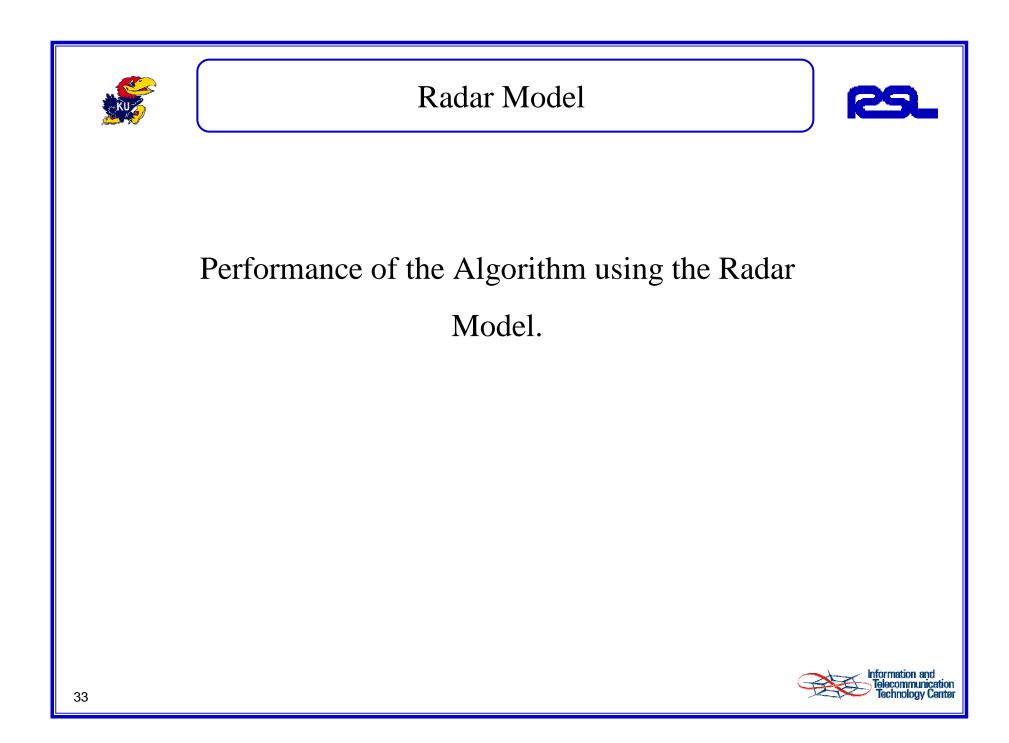
Max $\beta_t = 0.0175 = -18$ dB

• Genetic Algorithm is better by 12 dB compared to Algorithm-2

Disadvantages of Genetic Algorithm

- No mathematical basis for the Algorithm.
- Takes huge amount of time compared to Algorithm-2.
- Cannot be used where processing time is an important factor.







Radar Model



• The Radar Model has been defined in three major parts.

➤ Transmit signal.

≻ Target set

> Received Measurements.

• Transmit Signal

➢ Defined as a set of complex valued samples in a 5 dimensional space.

 $\overline{z}_{jk} = [x_j, y_j, z_j, t_k, \boldsymbol{\omega}_k]^T$

Where, $J \rightarrow$ total number of spatial samples (Total number of transmit elements).

 $K \rightarrow$ total number of temporal samples.





Radar Model



➤ Further, the transmit signal is defined as, a superposition of wide timewidth and wide bandwidth orthonormal basis functions.

> Slow time functions (P) and Fast time functions (Q)

 \succ Total # of orthonormal basis functions = PQ

 \succ The response vector is transformed as,

$\mathbf{\rho}_i = \mathbf{H}_i \mathbf{s} = \mathbf{H}'_i \mathbf{S}$

> The \mathbf{H}'_i matrices which relate the received signal to the weights of the basis functions are used as inputs to the Algorithm.

➢ Algorithm works to find the best weight vector for the basis functions.





Radar Model



• Target set

> A total of N_t number of targets are considered.

> A grid of $N_x x N_y$ is defined such that $N_t = N_x N_y$.

> The target position vector is defined in 4 dimensional subspace and is given as,

$$\overline{y}_t = [x_t, y_t, z_t, v_t]^T$$

> The spacing between the targets is set equal to the Doppler resolution and Range resolution in x and y directions respectively.





Radar Model



Received Measurements

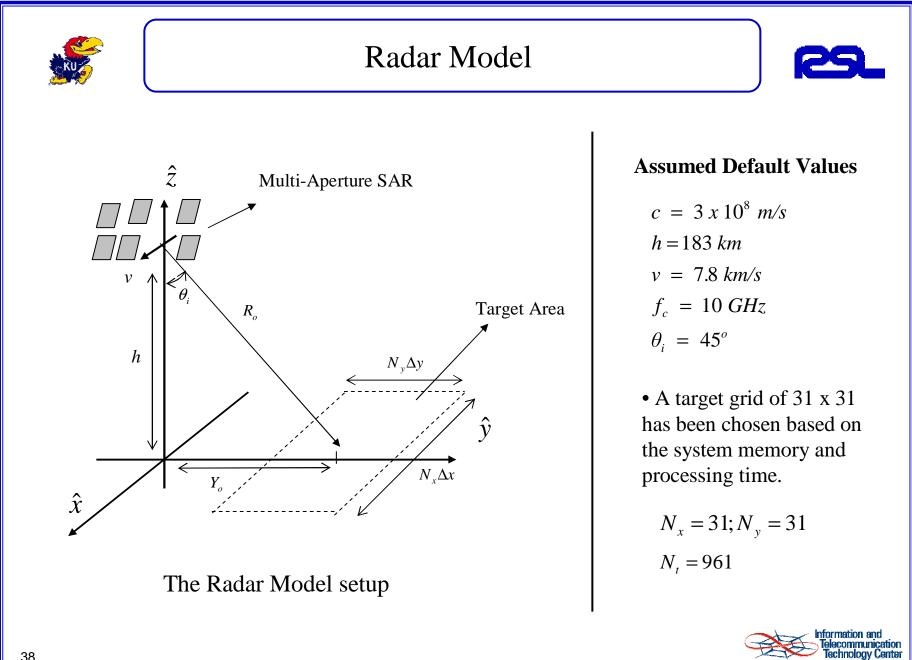
➤ Defined as a set of complex valued samples in a 5 dimensional space.

 $\overline{x}_{ik'} = [x_i, y_i, z_i, t_{k'}, \boldsymbol{\omega}_{k'}]^T$

Where, $I \rightarrow$ total number of spatial samples (Total number of Receive elements).

 $K' \rightarrow$ total number of temporal samples.



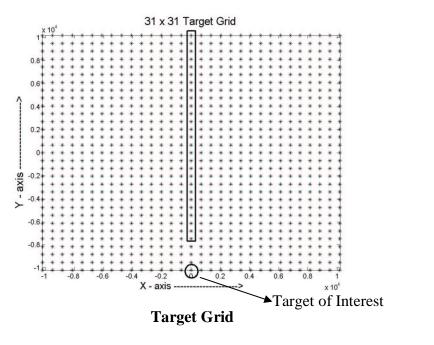




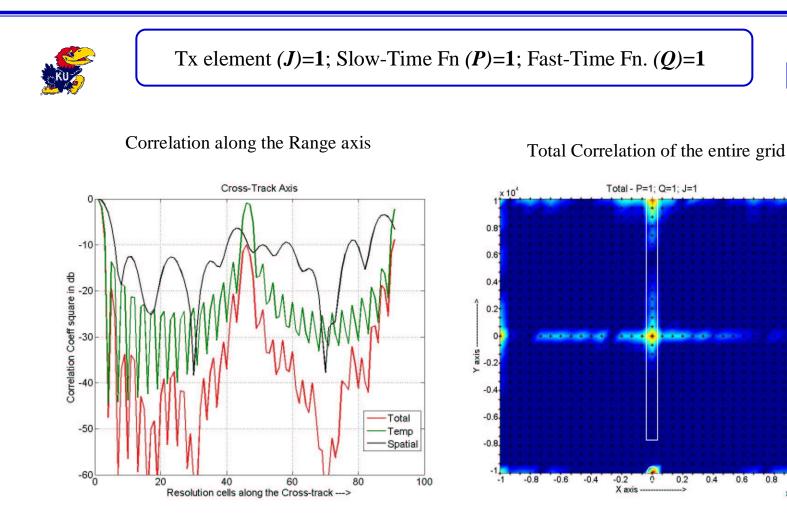
Analysis of the Algorithm using the Model.



- H-matrices from the Model are used as Inputs.
- •Total # of Transmit elements (J) = 1
- Total # of Receive elements (I) = 15
- Total number of slow-time functions (*P*) and Total number of fast-time functions (*Q*) P = 1; Q = 1 to P = 11; Q = 11
- Performance of the Algorithm with varying number of basis functions.







- Green \rightarrow Temporal; Black \rightarrow Spatial; Red \rightarrow Total
- Total Maximum Correlation = -9dB
- Not much freedom to the Algorithm.

-10

-15

-20

-25

-30

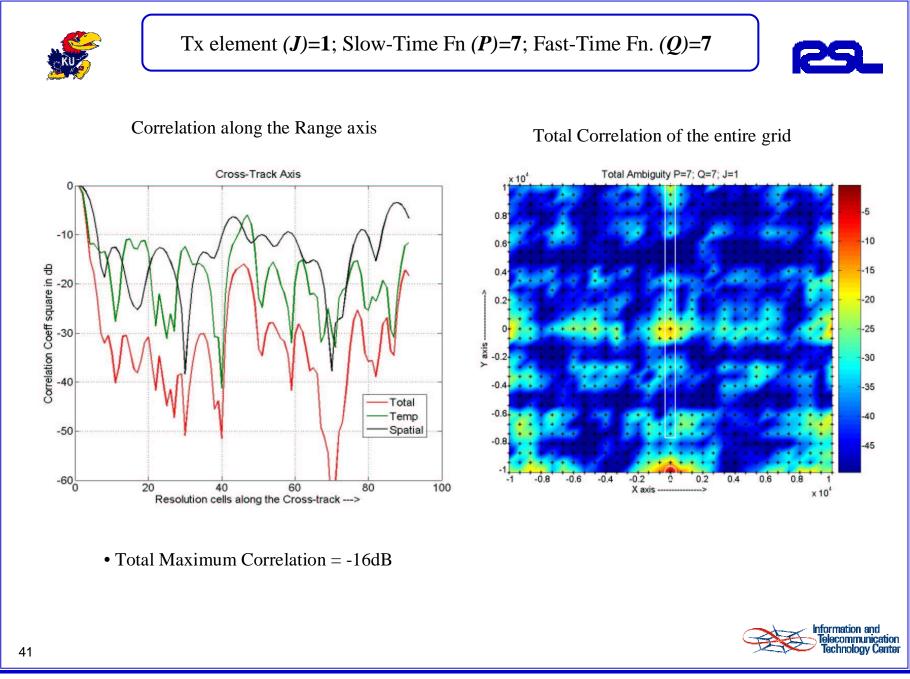
-35

40

-45

0.8

x 10⁴

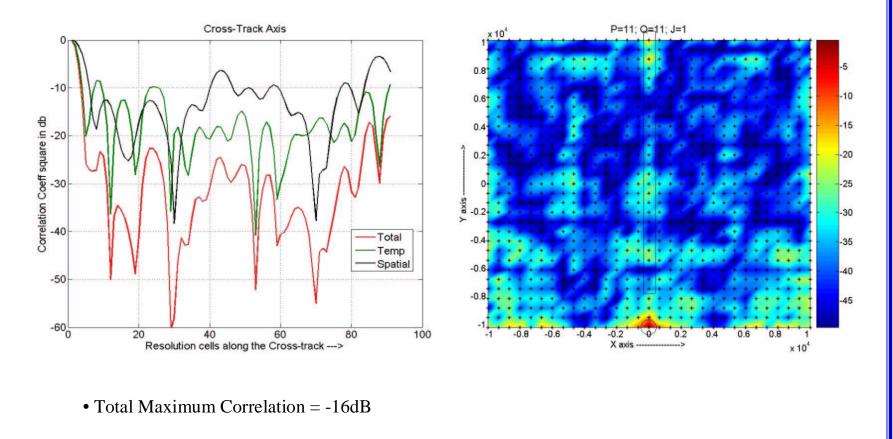






Correlation along the Range axis

Total Correlation of the entire grid







Transmit Elements (J)



• The Number of Transmit Elements (J) are increased from 1 to 2.

• Algorithm analyzed by varying the total number of basis functions.





Tx element (*J*)=2; Slow-Time Fn (*P*)=1; Fast-Time Fn. (*Q*)=1

x 10⁴

0.8

0.6

0.4

0.2

0

Xaxis

-0.4

-0.6

-0.8

-1] -1

-0.8

-0.6

-0.4

-0.2 X axis

.



-5

-10

-15

-20

-25

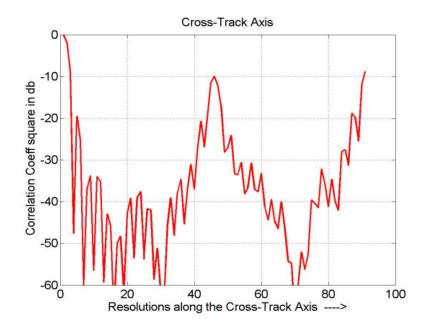
-30

-35

-40

-45

Correlation along the Range axis



• Total Maximum Correlation = -9dB



Total Correlation of the entire grid

Total Ambiguity P=1; Q=1; J=2

0.4

0.6

0.8

x 10⁴

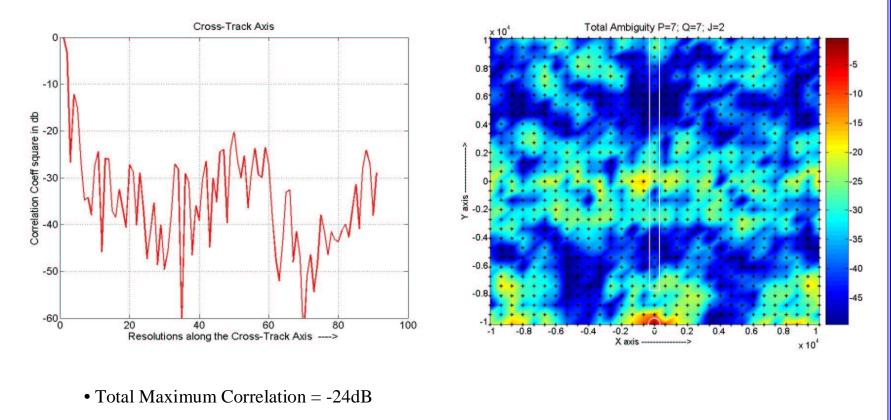


Tx element (J)=2; Slow-Time Fn (P)=7; Fast-Time Fn. (Q)=7



Correlation along the Range axis

Total Correlation of the entire grid



• Improvement of 8dB compared to J=1; P=7; Q=7 case.

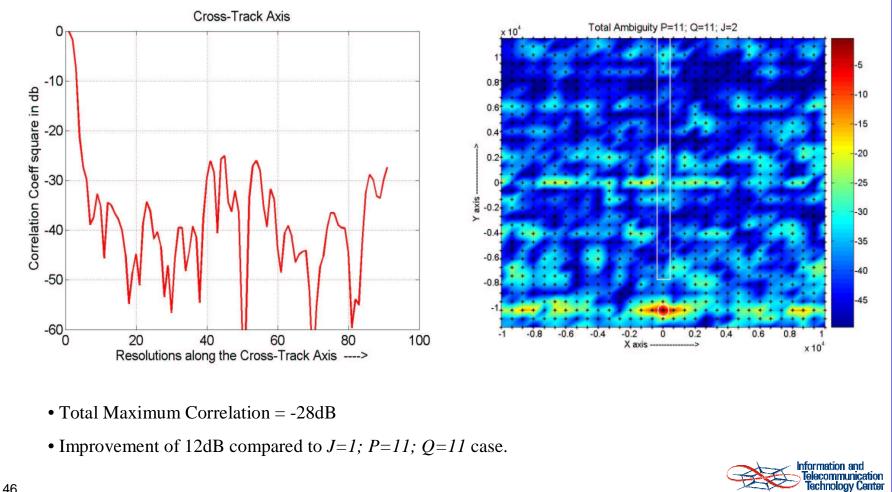






Correlation along the Range axis

Total Correlation of the entire grid

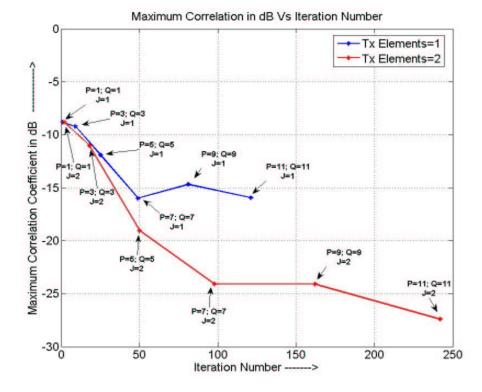




Summary of all the cases for J=1 and J=2



Maximum Correlation Vs Iteration Number



• Higher the total number of basis functions, more is the flexibility provided to the algorithm to come up with a better code





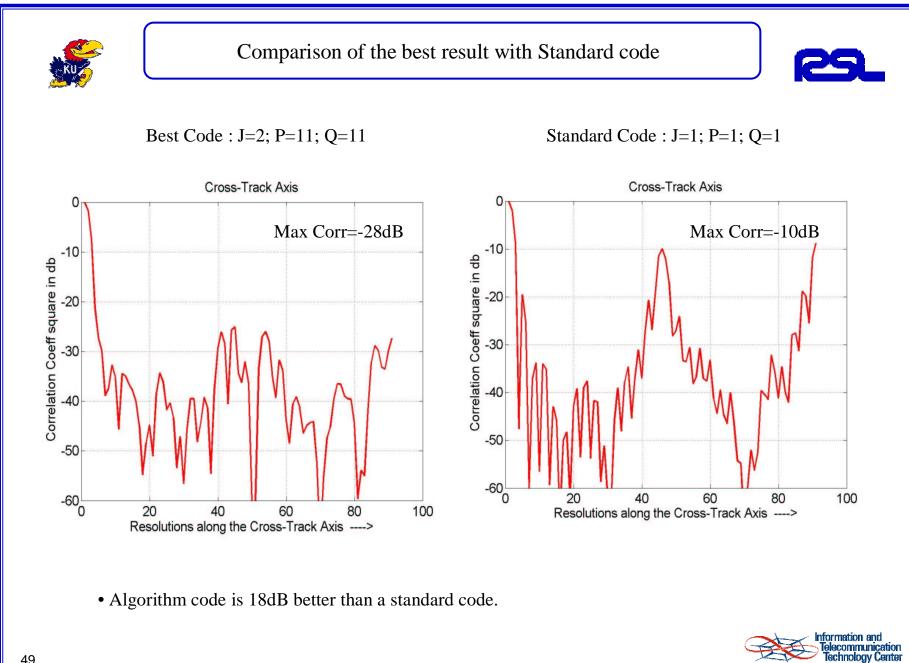
Efficacy of the best result.

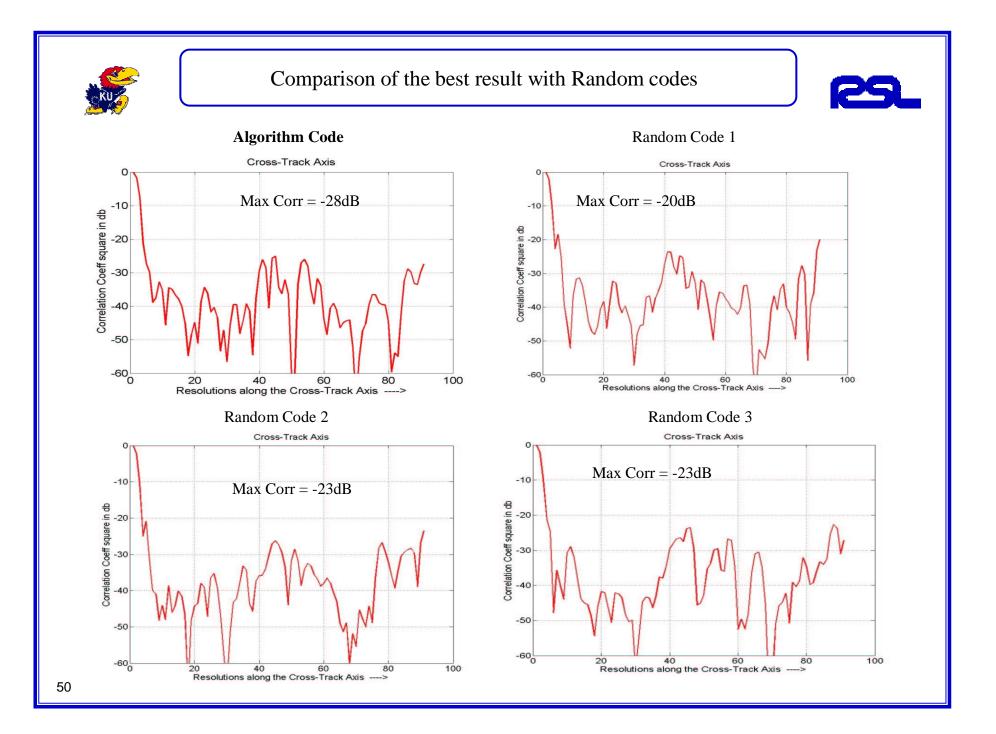


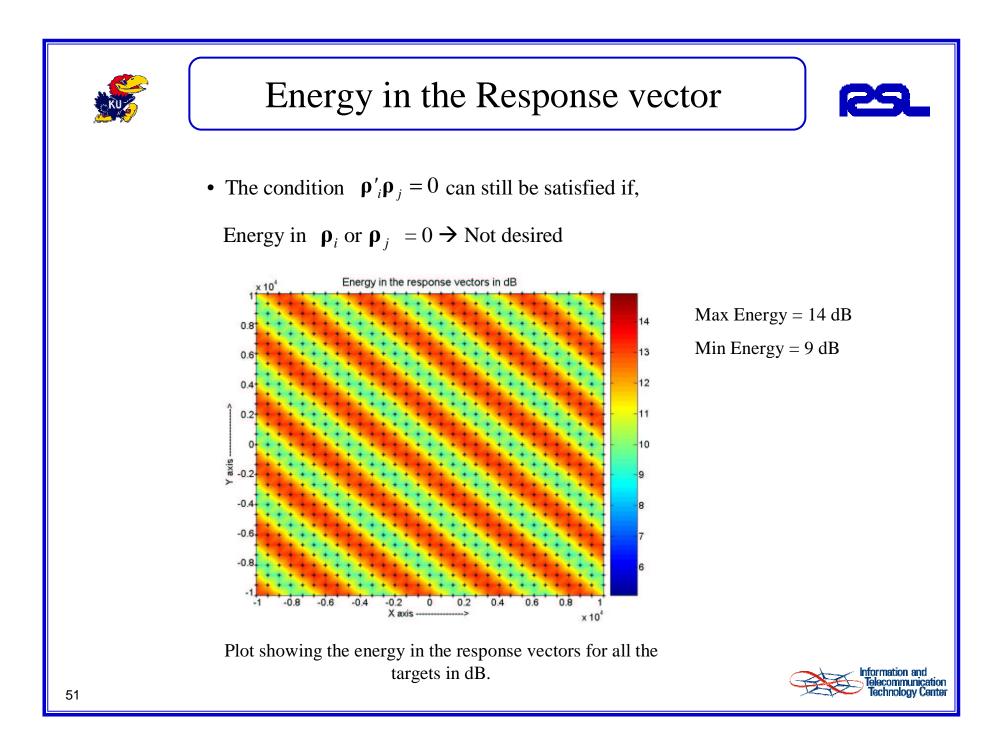
Question: How good is the best result that is obtained?

Answer: Comparison with Standard code and Random Codes











Conclusions



• Proved that Space-Time transmit signal can be designed in order to reduce the maximum correlation.

• Higher the total number of measurements (*M*), higher the total number of dimensions of the transmit signal (*N*), better is the performance of the Algorithms.

• The performance also depends on the structure of the **H**-matrices or, the radar scenario.

• As the total number of basis functions are increased, more flexibility is provided to the Algorithm to come up with the best code.

• As the total number of transmit elements are increased from 1 to 2, the maximum ambiguity is reduced to a great extent.





Future Work



- The performance needs to be evaluated in the Doppler direction.
- The ambiguity function is not invariant for J=2. Therefore a new algorithm needs to be developed to come up with a transmit code.
- The Algorithm needs to be modified accordingly when we use more than 2 transmit elements.



