



Master's Thesis Defense



Reduced Rank Filtering Techniques For Processing Multi-Aperture Radar

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Committee

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OUTLINE



- ❑ **Introduction**
 - The Non-Uniformly Distributed Aperture Radar System
 - The Radar model
 - Previous work done on Filtering Techniques
 - Thesis motivation
- ❑ **The Reduced Rank Square Root Filter Approach**
 - Design and Implementation
 - Choice of criterion determination
 - Discussion of Results
- ❑ **The Multi-Stage Wiener Filter Approach**
 - Design and Implementation
 - Discussion of Results
 - Innovative Implementations
- ❑ **Conclusions and Future work**



INTRODUCTION



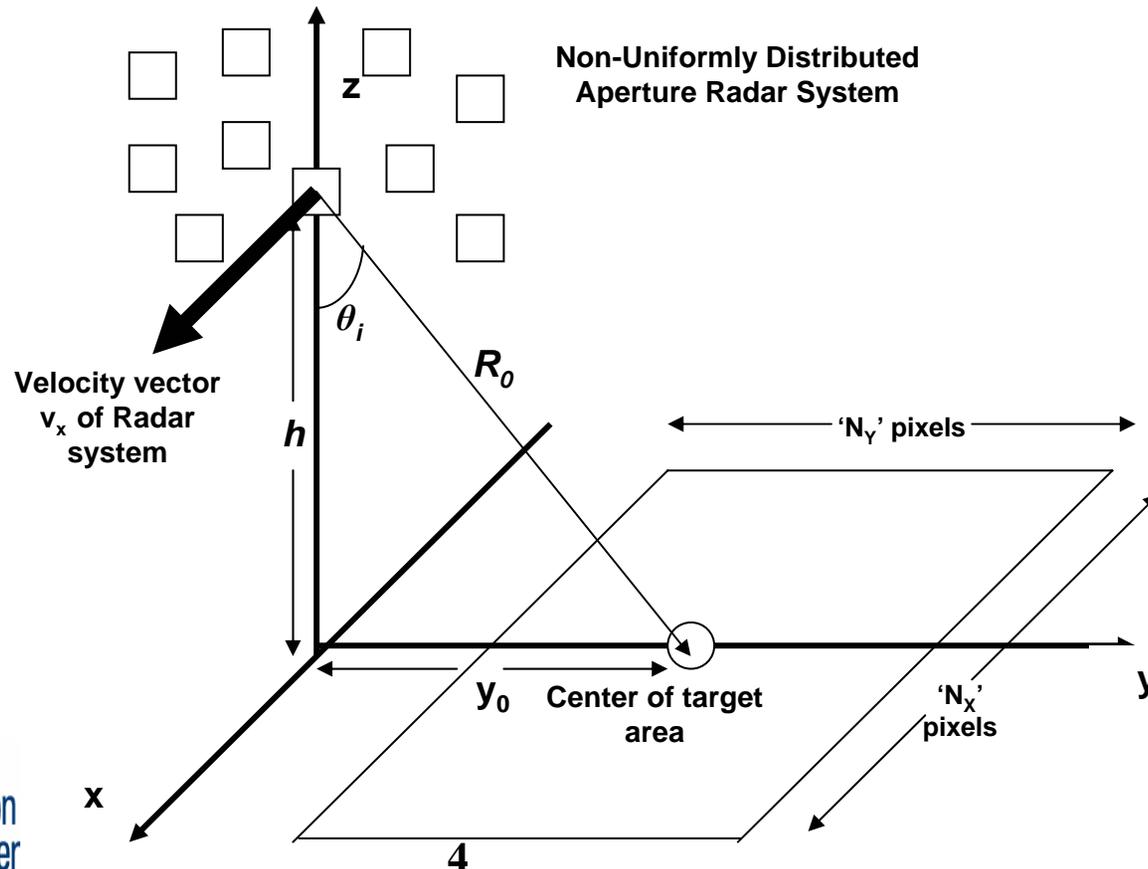
- ❑ **With advance in technology, the modern concept is to go for distributed sensor collection and processing as compared to a single sensor system**
- ❑ **In most cases where the sensors are not fixed mounted to their location, distribution of the sensors' locations is not uniform**
- ❑ **When using such a system, it also involves much more complex signal processing techniques to piece the data from different sensors into a complete picture**
- ❑ **For example, when using a Non-Uniformly Distributed Space borne Multiple Satellite System to collect SAR images, conventional processing technique like the Matched Filter will not function properly.**
- ❑ **Instead, a more complex technique is required in the form of the Wiener or Minimum Mean Square Error (MMSE) Filter**



RADAR MODEL USED



- Currently, as we do not have real data from a Non-Uniformly Distributed Aperture Radar System, a radar model is designed from Mathematical modeling and implemented using MATLAB

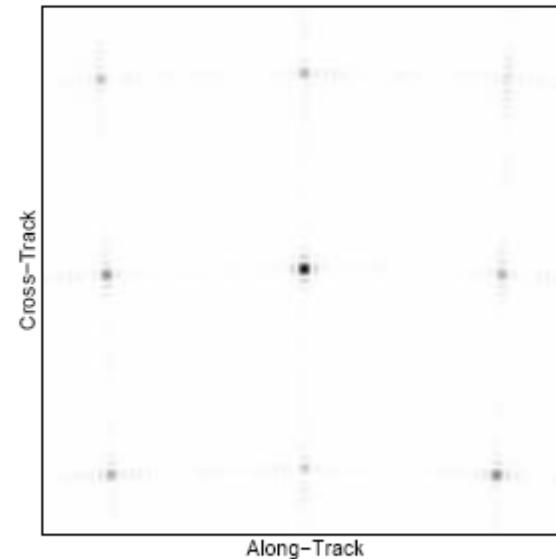




WIENER VERSUS MATCHED FILTER



- ❑ When using the radar model to simulate a image collected, the ambiguity function obtained for each pixel is as shown on the right
- ❑ In such a situation, due to the various ambiguities present in the system, matched filter processing will yield degraded results even in the midst of no measurement noise
- ❑ Based on work done by previous students, it is found that the MMSE filter will be able to provide a good answer even in low or moderate SNR situations

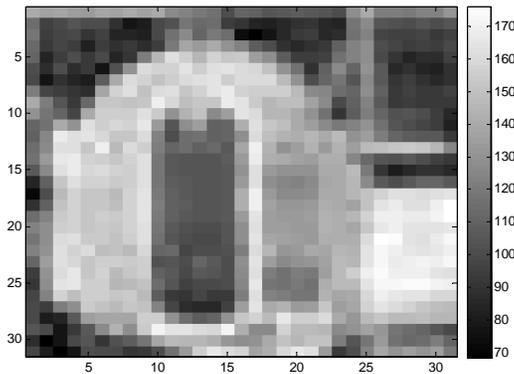




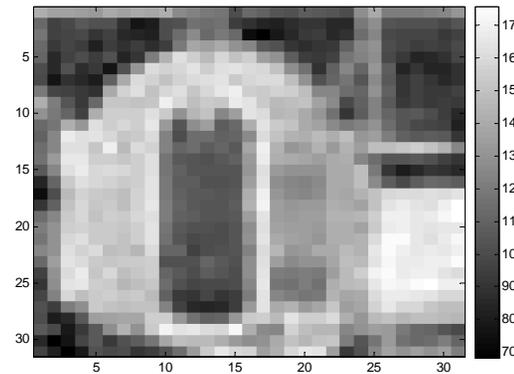
EFFICIENT WIENER IMPLEMENTATION



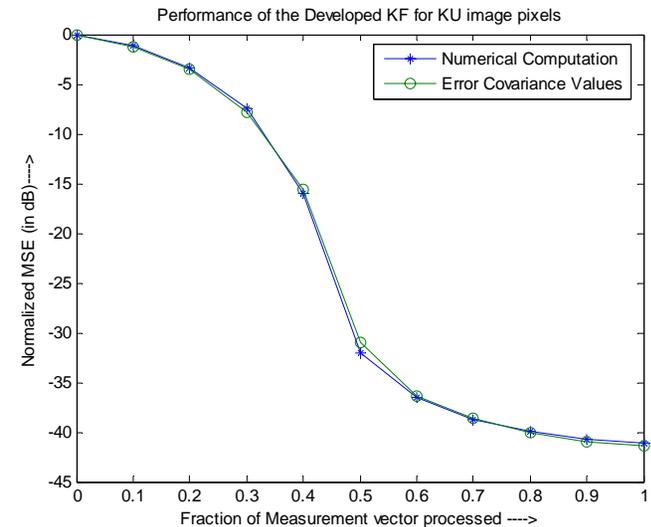
- ❑ As Wiener filter requires a computational expensive matrix inverse operation, thus it is possible to re-implement the filter in a recursive manner using the Kalman filter
- ❑ Previous work by other students have proven the functionality of the Kalman filter and sample results are shown below



Input Image



Output Image



MSE versus Data processed



MOTIVATION OF THESIS



- ❑ Although Kalman filter is efficient, it can suffer from instability as a result of finite machine precision
- ❑ Besides Kalman filter, there are other filters that are more efficient than the full rank Wiener filter when these filters are implemented in reduced rank echelon form
- ❑ Thus, the **1st research motivation** is to find alternative filters type that are **more robust** than Kalman filter in situations of finite machine precision
- ❑ **2nd research motivation** is to implement these alternative filters in a manner such that they are **more efficient** than the Wiener filter
- ❑ This give rise to the search for Reduced Rank Linear filtering Techniques in my thesis research



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❑ **The Reduced Rank Square Root Filter Approach** ←

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❑ **The Multi-Stage Wiener Filter Approach**

- Design and Implementation
- Discussion of Results
- Innovation Implementations

❑ **Conclusions and Future work**



SQUARE ROOT COVARIANCE FILTER



- ❑ Potter introduced the concept of Square Root Covariance filter (SRCF) in 1964 when dynamic driving noise is absent in the system
- ❑ The SRCF was developed for scalar measurement update implementation.
- ❑ The basic concept is to replace the propagation of the Error Covariance Matrix K_γ with its square root matrix S instead where the relationship between the 2 matrices are as follows:

$$K_\gamma = SS^H$$

- ❑ In this manner, the **positive semi-definiteness** of the Error Covariance Matrix will be maintained in all iterations
- ❑ Hence, it will no longer suffer from **instability problems**



POTTER'S SRCF MODEL



- ❑ In Potter's SRCF, the equations involved in each iteration step of the filter is shown below
- ❑ Also, Potter's SRCF is a **full rank filter** just like the Kalman filter

$$\mathbf{a}(l) = \mathbf{S}(l/l - 1)^H \mathbf{p}(l)^H$$

$$b(l) = \frac{1}{[\mathbf{a}(l)^H \mathbf{a}(l) + \sigma_n^2(l)]}$$

$$\eta(l) = \frac{1}{[1 + \{b(l)\sigma_n^2(l)\}^{1/2}]}$$

$$\mathbf{g}(l) = b(l)\mathbf{S}(l/l - 1)\mathbf{a}(l)$$

$$\begin{aligned}\hat{\gamma}(l/l) &= \hat{\gamma}(l - 1/l - 1) + \mathbf{g}(l)[r(l) - \mathbf{p}(l)\hat{\gamma}(l - 1/l - 1)] \\ &= \hat{\gamma}(l - 1/l - 1) + \mathbf{g}(l)v(l)\end{aligned}$$

$$\mathbf{S}(l/l) = \mathbf{S}(l/l - 1) - \eta(l)\mathbf{g}(l)\mathbf{a}(l)^H$$



Vector Format of SRCF



- ❑ As Potter's SRCF is using scalar measurement update which is not as efficient as a vector measurement update implementation, thus Andrew came up with the vector version of SRCF that is more efficient and it is defined as follows:

$$\mathbf{A}(l) = \mathbf{S}(l/l-1)^H \mathbf{P}(l)^H$$

$$\Sigma(l) = \sqrt{\mathbf{A}(l)^H \mathbf{A}(l) + \mathbf{K}_n(l)}^c$$

$$\begin{aligned} \hat{\gamma}(l/l) &= \hat{\gamma}(l-1/l-1) + \mathbf{S}(l/l-1) \mathbf{A}(l) [\Sigma(l)^{-1}]^H \Sigma(l)^{-1} \mathbf{v}(l) \\ &= \hat{\gamma}(l-1/l-1) + \mathbf{S}(l/l-1) \mathbf{A}(l) [\Sigma(l)^{-1}]^H \Sigma(l)^{-1} \mathbf{v}(l) \end{aligned}$$

$$\mathbf{S}(l/l) = \mathbf{S}(l/l-1) - \mathbf{S}(l/l-1) \mathbf{A}(l) [\Sigma(l)^{-1}]^H [\Sigma(l) + \sqrt{\mathbf{K}_n(l)}] \mathbf{A}(l)^H$$

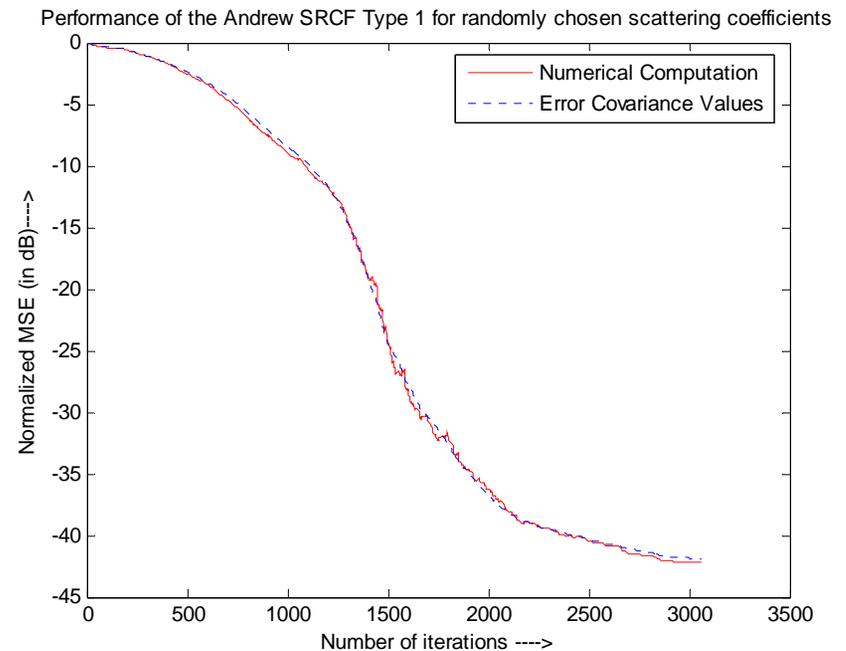
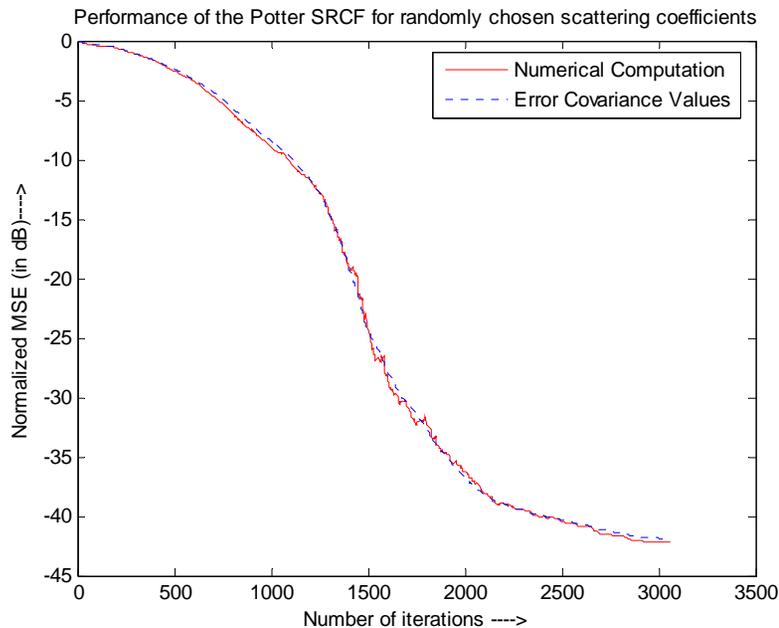
- ❑ Both scalar and vector version of the SRCF are then executed in simulation runs so as to examine the results obtained



RESULTS FROM SRCF



- ❑ Using data from the radar model simulator, the results of both Potter and Andrew's algorithm are plotted
- ❑ Both SRCF filters are able to achieve identical performance in accuracy with Kalman filter but requires longer computational time





REDUCED RANK SRCF (RRSQRT)



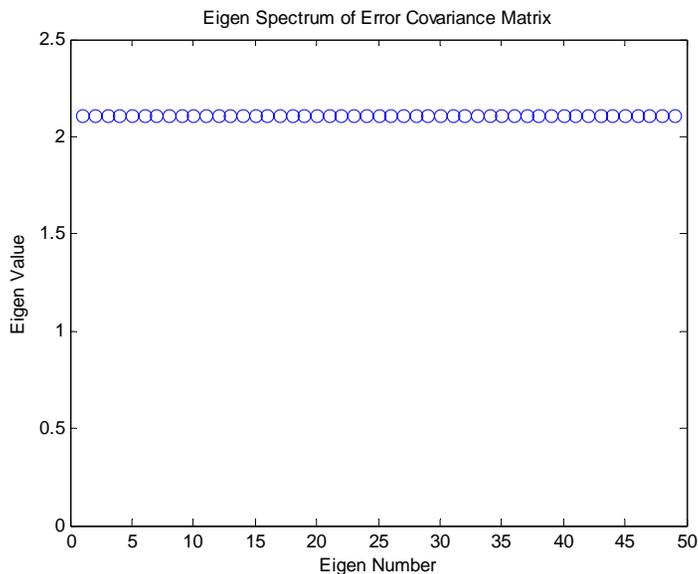
- ❑ Since the full rank SRCF is not as efficient as Kalman filter, thus the reduced rank version of Andrew SRCF is implemented to tackle this issue
- ❑ Rank reduction is achieved by **discarding non dominant Eigen vectors** as the iteration proceeds along
- ❑ However, we will need to **determine the criterion** to discard or keep the Eigen vectors
- ❑ Two approaches are attempted, 1st approach using just **guesswork** and 2nd approach based on the **variation of the Eigen Spectrum** of Error Covariance Matrix as the iteration proceeds
- ❑ Results obtained from guesswork are bad, therefore requiring the 2nd approach to determine the criterion



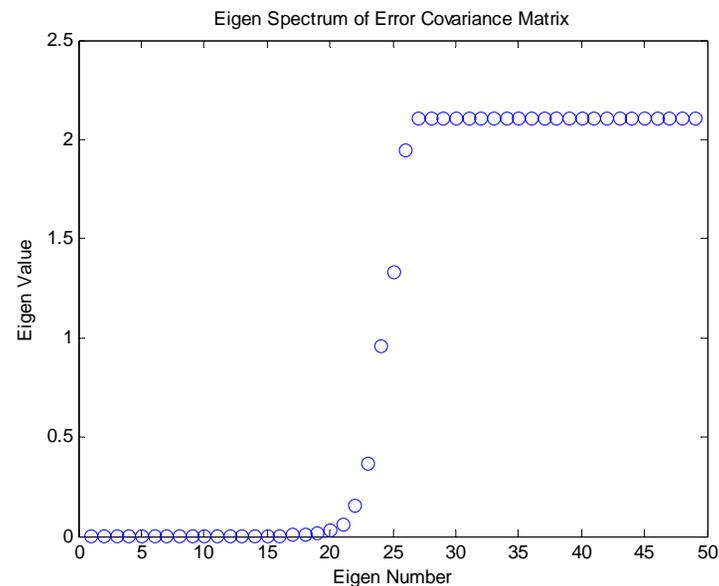
2ND APPROACH FOR RRSQRT CRITERION



- ❑ This approach requires the Eigen Spectrum of the SRCF at various stages of iteration to be make known
- ❑ The Eigen Spectrum for a sample measurement and target size are shown below



0 % of iteration process



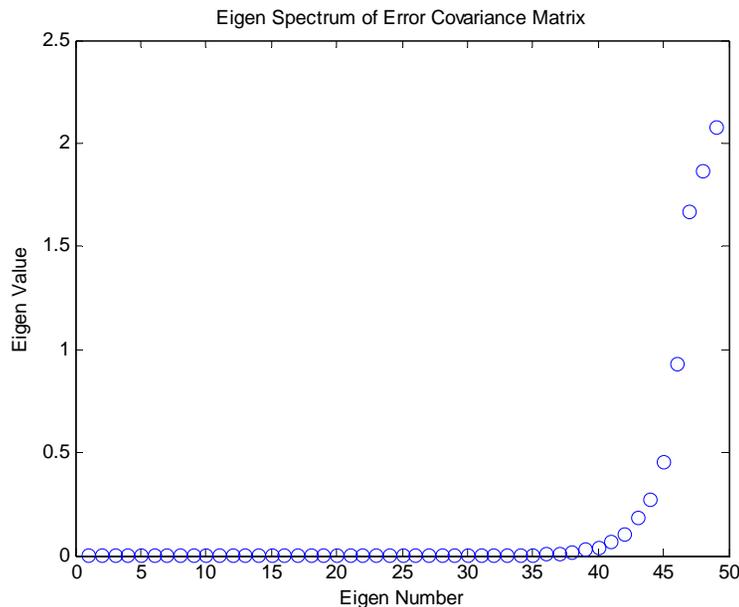
25 % of iteration process



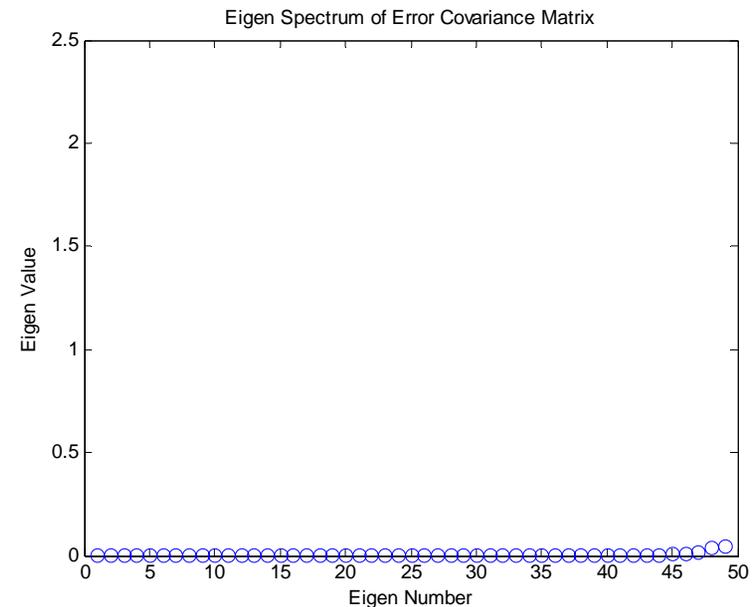
EIGEN SPECTRUM OF SRCF - CONTINUED



- ❑ As seen below, the dominant Eigen vectors decreases as the iteration proceeds until all Eigen vectors become insignificant
- ❑ This behavior is then used to develop the criterion for keeping the Eigen vectors in the iteration steps



50 % of iteration process



62.5 % of iteration process



FINAL CRITERION FOR RRSQRT



- ❑ Using the behavior of the SRCF Eigen Spectrum, the final criterion consists of 2 sub criteria shown below

% of initial Eigen Value used for Criteria 2	Step size in dB used for Criteria 1				
	4	8	12	16	20
0.001					
0.01					
0.05					
0.1					
0.5					
1.0					
4.0					



RESULTS FOR RRSQRT



- ❑ The MSE and Timing results obtained for the RRSQRT using the 2 criterions are shown below for comparisons with the Kalman filter

S/N	Description of Filter	Time	Final MSE
1	Kalman filter	37.206 sec	-42.216 dB

% of initial Eigen Value used for Criteria 2	Step size in dB used for Criteria 1				
	4	8	12	16	20
0.001	-42.216 dB / 961	-42.216 dB / 961	-42.216 dB / 961	-42.216 dB / 961	-42.216 dB / 961
0.01	-39.906 dB / 230	-40.649 dB / 245	-40.818 dB / 285	-41.210 dB / 313	-41.144 dB / 257
0.05	-32.726 dB / 26	-35.422 dB / 35	-36.855 dB / 54	-38.222 dB / 89	-38.148 dB / 40

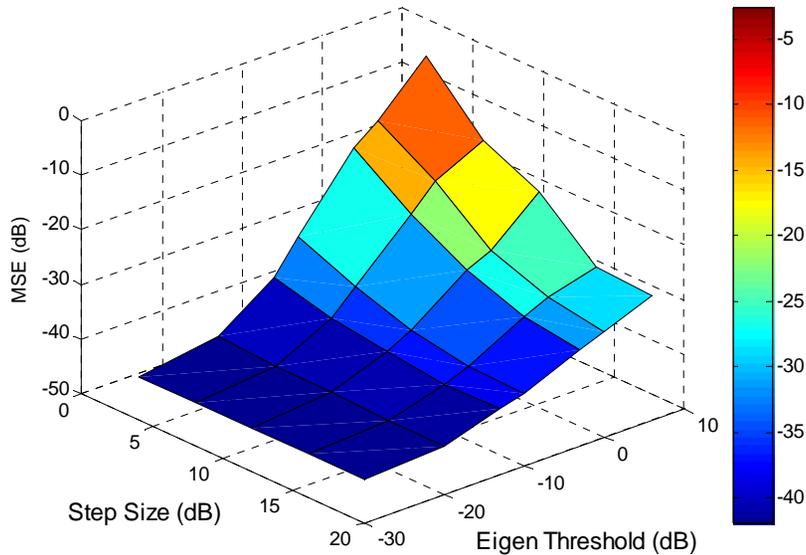
% of initial Eigen Value used for Criteria 2	Step size in dB used for Criteria 1				
	4	8	12	16	20
0.001	289.275 sec	168.691 sec	120.879 sec	95.145 sec	96.552 sec
0.01	78.994 sec	64.331 sec	60.305 sec	58.405 sec	57.667 sec
0.05	48.978 sec	46.594 sec	47.052 sec	49.748 sec	49.500 sec



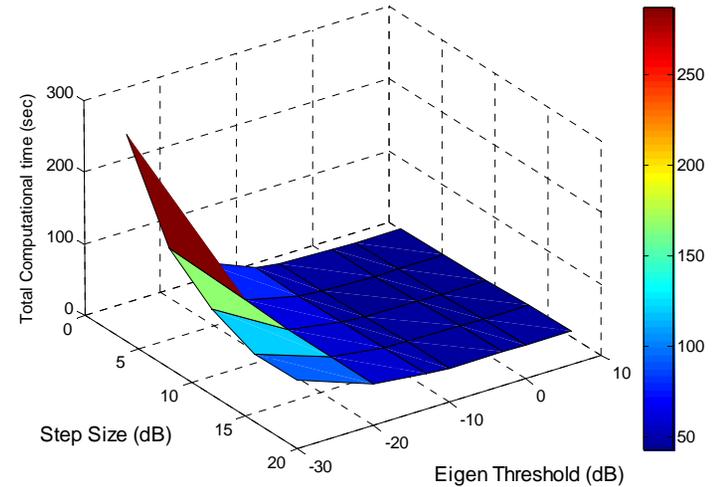
3D PLOT OF RRSQRT RESULTS



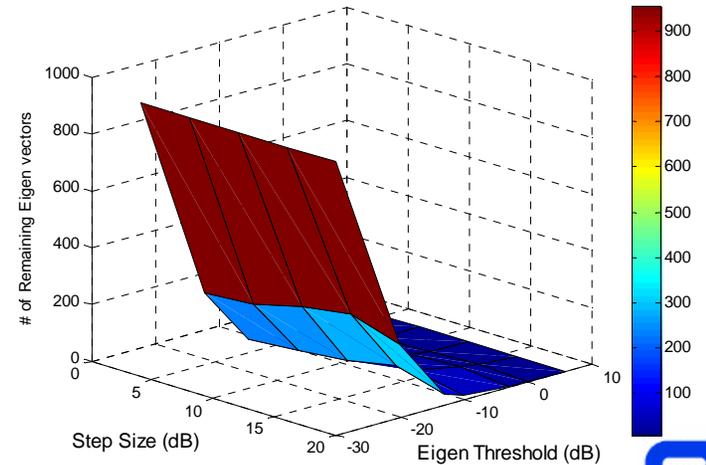
MSE obtained for RRSQRT versus the 2 criterions



Computational Time required for RRSQRT versus the 2 criterions



Remaining Eigen vectors for RRSQRT versus the 2 criterions





TIMING RESULTS W/O EIGEN OPERATION



- ❑ From investigation, the Eigen decomposition operation of the RRSQRT takes up a significant portion of the total computational time
- ❑ If this operation is ignored, the difference in the timing results are as shown below

% of initial Eigen Value used for Criteria 2	Step size in dB used for Criteria 1				
	4	8	12	16	20
0.001	289.275 sec	168.691 sec	120.879 sec	95.145 sec	96.552 sec
0.01	78.994 sec	64.331 sec	60.305 sec	58.405 sec	57.667 sec
0.05	48.978 sec	46.594 sec	47.052 sec	49.748 sec	49.500 sec

% of initial Eigen Value used for Criteria 2	Step size in dB used for Criteria 1				
	4	8	12	16	20
0.001	80.454 sec	63.785 sec	58.097 sec	52.691 sec	53.348 sec
0.01	35.367 sec	34.894 sec	34.945 sec	35.264 sec	34.432 sec
0.05	24.203 sec	25.734 sec	26.521 sec	28.857 sec	28.562 sec



SUMMING UP RRSQRT



- ❑ From the results obtained, it is shown that the RRSQRT is able to produce good estimation of the target pixels even when the rank is greatly reduced
- ❑ If a faster method is available for performing the Eigen Decomposition operation, then this technique will be as computational efficient as the Kalman filter while not suffering from instability problems



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MULTI-STAGE WIENER FILTER



- ❑ Besides the RRSQRT filter, there is also another filter that is also found to be much more efficient than the Wiener filter
- ❑ This filter was introduced in 1997 and is now gaining much popularity with the Communication and Radar community
- ❑ It is known as the Multi-Stage Wiener Filter (MSWF for short) based on orthogonal projections
- ❑ This filter can be also implemented in a full rank or reduced rank manner. But it has been shown in various applications that it is able to **achieve full rank performance even when implemented in reduced rank fashion**



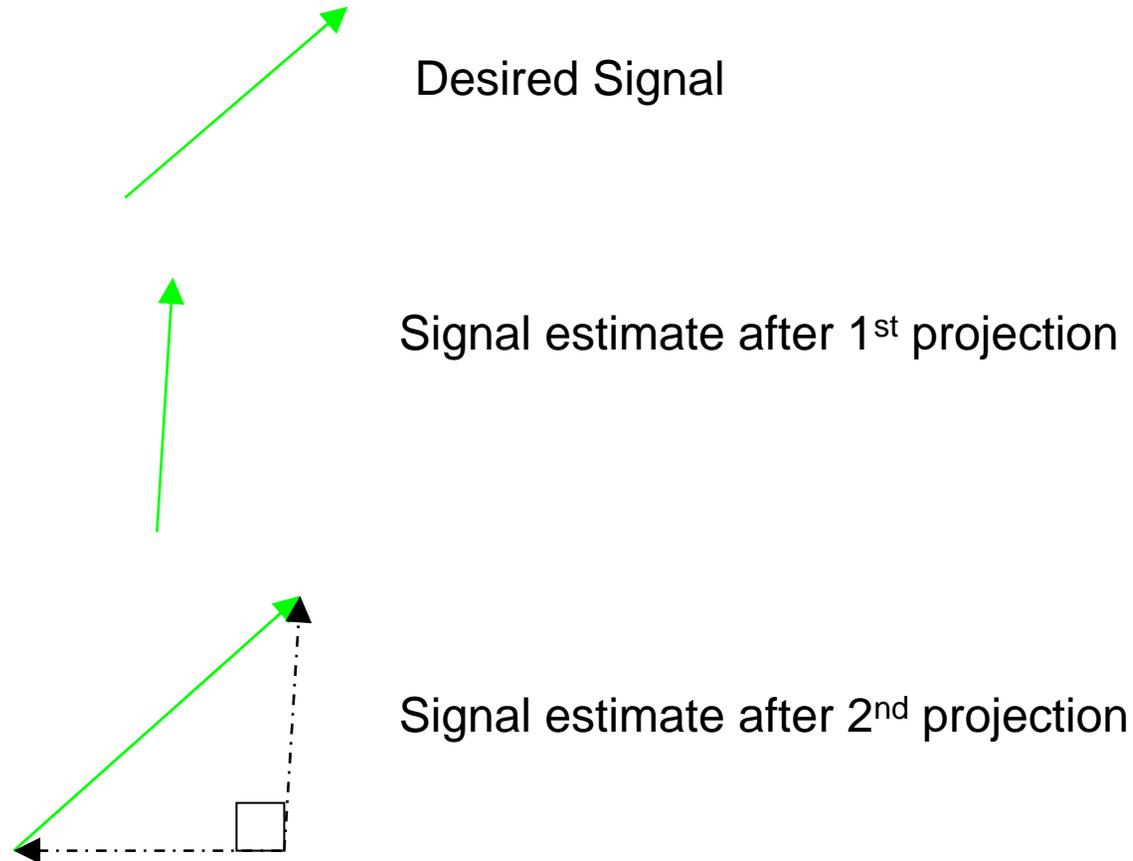
MULTI-STAGE WIENER FILTER - CONT



- ❑ The basic MSWF is used for estimating a scalar desired signal from a series of measurement data (called it scalar MSWF)
- ❑ It makes use of the cross correlation between the measurement data vector and the desired signal to project an initial estimate of the desired signal after the so called 1st stage of decomposition
- ❑ When there are **correlations** between the desired signal and the unwanted signals present in the measurement data, this 1st estimate will contain errors
- ❑ Thus, using a **blocking matrix orthogonal to the cross correlation vector**, a new measurement data and a new cross correlation vector is formed again
- ❑ This action is then repeated at the 2nd stage of decomposition and so forth until the error in the final estimate is diminished to an acceptable value



GRAPHICAL VISUALIZATION OF MSWF





EQUATIONS OF SCALAR MSWF



- ❑ Note that the MSWF consists of 3 main steps, namely the forward iteration step, turn-around step and the backward iteration step
- ❑ The equations for implementing the forward iteration step are shown

$$\delta_i = \sqrt{\mathbf{r}_{x_{i-1}d_{i-1}}^H \mathbf{r}_{x_{i-1}d_{i-1}}}$$

$$\mathbf{h}_i = \frac{\mathbf{r}_{x_{i-1}d_{i-1}}}{\delta_i}$$

$$d_i = \mathbf{h}_i^H \mathbf{x}_{i-1}$$

$$\sigma_{d_i}^2 = \mathbf{h}_i^H \mathbf{R}_{x_{i-1}} \mathbf{h}_i$$

$$\mathbf{B}_i = \text{null}\{\mathbf{h}_i\}$$

$$\mathbf{x}_i = \mathbf{B}_i \mathbf{x}_{i-1}$$

$$\mathbf{R}_{x_i} = \mathbf{B}_i \mathbf{R}_{x_{i-1}} \mathbf{B}_i^H$$

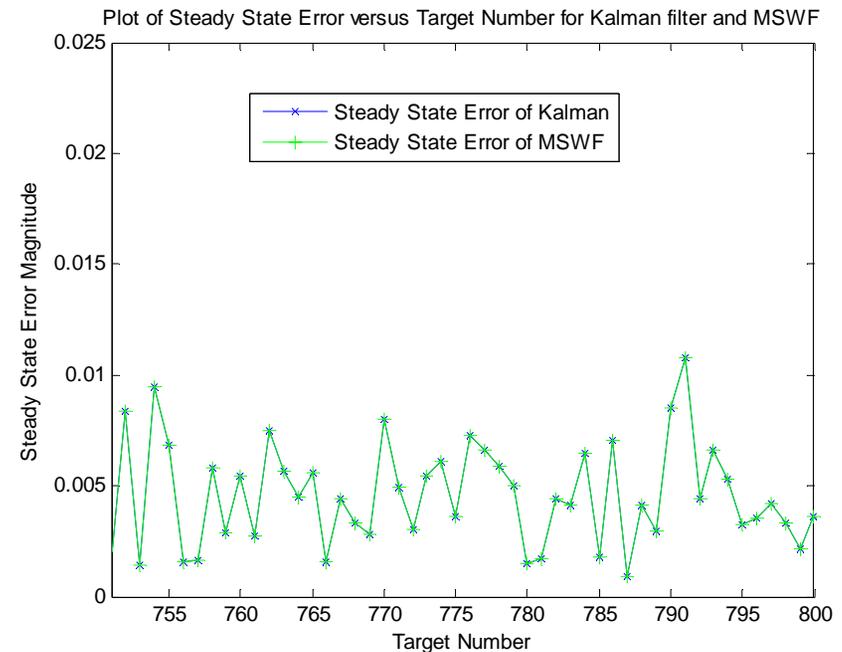
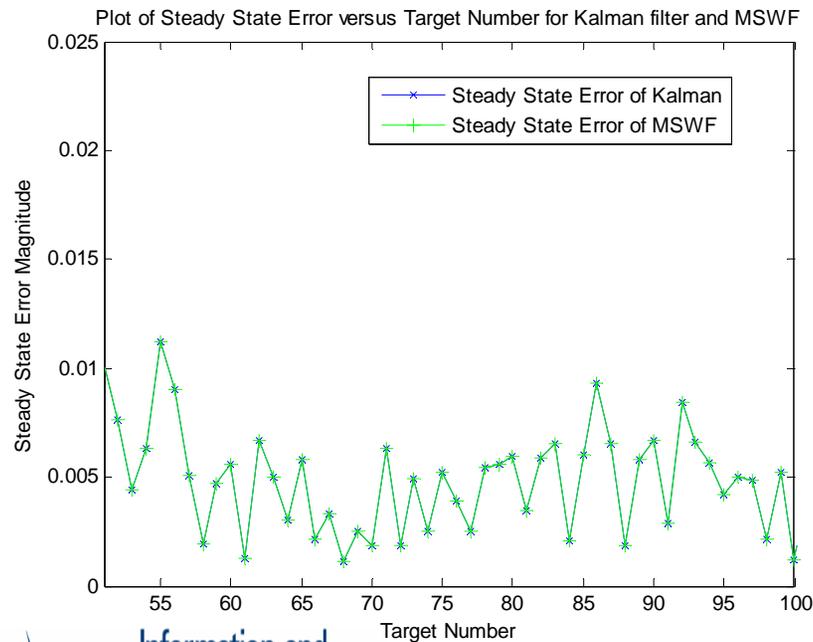
$$\mathbf{r}_{x_i d_i} = \mathbf{B}_i \mathbf{R}_{x_{i-1}} \mathbf{h}_i$$



RESULTS FROM SCALAR MSWF



- ❑ The scalar MSWF is then implemented and run sequentially to estimate all the target pixels in the SAR image using 40 stages of decomposition
- ❑ The final errors of the estimates using the scalar MSWF is then compared with that from the Kalman filter





SUMMING UP SCALAR MSWF



- ❑ From results obtained, the scalar MSWF is able to estimate the target pixels with as much accuracy as the Kalman filter
- ❑ Furthermore, the scalar MSWF only requires 40 stages of decomposition to achieve these results instead of full rank processing of 961 stages
- ❑ The computational time needed per target pixel is **much lesser than that of the Wiener filter**
- ❑ However, one undesirable feature is that it can only estimate 1 target pixel at a time, thus requires a long period to complete estimation of all target pixels if done in a serial manner, unless there are enough machines for parallel processing
- ❑ Another weak point is that the processing of each target pixel also takes a longer time than the Kalman filter



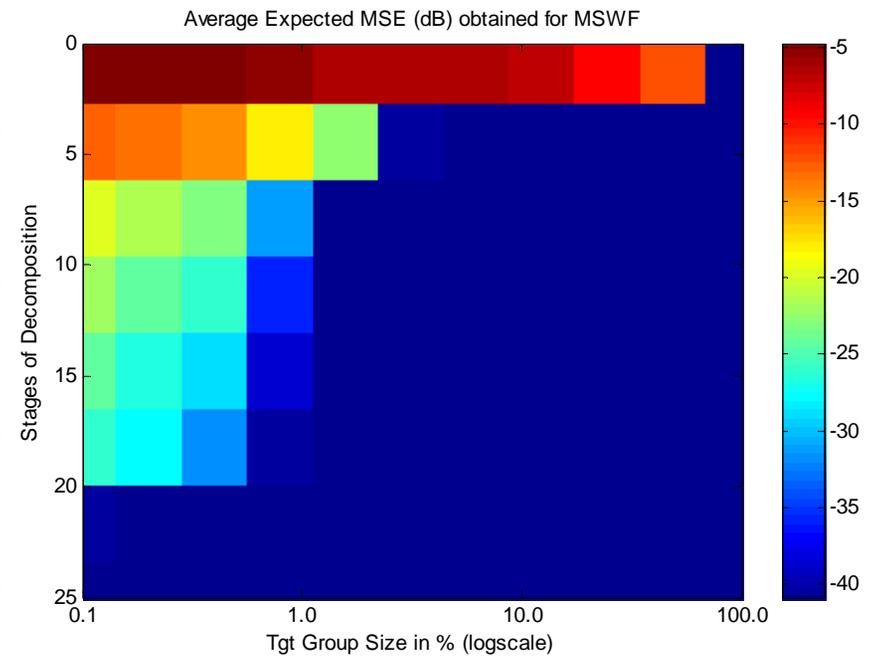
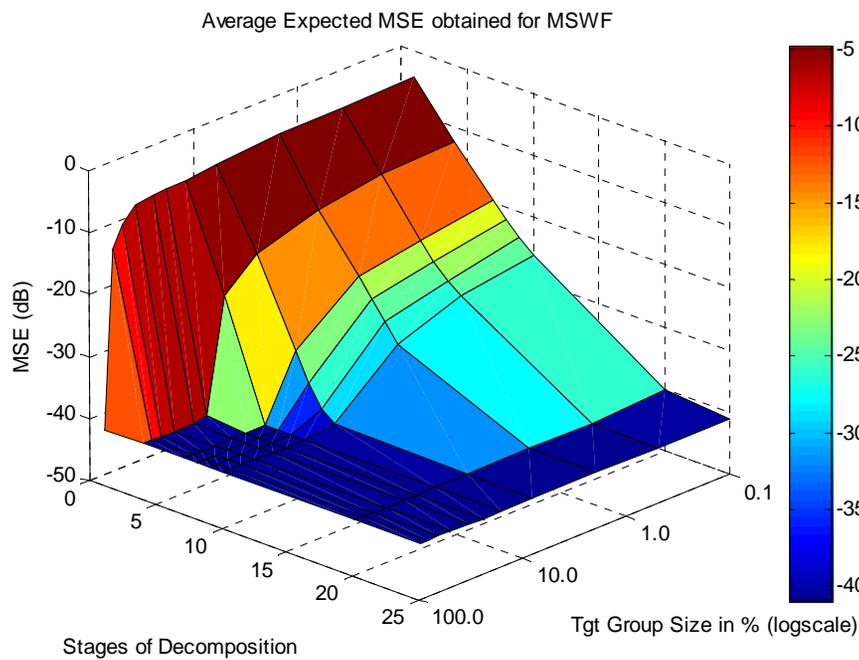
VECTOR MSWF



- ❑ Therefore, these shortcomings necessitates the development of the vector version of the MSWF that can **estimate more than 1 target pixel per target group** at a time
- ❑ Now, for the vector MSWF implementation, it can also be implemented by executing each group of targets in a serial manner for all targets or using parallel processing with 1 MSWF processor per target group
- ❑ Therefore, both options are explored and results are presented for discussion
- ❑ Note that the Mean Square Error (MSE) obtained for both methods are identical for the same combination of number of targets per group versus stages of decomposition

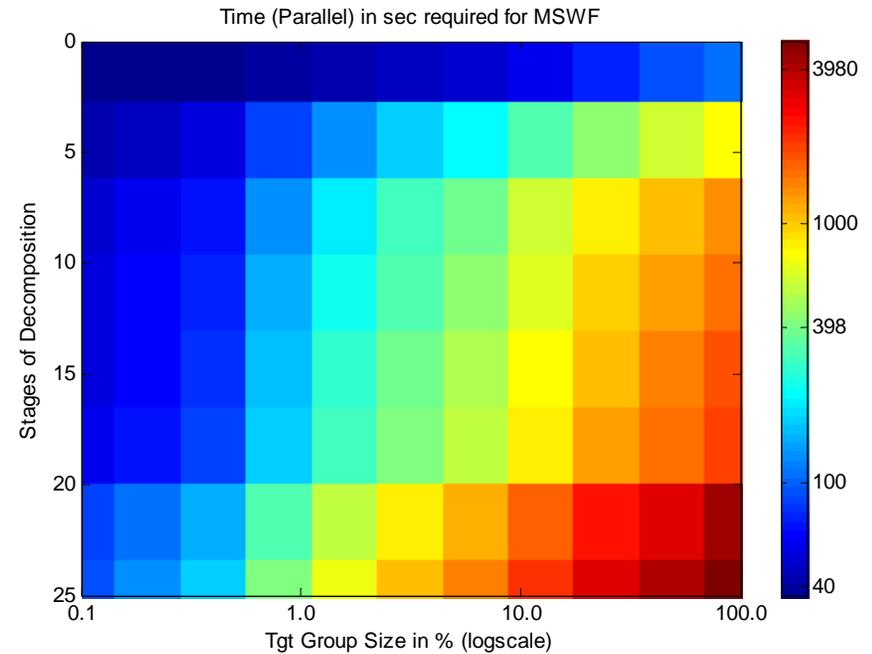
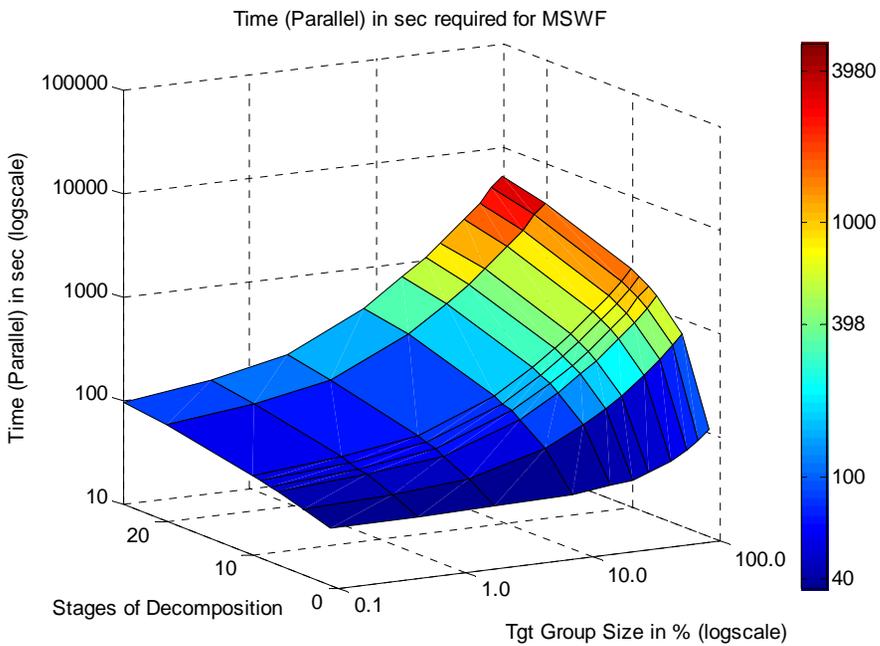


RESULTS OF MSE FOR VECTOR MSWF



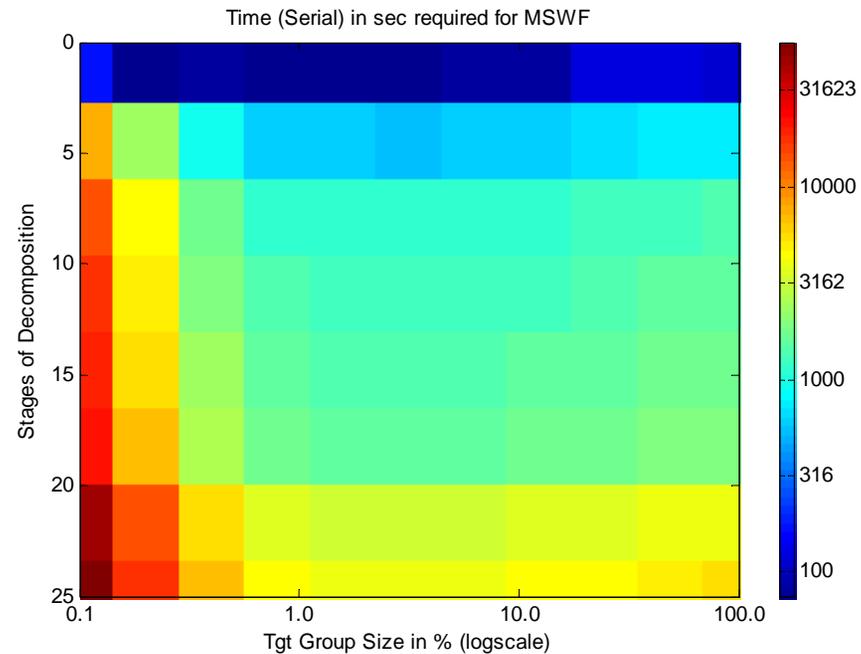
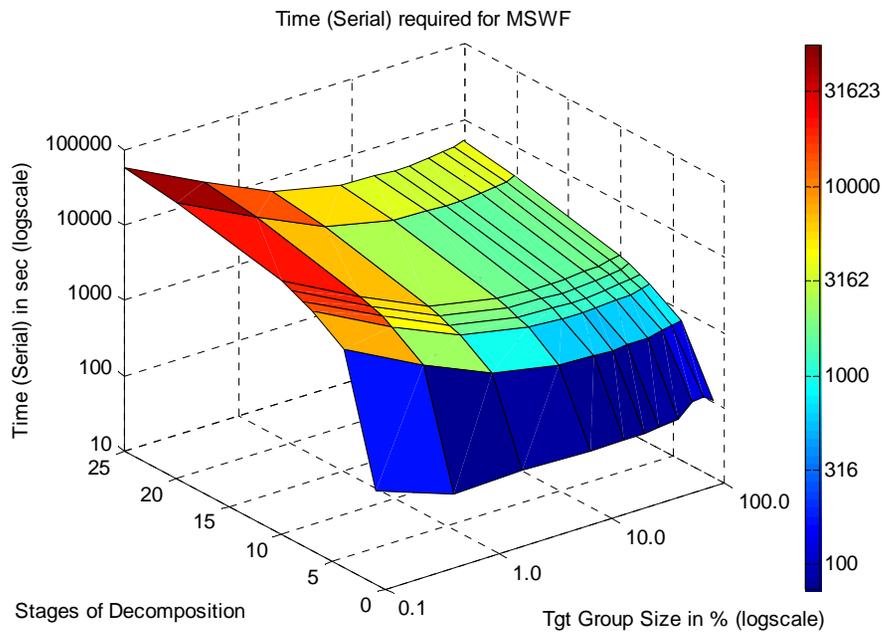


RESULTS OF EXECUTION TIME (PARALLEL)



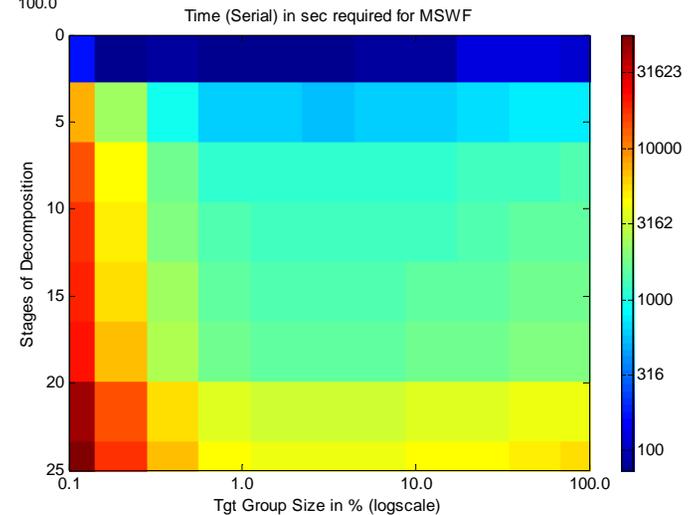
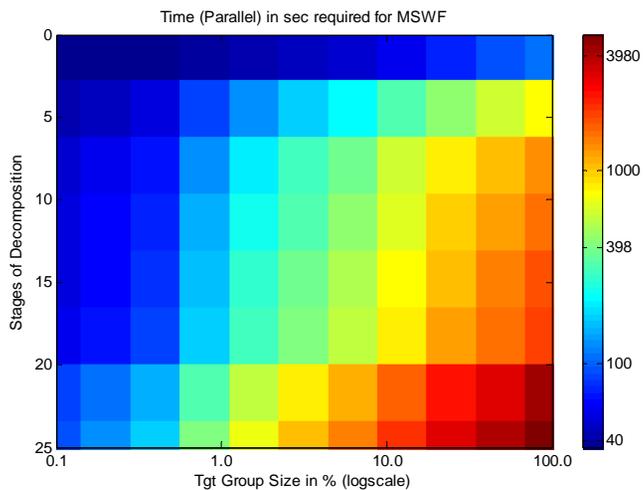
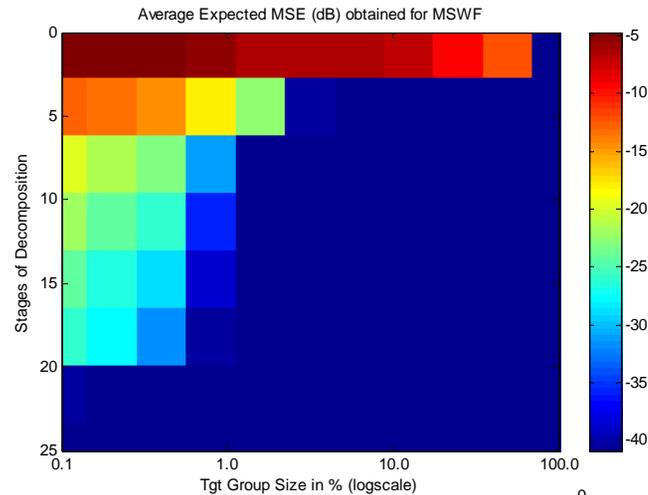


RESULTS OF EXECUTION TIME (SERIAL)





OVERALL COMBINATION OF RESULTS





SUMMING UP VECTOR MSWF



- ❑ The vector MSWF is able to provide pixel estimates with the same accuracy as Wiener and Kalman filter using **less than full rank processing**
- ❑ Some combinations of target group size and stages of decomposition will require lesser computational time than Wiener filter with an efficiency ratio of up to **1.65**
- ❑ For both parallel and serial method of implementation, **putting all targets in one group and using 1 stage of decomposition** currently provides the best deal in terms of computational time and accuracy of results
- ❑ However, this best deal is still not as efficient as that of Kalman filter. Therefore, will need to check whether the trend holds true for larger data set



LARGER DATA SET

- ❑ Next, the target size and the measurement size are each increased by about 4 times
- ❑ The results for the Wiener filter, Kalman filter and MSWF are shown below

Filter Type	Total Time /sec	Average $\bar{\sigma}_{\varepsilon_0}^2$ /dB	Average Computed MSE /dB
Wiener	9349.3	-39.483	-39.439
Kalman	1192.1	-39.483	-39.439
MSWF	3432.5	-39.479	-39.439

- ❑ We can see that the trend continues but **MSWF's edge over Wiener filter has increased significantly** over the smaller data set from **1.65 to 2.72**



INNOVATIVE MSWF IMPLEMENTATIONS



- ❑ We have observed that the MSWF is able to outperform the Wiener filter in terms of speed but it is not as efficient as Kalman filter
- ❑ Therefore, several variations are carried out to the standard implementation structure of the MSWF
- ❑ It is hoped that some of these variations will be able to speed up the MSWF but at little or no cost to its accuracy obtained for the results
- ❑ These approaches are termed as Innovative MSWF Implementations
- ❑ A total of 3 approaches are attempted



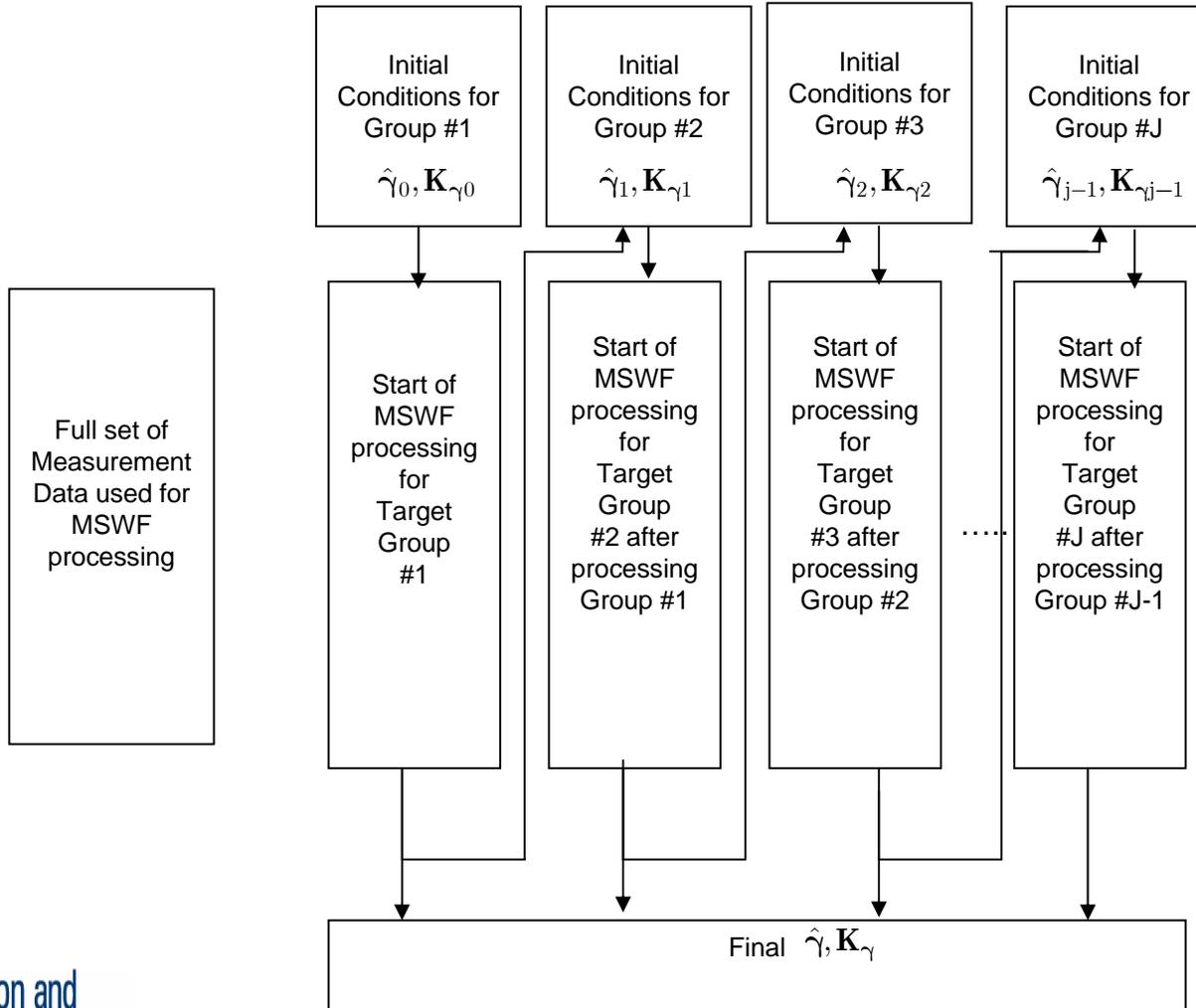
MODIFIED DATA INITIALIZATION



- ❑ In this approach, instead of using the same initial conditions for all target groups, the **results of the 1st target group** is used to fine tune the initial conditions of the 2nd target group
- ❑ Next, the results of the 1st and 2nd target group are used to modify the initial condition of the 3rd target group and so forth
- ❑ This form of initialization bears some similarity to the way the Kalman filter is being operated



STRUCTURE OF MODIFIED INITIALIZATION

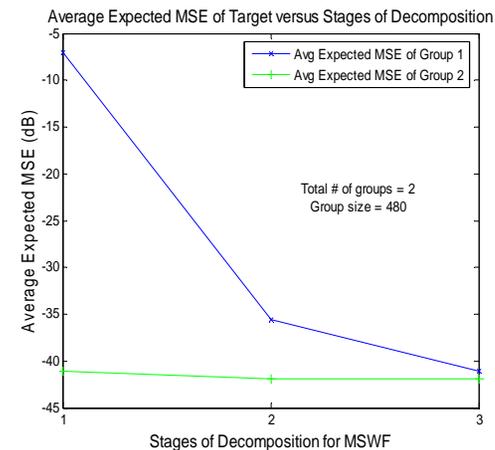
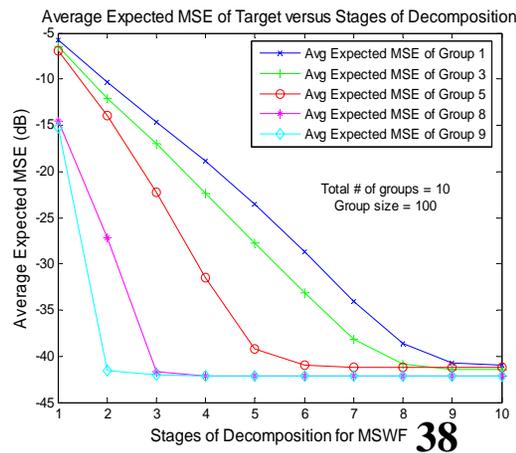
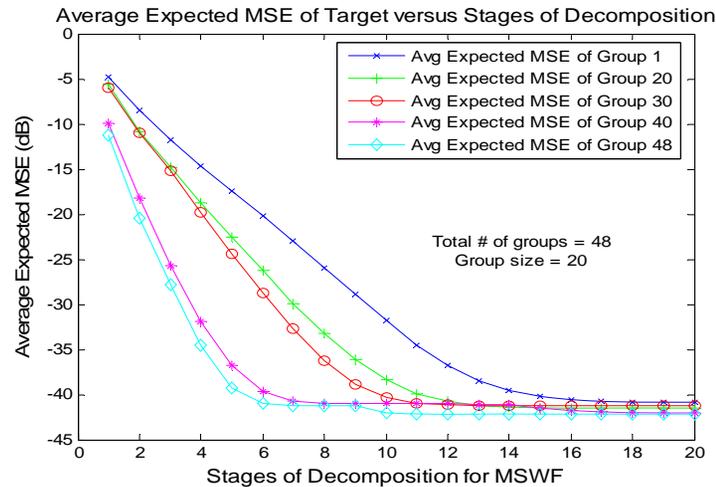




RESULTS FOR MODIFIED DATA APPROACH



- Using the modified initialization approach, the results obtained for a few target group sizes are obtained and shown below





SUMMING UP MODIFIED DATA APPROACH



- ❑ From the results obtained, we can see that the average number of decomposition stages required for any target group size decreases with the use of the modified data initialization approach
- ❑ After performing some computation and taking into account the overheads associated with this approach, the **net gain in the efficiency is about 28%**
- ❑ At the same time, there is no loss at all in the final results' accuracy
- ❑ Thus, this improvement is significant and this approach should be used whenever the serial MSWF implementation is chosen



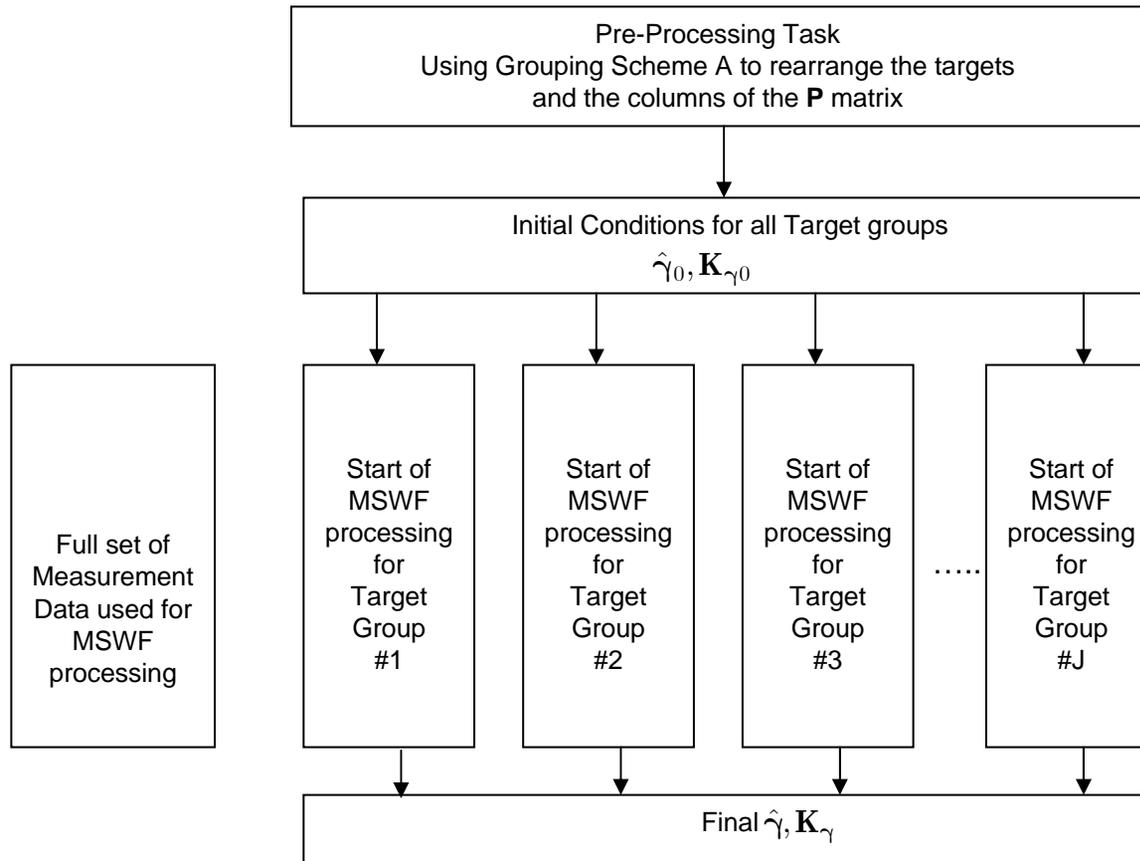
GROUPING TARGET BY CORRELATION



- ❑ The 2nd approach is to group the targets together using their cross correlation to one other as a criteria
- ❑ The criteria can be to group targets based on their **least correlation** to one other or their **highest correlation** to one another
- ❑ This grouping mechanism is applied before the start of the MSWF processing and can be applied to either serial MSWF implementation or parallel MSWF implementation
- ❑ Both groupings based on least correlation and highest correlation are attempted

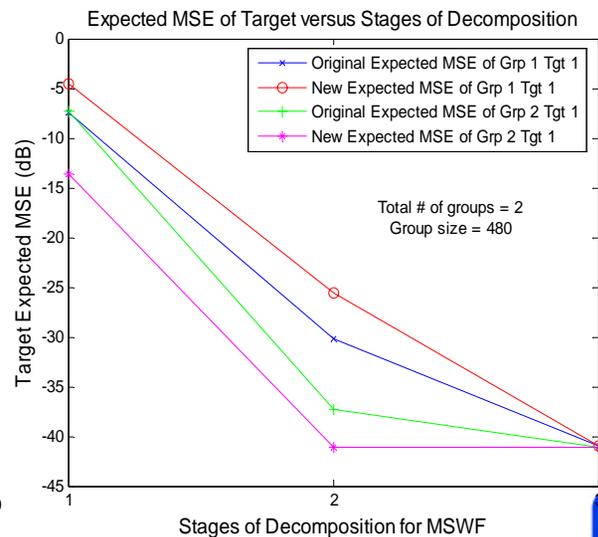
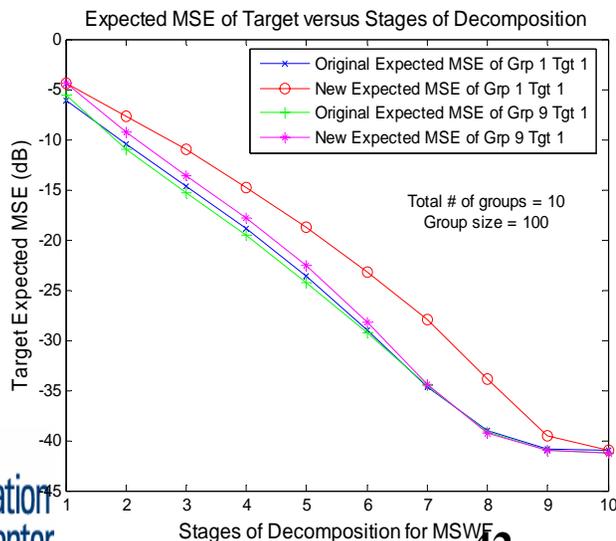
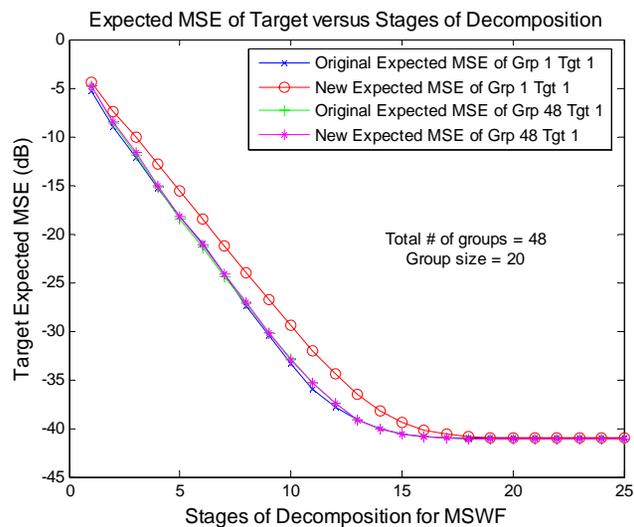


STRUCTURE FOR TARGET GROUPING



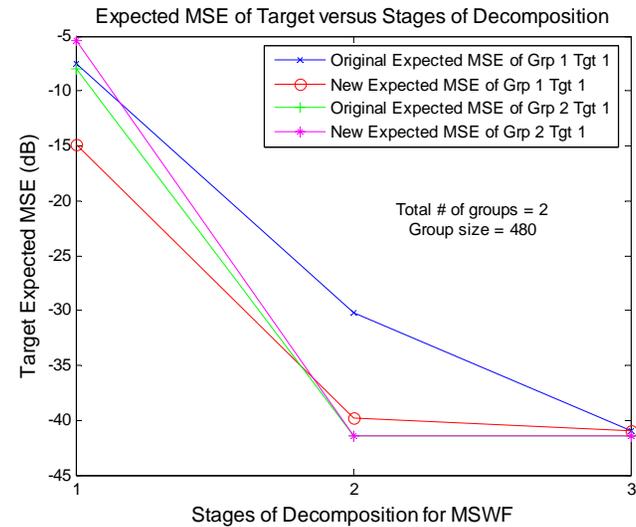
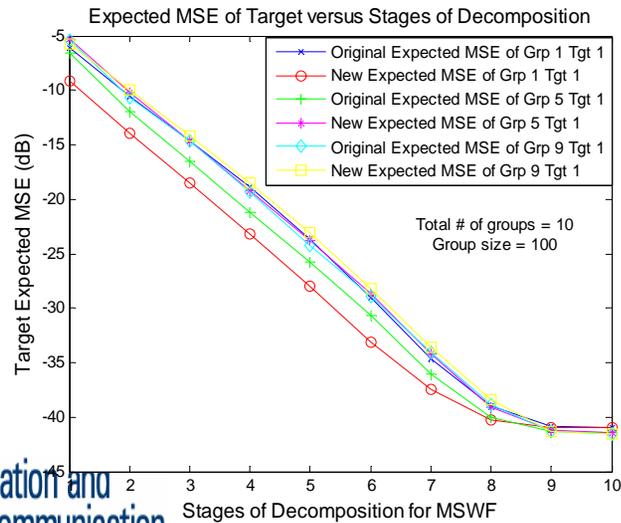
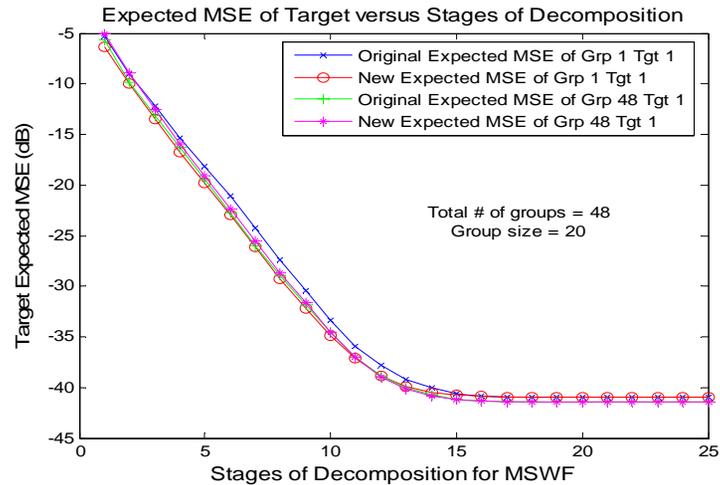


RESULTS FOR LEAST CORRELATION





RESULTS FOR HIGHEST CORRELATION





SUMMING UP TARGET GROUPING



- ❑ From the 2 trends observed, we can conclude that grouping targets that are highly correlated to one another into the same group will help to decrease the number of stages of decomposition required
- ❑ However, due to the imperfection of my grouping schemes, not all the targets are grouped based on this desired criteria and therefore no numerical figures are available to show the improvement
- ❑ Nevertheless, with a robust grouping scheme, this approach should bear fruitful results



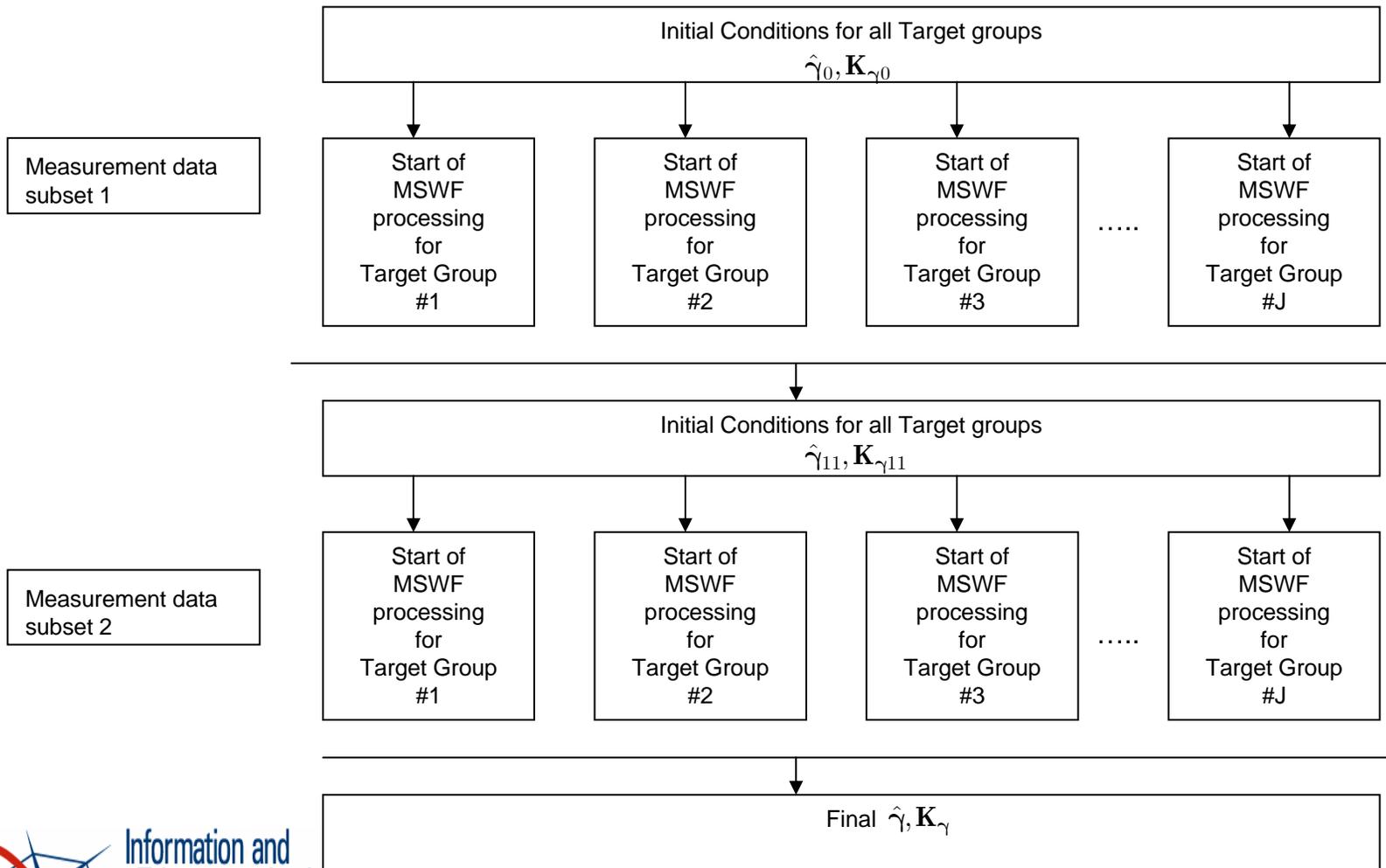
RECURSIVE MSWF



- ❑ Now, beside trying different approaches on the target space as in the last 2 approaches, we can also look into the measurement data space itself
- ❑ Drawing inspiration from the Kalman filter which is a recursive Wiener filter, we can also implement the MSWF in a recursive fashion by **breaking up the total measurement data into smaller subsets** and iteratively applying the MSWF on these data subsets.
- ❑ In this manner, it is hoped that computations involving smaller matrix dimensions in the forward iteration step will help to speed up the MSWF execution



STRUCTURE OF RECURSIVE MSWF





SETUP FOR RECURSIVE MSWF



- ❑ Now, to examine the performance of the recursive MSWF, various measurement subset and target group sizes are chosen so as to get a good picture of this approach
- ❑ The combination used in the simulation is as shown

Measurements per subset	Number of subsets	Target Group size = 961	Target Group size = 480	Target Group size = 320	Target Group size = 160
2856	1	√	√	√	√
1428	2	√	√	√	√
714	4	X	√	√	√
476	6	X	X	√	√

- ❑ Note that some combinations are not achievable because of the vector MSWF constraints



RESULTS FOR RECURSIVE MSWF



- ❑ Results obtained for the recursive MSWF are encouraging and some combination is able to achieve faster computational time with no loss of results' accuracy
- ❑ The results for the recursive parallel MSWF is shown here

Measurements per subset	Number of subsets	Target Group size = 961	Target Group size = 480	Target Group size = 320	Target Group size = 160
2856	1	113.203 sec	241.112 sec	222.498 sec	215.547 sec
1428	2	103.735 sec	151.110 sec	130.548 sec	103.331 sec
714	4	X	78.728 sec	69.301 sec	54.669 sec
476	6	X	X	43.549 sec	37.560 sec

Measurements per subset	Number of subsets	Target Group size = 961	Target Group size = 480	Target Group size = 320	Target Group size = 160
2856	1	-41.037 dB	-41.037 dB	-41.037 dB	-41.037 dB
1428	2	-41.037 dB	-39.880 dB	-39.350 dB	-38.959 dB
714	4	X	-37.093 dB	-34.392 dB	-30.052 dB
476	6	X	X	-25.015 dB	-21.524 dB



SUMMING UP RECURSIVE MSWF



- ❑ Implementing the MSWF as a recursive filter seems to be a good approach for both the parallel and serial MSWF implementation
- ❑ With certain combinations of measurement data subset and target group size, we can get improvements in the computational speed by up to 100% with little loss in accuracy
- ❑ This ability to process **new data iteratively also makes the MSWF to be more attractive** than the Wiener filter



OUTLINE



❑ Introduction

- The Non-Uniformly Distributed Aperture Radar System
- The Radar model
- Previous work done on Filtering Techniques
- Thesis motivation

❑ The Reduced Rank Square Root Filter Approach

- Design and Implementation
- Choice of criterion determination
- Discussion of Results

❑ The Multi-Stage Wiener Filter Approach

- Design and Implementation
- Discussion of Results
- Innovation Implementations

❑ Conclusions and Future work ←



CONCLUSIONS



- ❑ We are able to successfully implement Reduced Rank Filtering Techniques for the Non-Uniformly Distributed Aperture Radar System that are faster in execution than the Wiener filter while achieving the same accuracy in the final results
- ❑ With a more **efficient Eigen Decomposition engine**, the RRSQRT filter will be able to match or exceed the computational speed of the Kalman filter while avoiding the Kalman filter's pitfall of divergence or instability issue
- ❑ It has been shown that the Multi-Stage Wiener filter is able to achieve the same accuracy as the Wiener filter but requiring lesser rank in the filter processing
- ❑ Although the MSWF is not as fast as the Kalman filter, **innovative implementations will help to narrow the gap** between the two filters



FUTURE WORK



- ❑ For the RRSQRT filter, we can look into faster ways to compute the Eigen Decomposition as compared to the existing function in MATLAB
- ❑ We can also design some target scenarios where it is possible to start the RRSQRT filter processing with a smaller rank that will speed up its subsequent iteration process
- ❑ For the MSWF, we can also look into a more optimal grouping scheme for the targets such that the deficiencies in the current scheme can be corrected and then use the optimal scheme for implementation
- ❑ We can also look into alternative model of the MSWF that is more simple than the current MSWF model that is based on orthogonal projections. For example, one alternative model will be the Conjugate Gradient MSWF model



QUESTIONS



THANK YOU