Master’s Thesis Defense

Reduced Rank Filtering Techniques For Processing Multi-Aperture Radar

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Committee
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Dr. Shannon Blunt
OUTLINE

- Introduction
  - The Non-Uniformly Distributed Aperture Radar System
  - The Radar model
  - Previous work done on Filtering Techniques
  - Thesis motivation

- The Reduced Rank Square Root Filter Approach
  - Design and Implementation
  - Choice of criterion determination
  - Discussion of Results

- The Multi-Stage Wiener Filter Approach
  - Design and Implementation
  - Discussion of Results
  - Innovative Implementations

- Conclusions and Future work
INTRODUCTION

- With advance in technology, the modern concept is to go for distributed sensor collection and processing as compared to a single sensor system.
- In most cases where the sensors are not fixed mounted to their location, distribution of the sensors’ locations is not uniform.
- When using such a system, it also involves much more complex signal processing techniques to piece the data from different sensors into a complete picture.
- For example, when using a Non-Uniformly Distributed Space borne Multiple Satellite System to collect SAR images, conventional processing technique like the Matched Filter will not function properly.
- Instead, a more complex technique is required in the form of the Wiener or Minimum Mean Square Error (MMSE) Filter.
Currently, as we do not have real data from a Non-Uniformly Distributed Aperture Radar System, a radar model is designed from Mathematical modeling and implemented using MATLAB.
When using the radar model to simulate a image collected, the ambiguity function obtained for each pixel is as shown on the right.

In such a situation, due to the various ambiguities present in the system, matched filter processing will yield degraded results even in the midst of no measurement noise.

Based on work done by previous students, it is found that the MMSE filter will be able to provide a good answer even in low or moderate SNR situations.
EFFICIENT WIENER IMPLEMENTATION

- As Wiener filter requires a computational expensive matrix inverse operation, thus it is possible to re-implement the filter in a recursive manner using the Kalman filter.
- Previous work by other students have proven the functionality of the Kalman filter and sample results are shown below.

Input Image  |  Output Image

MSE versus Data processed
Although Kalman filter is efficient, it can suffer from instability as a result of finite machine precision.

Besides Kalman filter, there are other filters that are more efficient than the full rank Wiener filter when these filters are implemented in reduced rank echelon form.

Thus, the 1st research motivation is to find alternative filters type that are more robust than Kalman filter in situations of finite machine precision.

2nd research motivation is to implement these alternative filters in a manner such that they are more efficient than the Wiener filter.

This give rise to the search for Reduced Rank Linear filtering Techniques in my thesis research.
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  - Innovation Implementations

- **Conclusions and Future work**
Potter introduced the concept of Square Root Covariance filter (SRCF) in 1964 when dynamic driving noise is absent in the system. The SRCF was developed for scalar measurement update implementation. The basic concept is to replace the propagation of the Error Covariance Matrix $K_\gamma$ with its square root matrix $S$ instead where the relationship between the 2 matrices are as follows:

$$K_\gamma = SS^H$$

In this manner, the positive semi-definiteness of the Error Covariance Matrix will be maintained in all iterations. Hence, it will no longer suffer from instability problems.
In Potter’s SRCF, the equations involved in each iteration step of the filter is shown below

Also, Potter’s SRCF is a full rank filter just like the Kalman filter

\[
\begin{align*}
\mathbf{a}(l) &= \mathbf{S}(l / l - 1)^H \mathbf{p}(l)^H \\
\mathbf{b}(l) &= \frac{1}{[\mathbf{a}(l)^H \mathbf{a}(l) + \sigma^2_n(l)]} \\
\eta(l) &= \frac{1}{[1 + \{b(l)\sigma^2_n(l)\}^{1/2}]} \\
\mathbf{g}(l) &= b(l)\mathbf{S}(l / l - 1)\mathbf{a}(l) \\
\hat{\gamma}(l / l) &= \hat{\gamma}(l - 1 / l - 1) + \mathbf{g}(l)[\mathbf{r}(l) - \mathbf{p}(l)\hat{\gamma}(l - 1 / l - 1)] \\
&= \hat{\gamma}(l - 1 / l - 1) + \mathbf{g}(l)v(l) \\
\mathbf{S}(l / l) &= \mathbf{S}(l / l - 1) - \eta(l)\mathbf{g}(l)\mathbf{a}(l)^H
\end{align*}
\]
As Potter’s SRCF is using scalar measurement update which is not as efficient as a vector measurement update implementation, thus Andrew came up with the vector version of SRCF that is more efficient and it is defined as follows:

\[
A(l) = S(l/l - 1)^H P(l)^H \\
\Sigma(l) = \sqrt{A(l)^H A(l) + K_n(l)^c} \\
\hat{\gamma}(l/l) = \hat{\gamma}(l - 1/l - 1) + S(l/l - 1)A(l)[\Sigma(l)^{-1}]^H \Sigma(l)^{-1} v(l) \\
= \hat{\gamma}(l - 1/l - 1) + S(l/l - 1)A(l)[\Sigma(l)^{-1}]^H \Sigma(l)^{-1} v(l) \\
S(l/l) = S(l/l - 1) - S(l/l - 1)A(l)[\Sigma(l)^{-1}]^H [\Sigma(l) + \sqrt{K_n(l)}] A(l)^H
\]

Both scalar and vector version of the SRCF are then executed in simulation runs so as to examine the results obtained.
RESULTS FROM SRCF

- Using data from the radar model simulator, the results of both Potter and Andrew’s algorithm are plotted.
- Both SRCF filters are able to achieve identical performance in accuracy with Kalman filter but requires longer computational time.
Since the full rank SRCF is not as efficient as Kalman filter, thus the reduced rank version of Andrew SRCF is implemented to tackle this issue.

Rank reduction is achieved by discarding non dominant Eigen vectors as the iteration proceeds along.

However, we will need to determine the criterion to discard or keep the Eigen vectors.

Two approaches are attempted, 1st approach using just guesswork and 2nd approach based on the variation of the Eigen Spectrum of Error Covariance Matrix as the iteration proceeds.

Results obtained from guesswork are bad, therefore requiring the 2nd approach to determine the criterion.
2ND APPROACH FOR RRSQRT CRITERION

- This approach requires the Eigen Spectrum of the SRCF at various stages of iteration to be made known.
- The Eigen Spectrum for a sample measurement and target size are shown below.

![Eigen Spectrum of Error Covariance Matrix](image)

- 0% of iteration process
- 25% of iteration process
- As seen below, the dominant Eigen vectors decreases as the iteration proceeds until all Eigen vectors become insignificant.
- This behavior is then used to develop the criterion for keeping the Eigen vectors in the iteration steps.

50% of iteration process

50% of iteration process
FINAL CRITERION FOR RRSQRT

- Using the behavior of the SRCF Eigen Spectrum, the final criterion consists of 2 sub criterions shown below

<table>
<thead>
<tr>
<th>% of initial Eigen Value used for Criteria 2</th>
<th>Step size in dB used for Criteria 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>4.0</td>
<td></td>
</tr>
</tbody>
</table>
The MSE and Timing results obtained for the RRSQRT using the 2 criterions are shown below for comparisons with the Kalman filter.

<table>
<thead>
<tr>
<th>S/N</th>
<th>Description of Filter</th>
<th>Time</th>
<th>Final MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Kalman filter</td>
<td>37.206 sec</td>
<td>-42.216 dB</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>% of initial Eigen Value used for Criteria 2</th>
<th>Step size in dB used for Criteria 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td>0.001</td>
<td>-42.216 dB / 961</td>
</tr>
<tr>
<td>0.01</td>
<td>-39.906 dB / 230</td>
</tr>
<tr>
<td>0.05</td>
<td>-32.726 dB / 26</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>% of initial Eigen Value used for Criteria 2</th>
<th>Step size in dB used for Criteria 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td>0.001</td>
<td>289.275 sec</td>
</tr>
<tr>
<td>0.01</td>
<td>78.994 sec</td>
</tr>
<tr>
<td>0.05</td>
<td>48.978 sec</td>
</tr>
</tbody>
</table>
3D PLOT OF RRSQRT RESULTS

MSE obtained for RRSQRT versus the 2 criterions

Computational Time required for RRSQRT versus the 2 criterions

Remaining Eigen vectors for RRSQRT versus the 2 criterions
From investigation, the Eigen decomposition operation of the RRSQRT takes up a significant portion of the total computational time.

If this operation is ignored, the difference in the timing results are as shown below:

<table>
<thead>
<tr>
<th>% of initial Eigen Value used for Criteria 2</th>
<th>Step size in dB used for Criteria 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td>0.001</td>
<td>289.275 sec</td>
</tr>
<tr>
<td>0.01</td>
<td>78.994 sec</td>
</tr>
<tr>
<td>0.05</td>
<td>48.978 sec</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>% of initial Eigen Value used for Criteria 2</th>
<th>Step size in dB used for Criteria 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td>0.001</td>
<td>80.454 sec</td>
</tr>
<tr>
<td>0.01</td>
<td>35.367 sec</td>
</tr>
<tr>
<td>0.05</td>
<td>24.203 sec</td>
</tr>
</tbody>
</table>
SUMMING UP RRSQRT

- From the results obtained, it is shown that the RRSQRT is able to produce good estimation of the target pixels even when the rank is greatly reduced.
- If a faster method is available for performing the Eigen Decomposition operation, then this technique will be as computational efficient as the Kalman filter while not suffering from instability problems.
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Besides the RRSQRT filter, there is also another filter that is also found to be much more efficient than the Wiener filter.

This filter was introduced in 1997 and is now gaining much popularity with the Communication and Radar community.

It is known as the Multi-Stage Wiener Filter (MSWF for short) based on orthogonal projections.

This filter can be also implemented in a full rank or reduced rank manner. But it has been shown in various applications that it is able to **achieve full rank performance even when implemented in reduced rank fashion**.
The basic MSWF is used for estimating a scalar desired signal from a series of measurement data (called it scalar MSWF). It makes use of the cross correlation between the measurement data vector and the desired signal to project an initial estimate of the desired signal after the so called 1st stage of decomposition. When there are correlations between the desired signal and the unwanted signals present in the measurement data, this 1st estimate will contain errors. Thus, using a blocking matrix orthogonal to the cross correlation vector, a new measurement data and a new cross correlation vector is formed again. This action is then repeated at the 2nd stage of decomposition and so forth until the error in the final estimate is diminished to an acceptable value.
GRAPHICAL VISUALIZATION OF MSWF

Desired Signal

Signal estimate after 1\textsuperscript{st} projection

Signal estimate after 2\textsuperscript{nd} projection
Note that the MSWF consists of 3 main steps, namely the forward iteration step, turn-around step and the backward iteration step. The equations for implementing the forward iteration step are shown:

\[ \delta_i = \sqrt{r_{x_{i-1}d_{i-1}}^H r_{x_{i-1}d_{i-1}}} \]

\[ h_i = \frac{r_{x_{i-1}d_{i-1}}}{\delta_i} \]

\[ d_i = h_i^H x_{i-1} \]

\[ \sigma_{d_i}^2 = h_i^H R_{x_{i-1}} h_i \]

\[ B_i = \text{null}\{h_i\} \]

\[ x_i = B_i x_{i-1} \]

\[ R_{x_i} = B_i R_{x_{i-1}} B_i^H \]

\[ r_{x_id_i} = B_i R_{x_{i-1}} h_i \]
The scalar MSWF is then implemented and run sequentially to estimate all the target pixels in the SAR image using 40 stages of decomposition. The final errors of the estimates using the scalar MSWF is then compared with that from the Kalman filter.
SUMMING UP SCALAR MSWF

- From results obtained, the scalar MSWF is able to estimate the target pixels with as much accuracy as the Kalman filter.
- Furthermore, the scalar MSWF only requires 40 stages of decomposition to achieve these results instead of full rank processing of 961 stages.
- The computational time needed per target pixel is much lesser than that of the Wiener filter.
- However, one undesirable feature is that it can only estimate 1 target pixel at a time, thus requires a long period to complete estimation of all target pixels if done in a serial manner, unless there are enough machines for parallel processing.
- Another weak point is that the processing of each target pixel also takes a longer time than the Kalman filter.
Therefore, these shortcomings necessitates the development of the vector version of the MSWF that can estimate more than 1 target pixel per target group at a time.

Now, for the vector MSWF implementation, it can also be implemented by executing each group of targets in a serial manner for all targets or using parallel processing with 1 MSWF processor per target group.

Therefore, both options are explored and results are presented for discussion.

Note that the Mean Square Error (MSE) obtained for both methods are identical for the same combination of number of targets per group versus stages of decomposition.
RESULTS OF MSE FOR VECTOR MSWF

Average Expected MSE obtained for MSWF

MSE (dB)

Stages of Decomposition

Tgt Group Size in % (logscale)

Average Expected MSE (dB) obtained for MSWF

MSE (dB)

Stages of Decomposition

Tgt Group Size in % (logscale)
RESULTS OF EXECUTION TIME (PARALLEL)

Time (Parallel) in sec required for MSWF

Stages of Decomposition

Tgt Group Size in % (logscale)
RESULTS OF EXECUTION TIME (SERIAL)
OVERALL COMBINATION OF RESULTS
The vector MSWF is able to provide pixel estimates with the same accuracy as Wiener and Kalman filter using less than full rank processing.

Some combinations of target group size and stages of decomposition will require lesser computational time than Wiener filter with an efficiency ratio of up to 1.65.

For both parallel and serial method of implementation, putting all targets in one group and using 1 stage of decomposition currently provides the best deal in terms of computational time and accuracy of results.

However, this best deal is still not as efficient as that of Kalman filter. Therefore, will need to check whether the trend holds true for larger data set.
Next, the target size and the measurement size are each increased by about 4 times

The results for the Wiener filter, Kalman filter and MSWF are shown below

<table>
<thead>
<tr>
<th>Filter Type</th>
<th>Total Time /sec</th>
<th>Average $\frac{\sigma}{\sigma_0}$/dB</th>
<th>Average Computed MSE /dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wiener</td>
<td>9349.3</td>
<td>-39.483</td>
<td>-39.439</td>
</tr>
<tr>
<td>MSWF</td>
<td>3432.5</td>
<td>-39.479</td>
<td>-39.439</td>
</tr>
</tbody>
</table>

We can see that the trend continues but MSWF’s edge over Wiener filter has increased significantly over the smaller data set from 1.65 to 2.72
We have observed that the MSWF is able to outperform the Wiener filter in terms of speed but it is not as efficient as Kalman filter.

Therefore, several variations are carried out to the standard implementation structure of the MSWF.

It is hoped that some of these variations will be able to speed up the MSWF but at little or no cost to its accuracy obtained for the results.

These approaches are termed as Innovative MSWF Implementations.

A total of 3 approaches are attempted.
In this approach, instead of using the same initial conditions for all target groups, the results of the 1st target group is used to fine tune the initial conditions of the 2nd target group.

Next, the results of the 1st and 2nd target group are used to modify the initial condition of the 3rd target group and so forth.

This form of initialization bears some similarity to the way the Kalman filter is being operated.
STRUCTURE OF MODIFIED INITIALIZATION

- Full set of Measurement Data used for MSWF processing

- Initial Conditions for Group #1: \( \hat{\gamma}_0, K_{\gamma 0} \)
- Initial Conditions for Group #2: \( \hat{\gamma}_1, K_{\gamma 1} \)
- Initial Conditions for Group #3: \( \hat{\gamma}_2, K_{\gamma 2} \)
- Initial Conditions for Group #J: \( \hat{\gamma}_{j-1}, K_{\gamma j-1} \)

- Start of MSWF processing for Target Group #1
- Start of MSWF processing for Target Group #2 after processing Group #1
- Start of MSWF processing for Target Group #3 after processing Group #2
- Start of MSWF processing for Target Group #J after processing Group #J-1

- Final Initial Conditions: \( \hat{\gamma}_J, K_{\gamma J} \)
RESULTS FOR MODIFIED DATA APPROACH

- Using the modified initialization approach, the results obtained for a few target group sizes are obtained and shown below.
SUMMING UP MODIFIED DATA APPROACH

- From the results obtained, we can see that the average number of decomposition stages required for any target group size decreases with the use of the modified data initialization approach.
- After performing some computation and taking into account the overheads associated with this approach, the net gain in the efficiency is about 28%.
- At the same time, there is no loss at all in the final results’ accuracy.
- Thus, this improvement is significant and this approach should be used whenever the serial MSWF implementation is chosen.
The 2nd approach is to group the targets together using their cross correlation to one other as a criteria.

The criteria can be to group targets based on their least correlation to one other or their highest correlation to one another.

This grouping mechanism is applied before the start of the MSWF processing and can be applied to either serial MSWF implementation or parallel MSWF implementation.

Both groupings based on least correlation and highest correlation are attempted.
STRUCTURE FOR TARGET GROUPING

- Full set of Measurement Data used for MSWF processing
- Initial Conditions for all Target groups $\gamma(0), K_{\gamma(0)}$
- Start of MSWF processing for Target Group #1
- Start of MSWF processing for Target Group #2
- Start of MSWF processing for Target Group #3
- Start of MSWF processing for Target Group #J
- Final $\gamma, K_{\gamma}$

Pre-Processing Task
Using Grouping Scheme A to rearrange the targets and the columns of the $P$ matrix
RESULTS FOR LEAST CORRELATION

Expected MSE of Target versus Stages of Decomposition

- Original Expected MSE of Grp 1 Tgt 1
- New Expected MSE of Grp 1 Tgt 1
- Original Expected MSE of Grp 48 Tgt 1
- New Expected MSE of Grp 48 Tgt 1

Total # of groups = 48
Group size = 20

Expected MSE of Target versus Stages of Decomposition

- Original Expected MSE of Grp 1 Tgt 1
- New Expected MSE of Grp 1 Tgt 1
- Original Expected MSE of Grp 9 Tgt 1
- New Expected MSE of Grp 9 Tgt 1

Total # of groups = 10
Group size = 100

Expected MSE of Target versus Stages of Decomposition

- Original Expected MSE of Grp 1 Tgt 1
- New Expected MSE of Grp 1 Tgt 1
- Original Expected MSE of Grp 2 Tgt 1
- New Expected MSE of Grp 2 Tgt 1

Total # of groups = 2
Group size = 480
RESULTS FOR HIGHEST CORRELATION

Expected MSE of Target versus Stages of Decomposition

- Original Expected MSE of Grp 1 Tgt 1
- New Expected MSE of Grp 1 Tgt 1
- Original Expected MSE of Grp 48 Tgt 1
- New Expected MSE of Grp 48 Tgt 1

Total # of groups = 48
Group size = 20

Expected MSE of Target versus Stages of Decomposition

- Original Expected MSE of Grp 1 Tgt 1
- New Expected MSE of Grp 1 Tgt 1
- Original Expected MSE of Grp 5 Tgt 1
- New Expected MSE of Grp 5 Tgt 1
- Original Expected MSE of Grp 9 Tgt 1
- New Expected MSE of Grp 9 Tgt 1

Total # of groups = 10
Group size = 100

Expected MSE of Target versus Stages of Decomposition

- Original Expected MSE of Grp 1 Tgt 1
- New Expected MSE of Grp 1 Tgt 1
- Original Expected MSE of Grp 2 Tgt 1
- New Expected MSE of Grp 2 Tgt 1
- Original Expected MSE of Grp 2 Tgt 1
- New Expected MSE of Grp 2 Tgt 1

Total # of groups = 2
Group size = 480
SUMMING UP TARGET GROUPING

- From the 2 trends observed, we can conclude that grouping targets that are highly correlated to one another into the same group will help to decrease the number of stages of decomposition required.
- However, due to the imperfection of my grouping schemes, not all the targets are grouped based on this desired criteria and therefore no numerical figures are available to show the improvement.
- Nevertheless, with a robust grouping scheme, this approach should bear fruitful results.
Now, beside trying different approaches on the target space as in the last 2 approaches, we can also look into the measurement data space itself. Drawing inspiration from the Kalman filter which is a recursive Wiener filter, we can also implement the MSWF in a recursive fashion by breaking up the total measurement data into smaller subsets and iteratively applying the MSWF on these data subsets. In this manner, it is hoped that computations involving smaller matrix dimensions in the forward iteration step will help to speed up the MSWF execution.
STRUCTURE OF RECURSIVE MSWF

Initial Conditions for all Target groups

\[ \hat{\gamma}_{0}^{1}, K_{\hat{\gamma}_{0}} \]

Start of MSWF processing for Target Group #1

Start of MSWF processing for Target Group #2

Start of MSWF processing for Target Group #3

Start of MSWF processing for Target Group #J

Initial Conditions for all Target groups

\[ \hat{\gamma}_{11}^{11}, K_{\hat{\gamma}_{11}} \]

Start of MSWF processing for Target Group #1

Start of MSWF processing for Target Group #2

Start of MSWF processing for Target Group #3

Start of MSWF processing for Target Group #J

Final \[ \hat{\gamma}, K_{\gamma} \]
Now, to examine the performance of the recursive MSWF, various measurement subset and target group sizes are chosen so as to get a good picture of this approach.

The combination used in the simulation is as shown:

<table>
<thead>
<tr>
<th>Measurements per subset</th>
<th>Number of subsets</th>
<th>Target Group size = 961</th>
<th>Target Group size = 480</th>
<th>Target Group size = 320</th>
<th>Target Group size = 160</th>
</tr>
</thead>
<tbody>
<tr>
<td>2856</td>
<td>1</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>1428</td>
<td>2</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>714</td>
<td>4</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>476</td>
<td>6</td>
<td>X</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

Note that some combinations are not achievable because of the vector MSWF constraints.
RESULTS FOR RECURSIVE MSWF

- Results obtained for the recursive MSWF are encouraging and some combination is able to achieve faster computational time with no loss of results’ accuracy.
- The results for the recursive parallel MSWF is shown here

<table>
<thead>
<tr>
<th>Measurements per subset</th>
<th>Number of subsets</th>
<th>Target Group size = 961</th>
<th>Target Group size = 480</th>
<th>Target Group size = 320</th>
<th>Target Group size = 160</th>
</tr>
</thead>
<tbody>
<tr>
<td>2856</td>
<td>1</td>
<td>113.203 sec</td>
<td>241.112 sec</td>
<td>222.498 sec</td>
<td>215.547 sec</td>
</tr>
<tr>
<td>1428</td>
<td>2</td>
<td>103.735 sec</td>
<td>151.110 sec</td>
<td>130.548 sec</td>
<td>103.331 sec</td>
</tr>
<tr>
<td>714</td>
<td>4</td>
<td>X</td>
<td>78.728 sec</td>
<td>69.301 sec</td>
<td>54.669 sec</td>
</tr>
<tr>
<td>476</td>
<td>6</td>
<td>X</td>
<td>X</td>
<td>43.549 sec</td>
<td>37.560 sec</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Measurements per subset</th>
<th>Number of subsets</th>
<th>Target Group size = 961</th>
<th>Target Group size = 480</th>
<th>Target Group size = 320</th>
<th>Target Group size = 160</th>
</tr>
</thead>
<tbody>
<tr>
<td>2856</td>
<td>1</td>
<td>-41.037 dB</td>
<td>-41.037 dB</td>
<td>-41.037 dB</td>
<td>-41.037 dB</td>
</tr>
<tr>
<td>1428</td>
<td>2</td>
<td>-41.037 dB</td>
<td>-39.880 dB</td>
<td>-39.350 dB</td>
<td>-38.959 dB</td>
</tr>
<tr>
<td>714</td>
<td>4</td>
<td>X</td>
<td>-37.093 dB</td>
<td>-34.392 dB</td>
<td>-30.052 dB</td>
</tr>
<tr>
<td>476</td>
<td>6</td>
<td>X</td>
<td>X</td>
<td>-25.015 dB</td>
<td>-21.524 dB</td>
</tr>
</tbody>
</table>
Implementing the MSWF as a recursive filter seems to be a good approach for both the parallel and serial MSWF implementation.

With certain combinations of measurement data subset and target group size, we can get improvements in the computational speed by up to 100% with little loss in accuracy.

This ability to process new data iteratively also makes the MSWF to be more attractive than the Wiener filter.
OUTLINE

- Introduction
  - The Non-Uniformly Distributed Aperture Radar System
  - The Radar model
  - Previous work done on Filtering Techniques
  - Thesis motivation

- The Reduced Rank Square Root Filter Approach
  - Design and Implementation
  - Choice of criterion determination
  - Discussion of Results

- The Multi-Stage Wiener Filter Approach
  - Design and Implementation
  - Discussion of Results
  - Innovation Implementations

- Conclusions and Future work
CONCLUSIONS

- We are able to successfully implement Reduced Rank Filtering Techniques for the Non-Uniformly Distributed Aperture Radar System that are faster in execution than the Wiener filter while achieving the same accuracy in the final results.
- With a more efficient Eigen Decomposition engine, the RRSQRT filter will be able to match or exceed the computational speed of the Kalman filter while avoiding the Kalman filter’s pitfall of divergence or instability issue.
- It has been shown that the Multi-Stage Wiener filter is able to achieve the same accuracy as the Wiener filter but requiring lesser rank in the filter processing.
- Although the MSWF is not as fast as the Kalman filter, innovative implementations will help to narrow the gap between the two filters.
FUTURE WORK

- For the RRSQRT filter, we can look into faster ways to compute the Eigen Decomposition as compared to the existing function in MATLAB.
- We can also design some target scenarios where it is possible to start the RRSQRT filter processing with a smaller rank that will speed up its subsequent iteration process.
- For the MSWF, we can also look into a more optimal grouping scheme for the targets such that the deficiencies in the current scheme can be corrected and then use the optimal scheme for implementation.
- We can also look into alternative model of the MSWF that is more simple than the current MSWF model that is based on orthogonal projections. For example, one alternative model will be the Conjugate Gradient MSWF model.
QUESTIONS

THANK YOU