Reduced-Complexity Joint Frequency, Timing and Phase Recovery for PAM Based CPM Receivers

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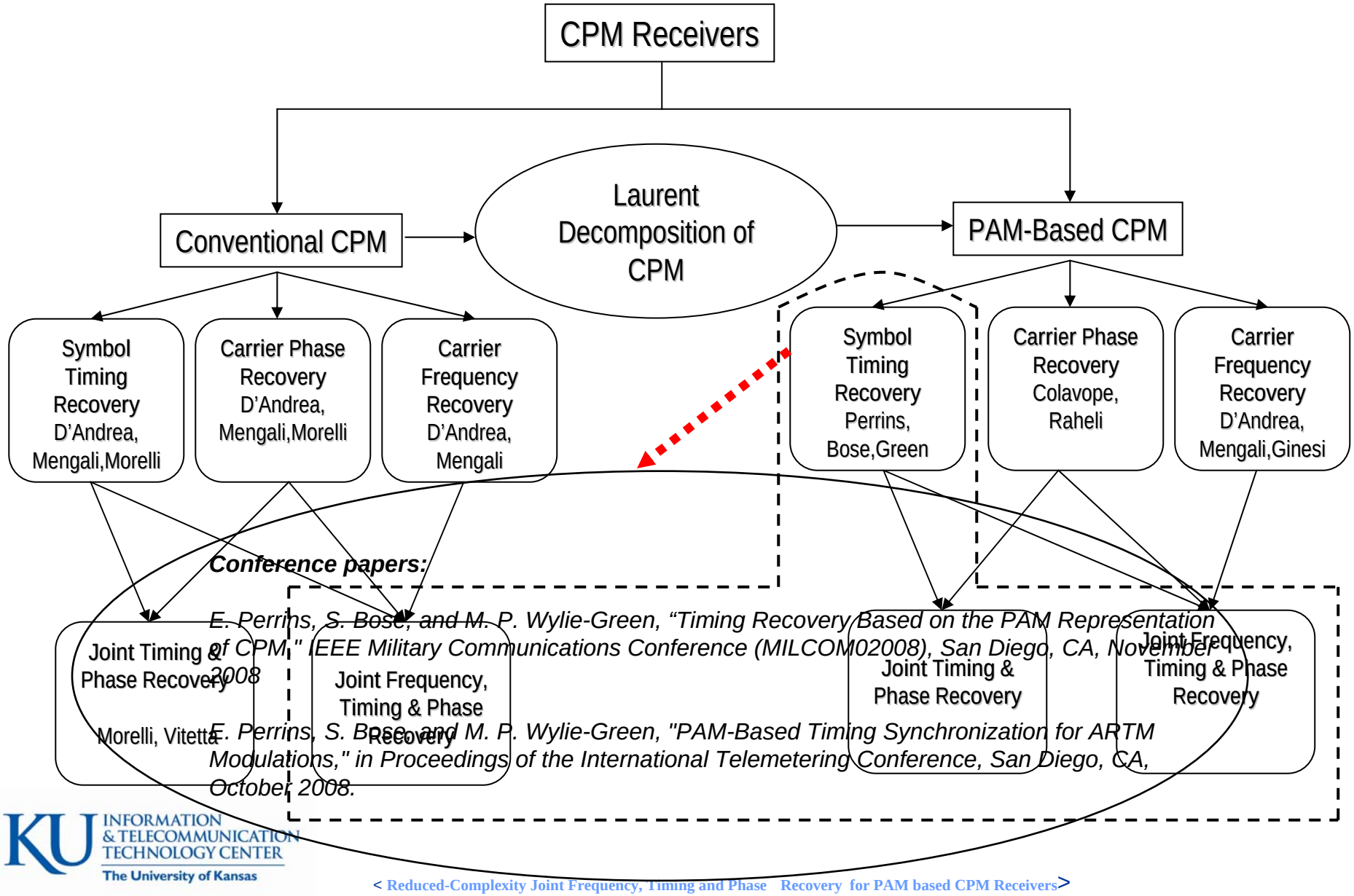
Overview

- Introduction to Continuous Phase Modulation (CPM) related work
- Motivation of research
- Signal models and complexity reduction principle
- Joint timing and phase error detector (TED & PED)
- Effect of large frequency offsets on TED and PED
- Performance analysis metrics and bounds
- Simulation results
- False lock recovery using reduced complexity detector configurations
- Conclusions and future work

Introduction to Continuous Phase Modulation (CPM) related work



Conference papers:

E. Perrins, S. Bose, and M. P. Wylie-Green, "Timing Recovery Based on the PAM Representation of CPM," IEEE Military Communications Conference (MILCOM02008), San Diego, CA, November 2008

E. Perrins, S. Bose, and M. P. Wylie-Green, "PAM-Based Timing Synchronization for ARTM Modulations," in Proceedings of the International Telemetering Conference, San Diego, CA, October 2008.

Motivation of research

Why CPM?

– Power and bandwidth efficient.

– Easy to use with low-cost PAs.

Problems with CPM

– Receiver complexity.

– Receiver synchronization.

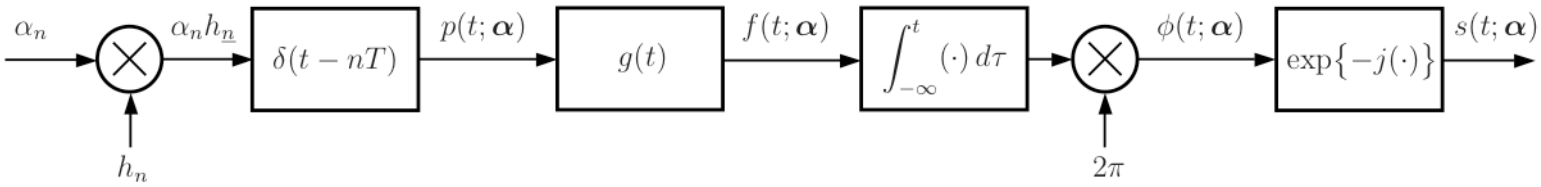
–**Motivation for using Pulse Amplitude Modulation (PAM)**

– Linearize CPM; first proposed for binary CPMs in the well-known paper by Laurent.

– Reduce receiver complexity by discarding low-energy PAM pulses.

– Recover symbol timing using simple algorithms.

Conventional CPM signal model



$$s(t;\alpha) = A\exp\left\{ j2\pi h\sum_{i=0}^{n}\alpha_i q(t-iT)\right\}$$

- f(t) is the frequency pulse, it has a finite duration of L symbol times and an area of 1/2.
- q(t) is the time-integral of f(t)
- h is the modulation index, it is typically a rational number
- α_n are drawn from an M-ary alphabet

Conventional CPM signal model...contd.

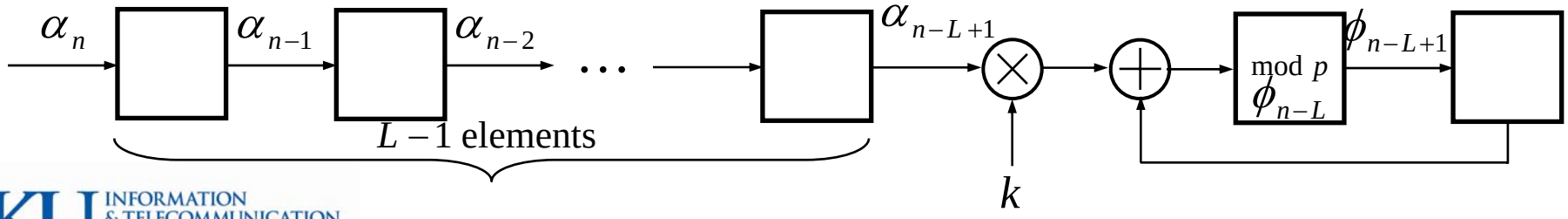
- The phase can be grouped into two terms since q(t) = 1/2 for t>LT:

$$\phi(t;\alpha) = 2\pi h \sum_{i=0}^{n} \alpha_i q(t - iT)$$

$$= \underbrace{2\pi h \sum_{i=n-L+1}^{n} \alpha_i q(t - iT)}_{\phi(t;\underline{\alpha}_n)} + \underbrace{\pi h \sum_{i=0}^{n-L} \alpha_i}_{\phi_{n-L}}$$

- Since the modulation indexes are rational numbers, h=k/p, we can describe the signal with a finite state machine:

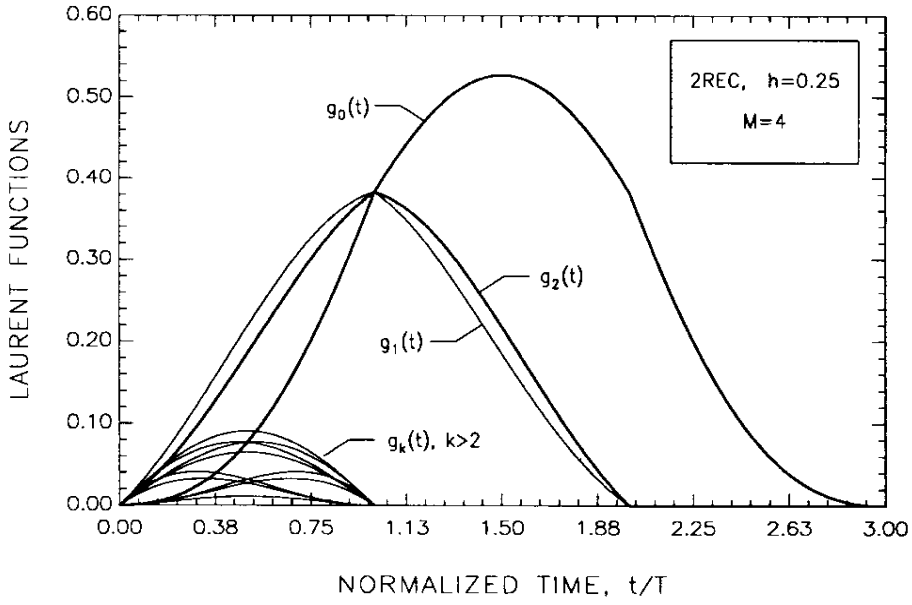
$$\sigma_n = (\cdots, \alpha_{n-3}, \alpha_{n-2}, \alpha_{n-1}, \alpha_n) \quad \leftrightarrow \quad \sigma_n = \underbrace{(\phi_{n-L}, \alpha_{n-L+1}, ..., \alpha_{n-1}, \alpha_n)}_{pM^{L-1} \text{ states}}$$



Pulse Amplitude Modulation (PAM) based CPM model

- M-ary Single-h

$$s(t;\alpha) = \sqrt{\frac{E_s}{T_s}} \sum_{k=0}^{N-1} \sum_{i} b_{k,i} g_k (t - iT_s)$$



- PAM complexity reduction principle

Number of PAM Components $N = 2^{P(L-1)}(M-1)$ and $P = \log_2 M$

$$s(t;\alpha) = \sqrt{\frac{E_s}{T_s}} \sum_{i} e^{j\phi_{i-L}} \sum_{k=0}^{N-1} b_k(c_i) g_k(t - iT_s) \qquad \kappa \subseteq \{0,1,\cdots,N-1\}$$

- Subset the largest amplitude pulses to reduce the number of matched filters (MF)
- Reduce number of trellis states in the detector

PAM based joint timing and phase error detector

- Received signal model in AWGN channel

$$r(t) = e^{j\theta}\sqrt{\frac{E_s}{T_s}}\sum_{k=0}^{N-1}\sum_{i} b_{k,i}\, g_k(t-\tau-iT_s) + w(t)$$

- Coherent detection
  - Symbol detection using the Viterbi algorithm (VA)
  - Decision-directed timing recovery
  - Decision-directed phase recovery
- Noncoherent detection
  - Symbol detection using the Viterbi algorithm
  - Decision-directed timing recovery without explicit phase information

  Note:
  - Timing and phase recovery uses decisions from the receiver.
  - Symbol detection and signal recovery are based on maximum likelihood principle.

PAM based joint timing and phase error detector

- Metric increment in VA for sequence detection

$$\sum_{i=0}^{L_0-1} \mathrm{Re}\left\{ y_i(\tilde{c}_i, \tilde{\phi}_{i-L'}, \tau) e^{-j\theta} \right\} = 0 \qquad 0 \le t \le L_0 T_s$$

- PAM-based TED is given by $\dot{y}_i(c_i, \phi_{i-L}, \tilde{\tau}) e^{-j\theta} = \sum_{k \in k_{TED}} b^*_{k,i} \dot{x}_{k,i}(\tilde{\tau})$

  where the TED increment $\sum_{i=0}^{L_0-1} \mathrm{Re}\{ \dot{y}_i(c_i, \phi_{i-L}, \tilde{\tau}) e^{-j\theta} \} = 0$

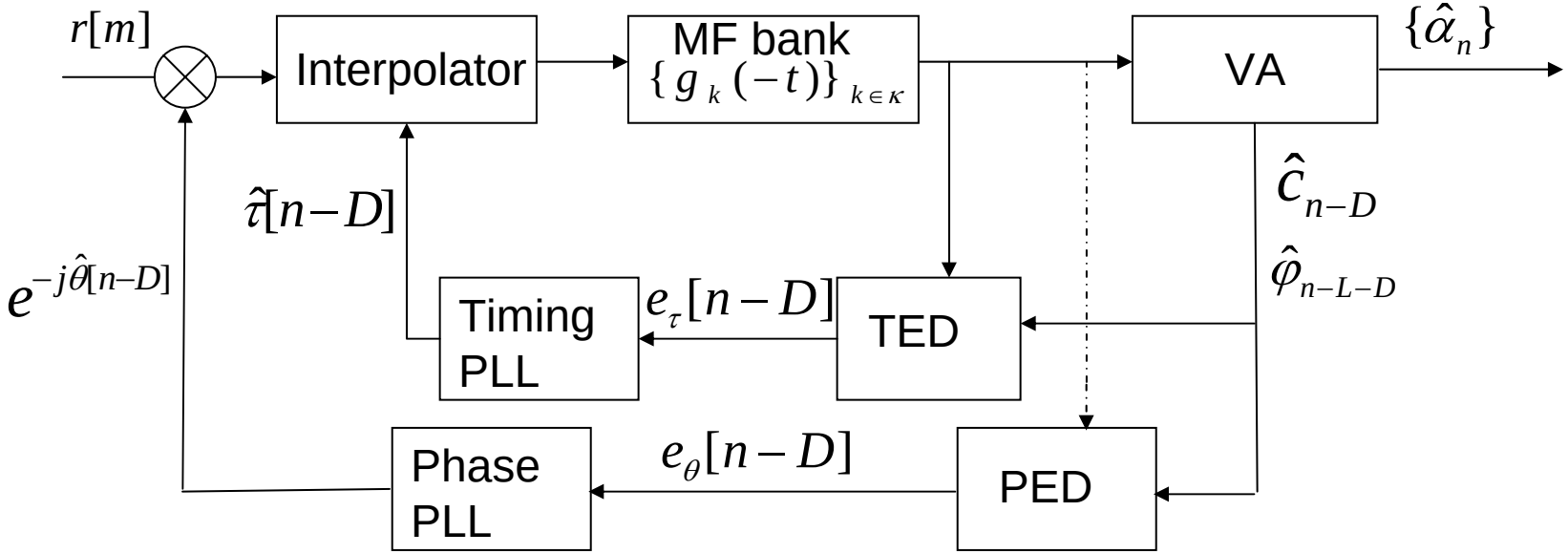
- PAM-based PED is given by $z_i(c_i, \phi_{i-L}, \tau) e^{-j\tilde{\theta}} = \sum_{k \in k_{PED}} b^*_{k,i} x_{k,i}(\tau)$

  where the PED increment $\sum_{i=0}^{L_0-1} \mathrm{Im}\{ z_i(c_i, \phi_{i-L}, \tau) e^{-j\tilde{\theta}} \} = 0$

MF bank filter output $x_{k,i}(\tau) \cong \int_{\tau+iT_s}^{\tau+(i+D_k)T_s} r(t) e^{-j\theta} g_k(t - \tau - iT_s) dt$

and $1 \le D_k \le L+1$

Error detector implementation



Error signal from the TED $e_\tau[n-D] = \mathrm{Re}\left\{\dot{y}_{n-D}(\hat{c}_{n-D}, \hat{\phi}_{n-L-D}, \hat{\tau}[n-D])e^{-j\hat{\theta}[n-D]}\right\}$

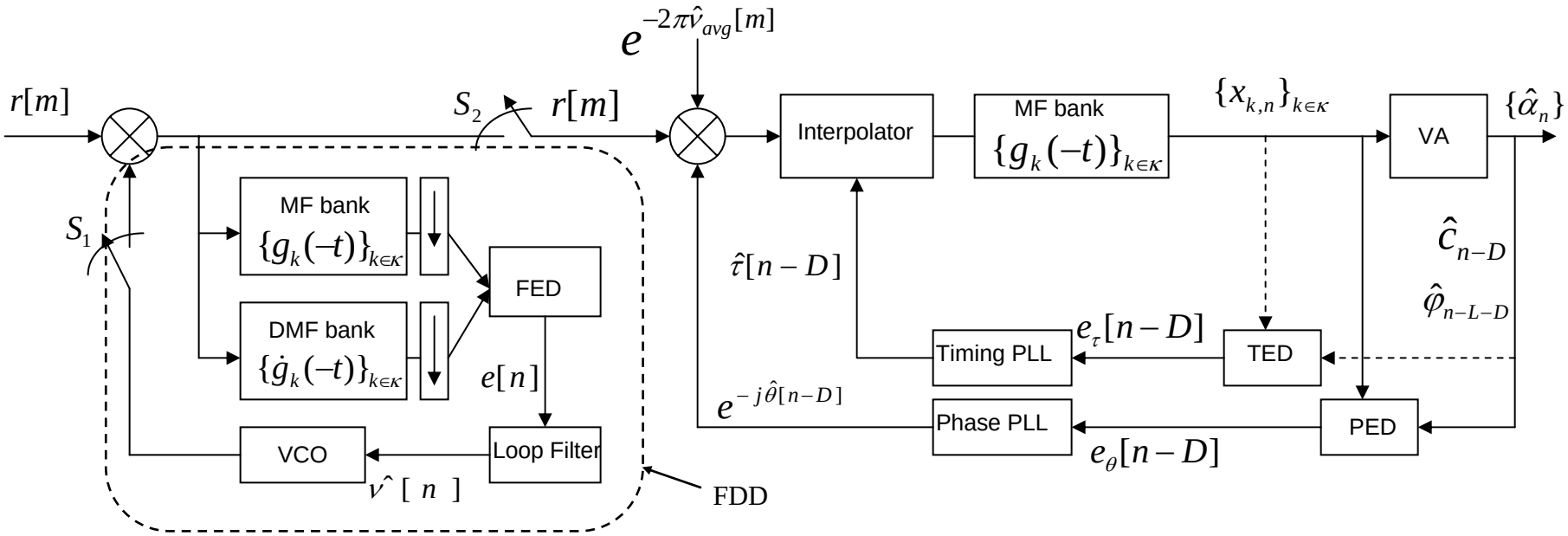
Error signal from the PED $e_\theta[n-D] = \mathrm{Im}\left\{z_{n-D}(\hat{c}_{n-D}, \hat{\phi}_{n-L-D}, \hat{\tau}[n-D])e^{-j\hat{\theta}[n-D]}\right\}$

Note:

- Matched filters estimate data symbols through VA implementation.
- Derivative matched filters generate TED Error hence timing estimate.

A discrete-time differentiator approximates the derivative.

Effect of large frequency offsets on TED and PED



- Frequency offset on the order of the symbol rate 1/Ts

  - A non-data-aided (NDA) frequency recovery is done before attempting symbol sequence, timing and phase recovery. A Frequency Difference Detector (FDD) is employed for this purpose.

  - Timing recovery without the explicit recovery of phase (noncoherent) is more suitable as phase recovery is still difficult due to the average residual frequency jitter $\hat{v}_{avg}$.

Performance analysis metrics and bounds

• We use modified Cramer-Rao bound (MCRB) to establish a lower bound on the degree of accuracy to which τ , θ and ν can be estimated.

- Normalized MCRB - timing $\frac{1}{T_s^2} \times MCRB(\tau) = \frac{1}{8\pi^2 h^2 C_\alpha C_f L_0} \times \frac{1}{E_s / N_0}$

  where $C_\alpha \cong E\{\alpha_n{}^2\} = (M^2 - 1)/3$ for uncorrelated data symbols

  special cases : 1) LREC: $C_f = C_{LREC} \cong 1/(4L)$

  2) LRC: $C_f = C_{LRC} \cong 3/(8L)$

  - Normalized timing error variance $\frac{1}{T_s^2} \times \sigma_\tau^2 \cong \frac{1}{T_s^2} \times Var\{\hat{\tau}[n] - \tau\}$

- MCRB – phase $MCRB(\theta) = \frac{1}{2L_0} \times \frac{1}{E_s / N_0}$

  - Phase error variance $\sigma_\theta^2 \cong Var\{\hat{\theta}[n] - \theta\}$

- MCRB – frequency $T_s^2 \times MCRB(\nu) = \frac{3}{2\pi^2 L_0^3} \times \frac{1}{E_s / N_0}$

  - Normalized frequency error variance $T_s^2 \times \sigma_\nu^2 \cong T_s^2 \times Var\{\hat{\nu}[n] - \nu\}$

Performance analysis metrics and bounds…contd.

- Phase Locked Loop (PLL) Considerations
  - Error detector outputs are refined into suitable offset estimates
  - The loop bandwidth BT_s is an important parameter determining the performance of the synchronizers.
- Timing PLL
  - First order timing PLL implementation refines the TED output $e_\tau[n]$ after every T_s, $\hat{\tau}[n] \cong \hat{\tau}[n-1] + \gamma_\tau e_\tau[n]$. PLL step size is $\gamma_\tau \cong \frac{4B_\tau T_s}{k_{p\tau}}$
- Phase PLL
  - The new phase estimate from the PLL is obtained as $\hat{\theta}[n] \cong \hat{\theta}[n-1] + \gamma_\theta \xi[n]$
    - First order PLL with no carrier frequency offset and $\xi[n] = e_\theta[n]$
    - Second order PLL with residual carrier frequency offset

      $\xi[n] = \xi[n-1] + (K1 + K2)\kappa_{p\theta} e_\theta[n] - K2\kappa_{p\theta} e_\theta[n-1]$

      K1 and K2 are proportional and integration constants respectively
- Frequency PLL
  - First order frequency PLL refines FDD output after every T_s as $\hat{v}[n] \cong \hat{v}[n-1] + \gamma_v e_v[n]$. PLL step size is $\gamma_v \cong \frac{4B_v T_s}{k_{pv}}$

Performance analysis metrics and bounds…contd.

- S-Curve identifies the stable lock points
  - These are the zero-crossing positive slope points on the curve.
  - We want the such a point at zero error, otherwise it is a false lock point
  - Decision directed M-ary TED and PED have false lock points
  - FDD is NDA, therefore, free of false lock points.

- S-curve for TED
  - $S(\delta_\tau) \cong \sqrt{E_s / T_s}.E\{e_\tau[n] \mid \delta_\tau\}$

  where $\delta_\tau \cong \tau - \hat{\tau}$ is timing offset

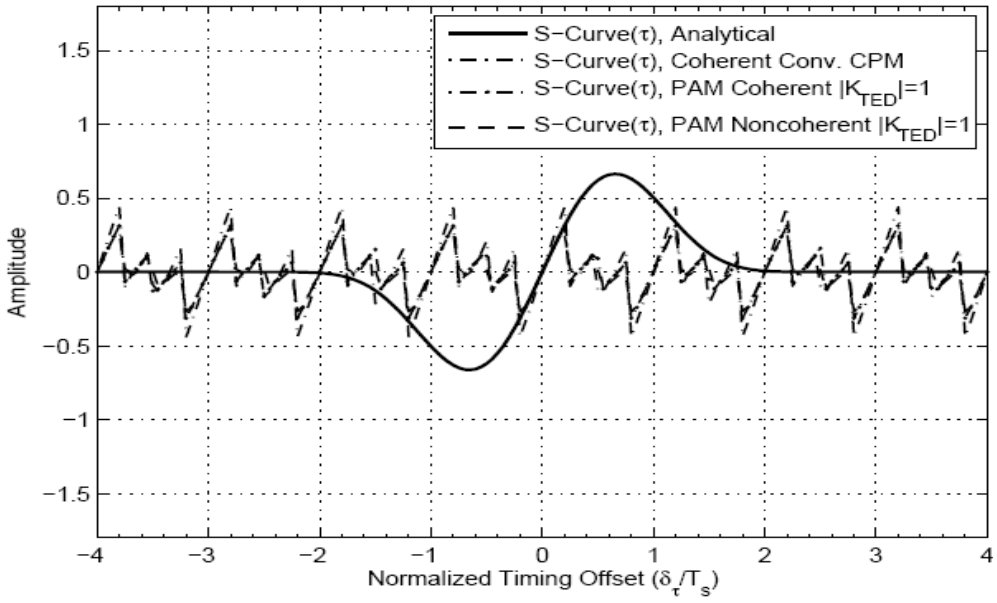


Figure 6.6. S-Curves for the TED. The modulation scheme is Quaternary CPM ($M = 4, L = 2, h = 1/4$).

Performance analysis metrics and bounds…contd.

S-curve for PED

- $S(\delta_\theta) \cong \sqrt{E_s / T_s}.E\{e_\theta[n] \mid \delta_\theta\}$

where $\delta_\theta \cong \theta - \hat{\theta}$ is the phase offset

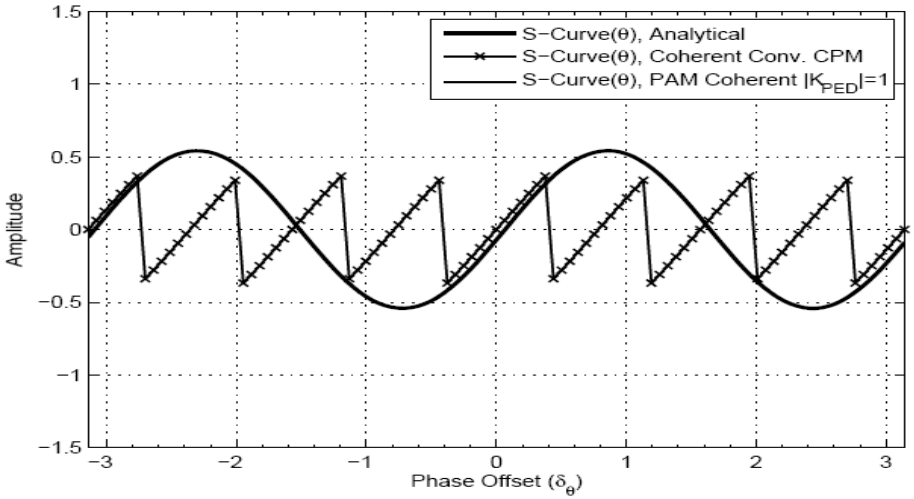


Figure 6.7. S-Curves for the PED. The modulation scheme is Quaternary CPM ($M = 4, L = 2, h = 1/4$).

•S-curve for FDD

- $S(\delta_\nu) \cong \sqrt{E_s / T_s}.E\{e_\nu[n] \mid \delta_\nu\}$

where $\delta_\nu \cong \nu - \hat{\nu}$ is the frequency offset

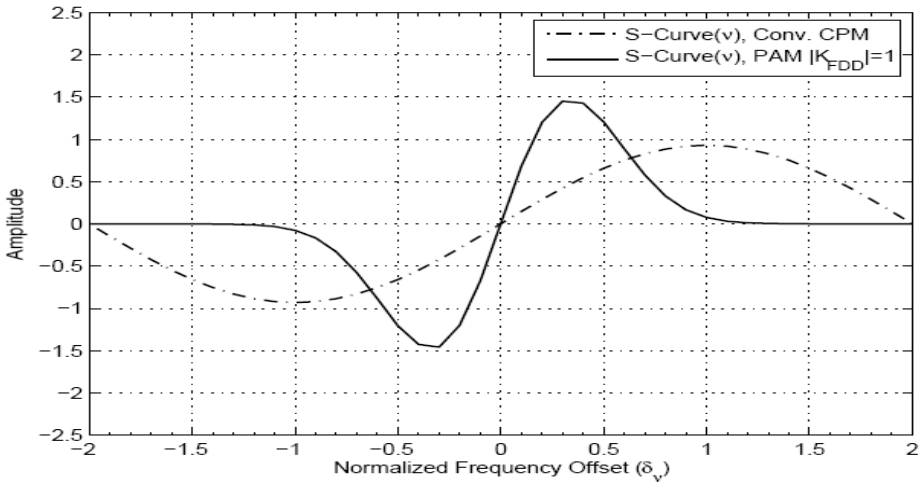


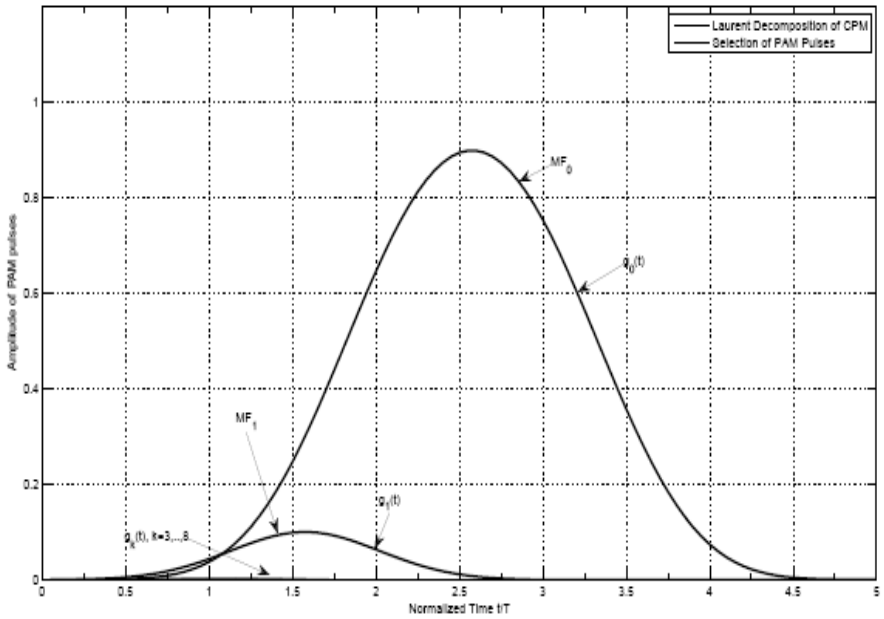
Figure 6.15. S-Curves for the FDD. The modulation scheme is M-ary CPM ($M = 4$, 2RC, $h = 1/4$).

Simulation results

• Binary GMSK system with Gaussian pulses (Gaussian Minimum Shift Keying)

M=2, 4G, h=1/2

- Optimal PAM based detector
  - Trellis state = 16
  - 8 single-h MFs/Pulses



- Reduced complexity detectors chosen for this example
  - 4 state detector with L' = 2
  - $|\kappa| = |\kappa_{TED}| = |\kappa_{PED}| = 2$ MFs/pulses.
  - $|\kappa| = 2$ MFs and $|\kappa_{TED}| = |\kappa_{PED}| = 1$ pulse.

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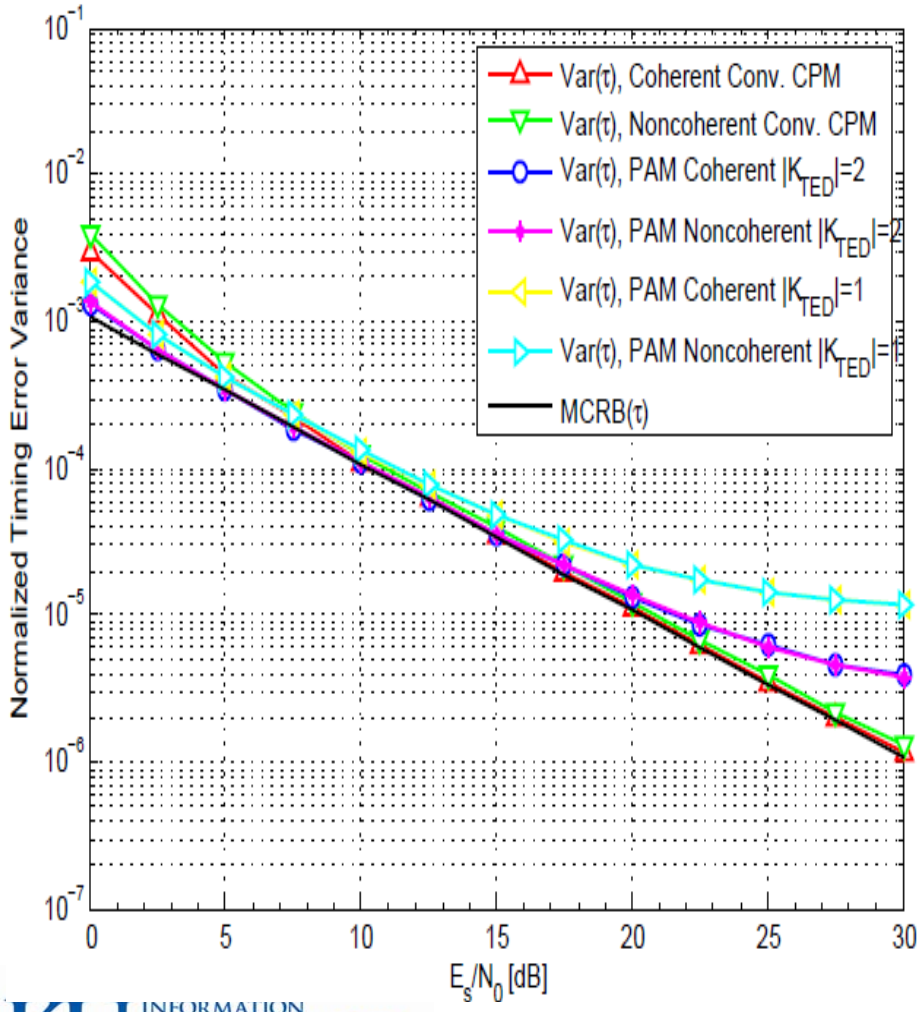
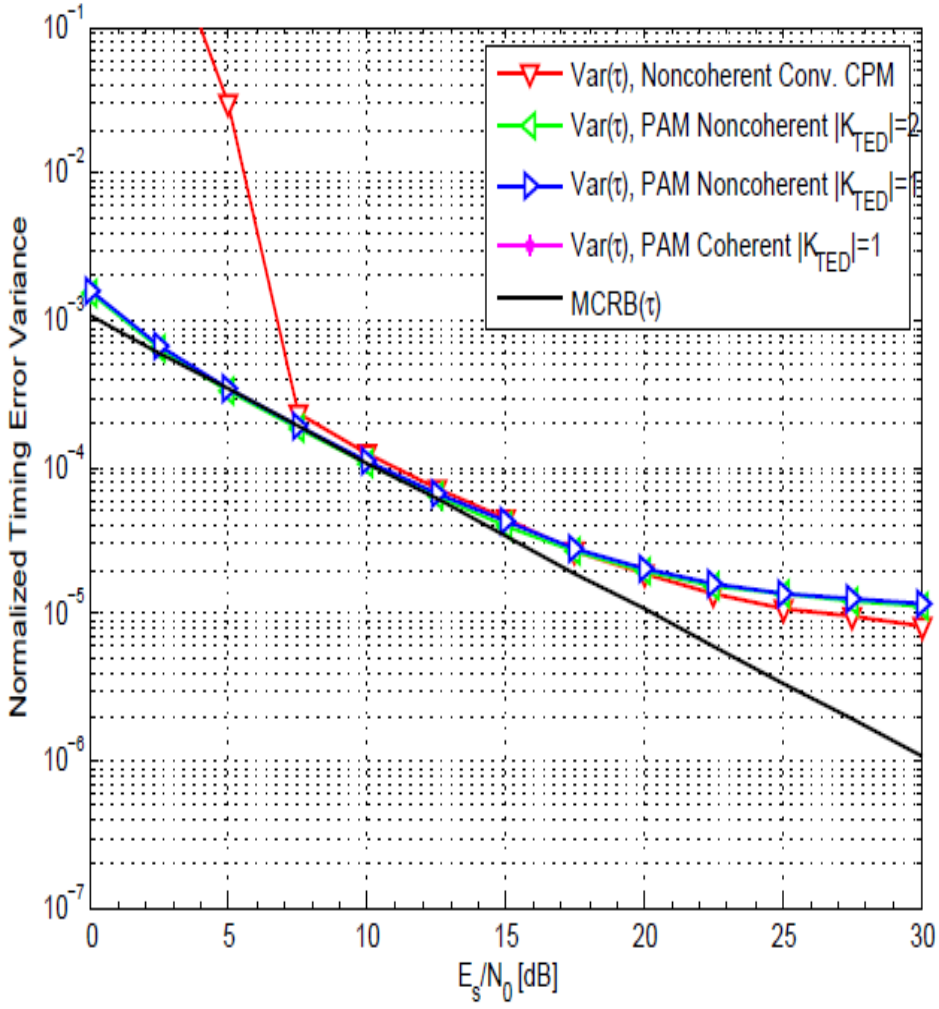
Simulation results (Binary GMSK…)

•*Timing error variance with no carrier frequency offset*

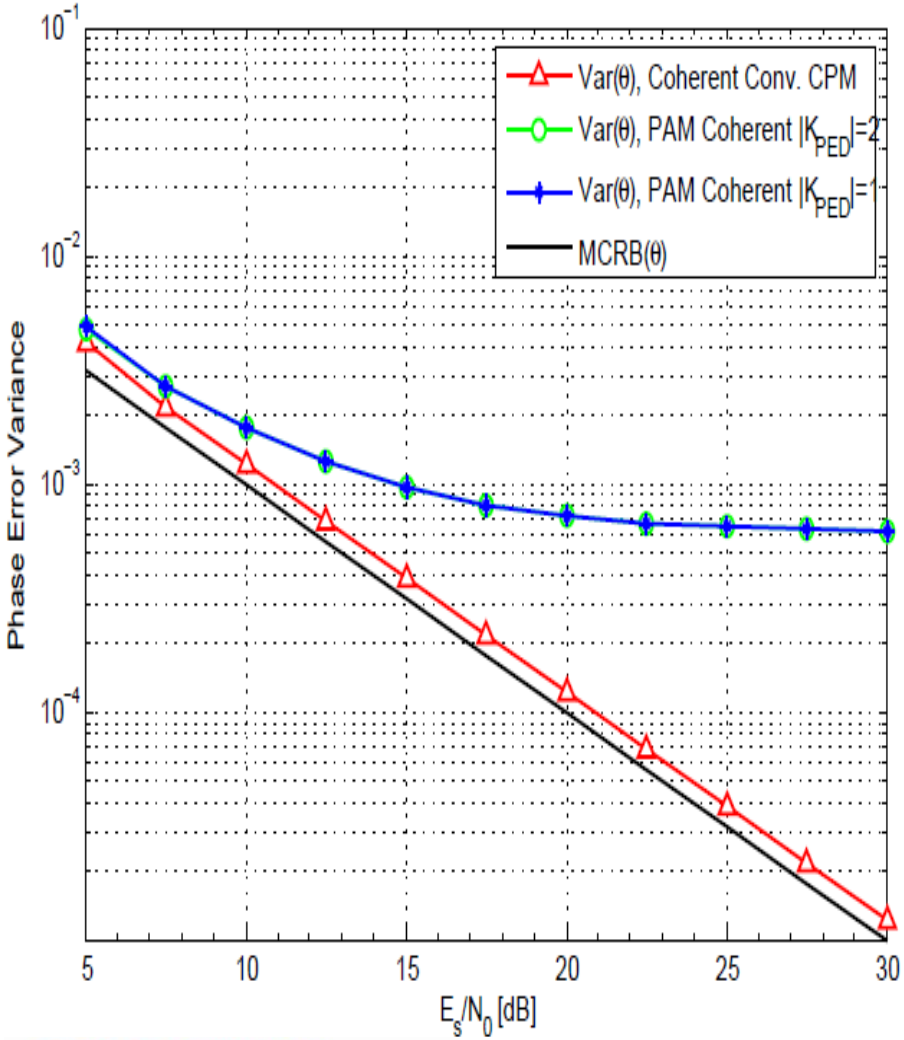


•*Timing error variance with a large carrier frequency offset*

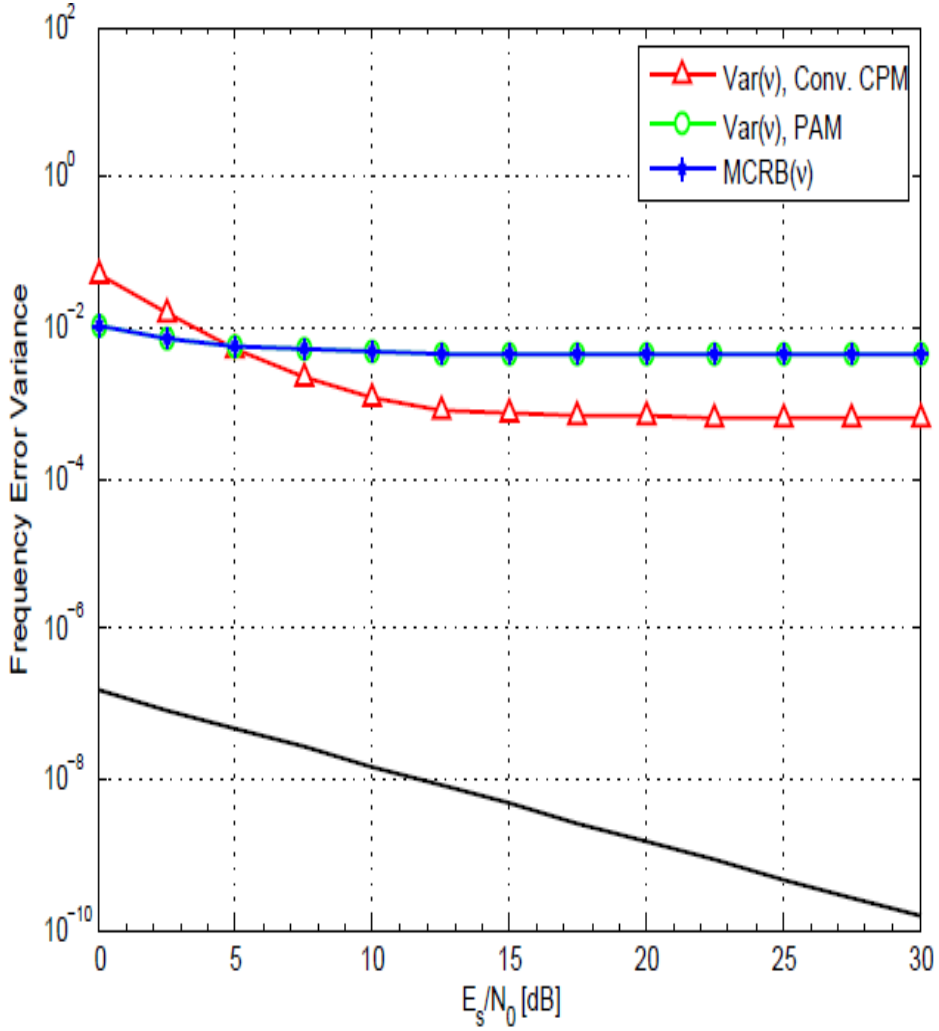


Simulation results (Binary GMSK)

•*Phase error variance with no carrier frequency offset*



•*Frequency error variance with a large carrier frequency offset*



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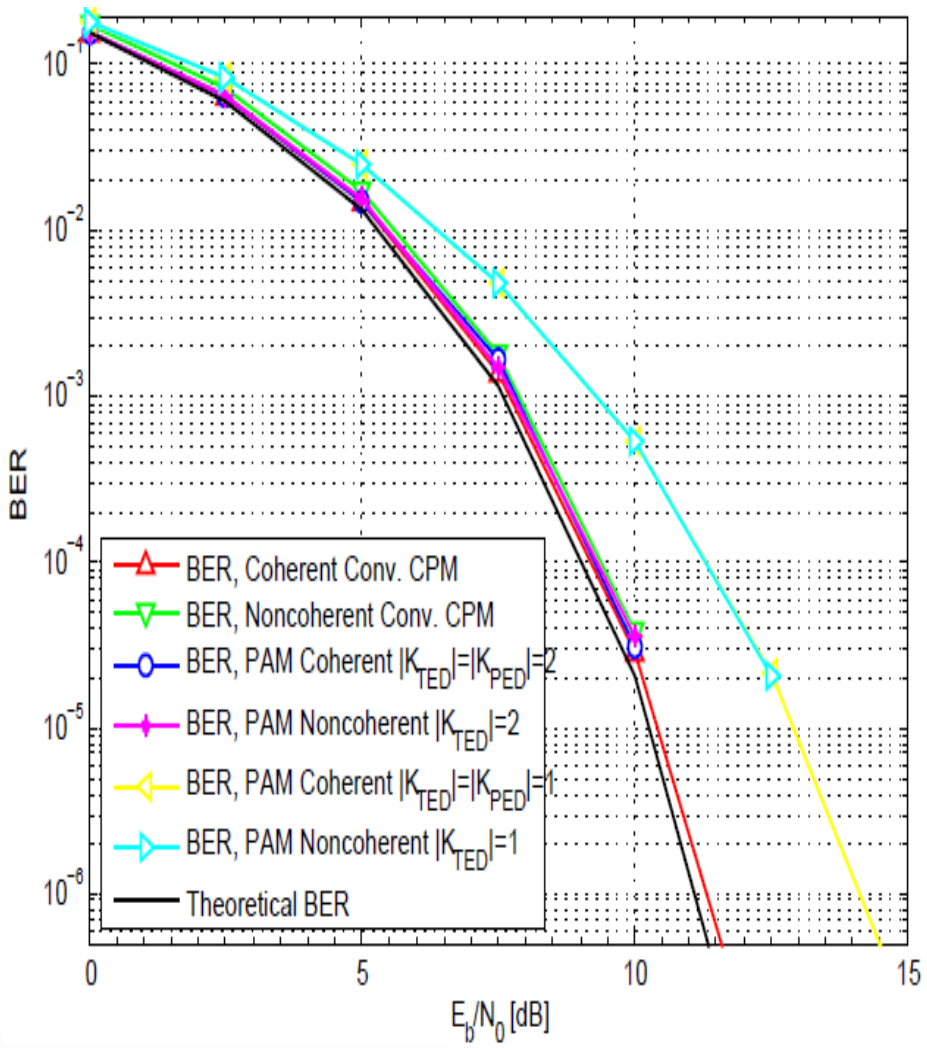
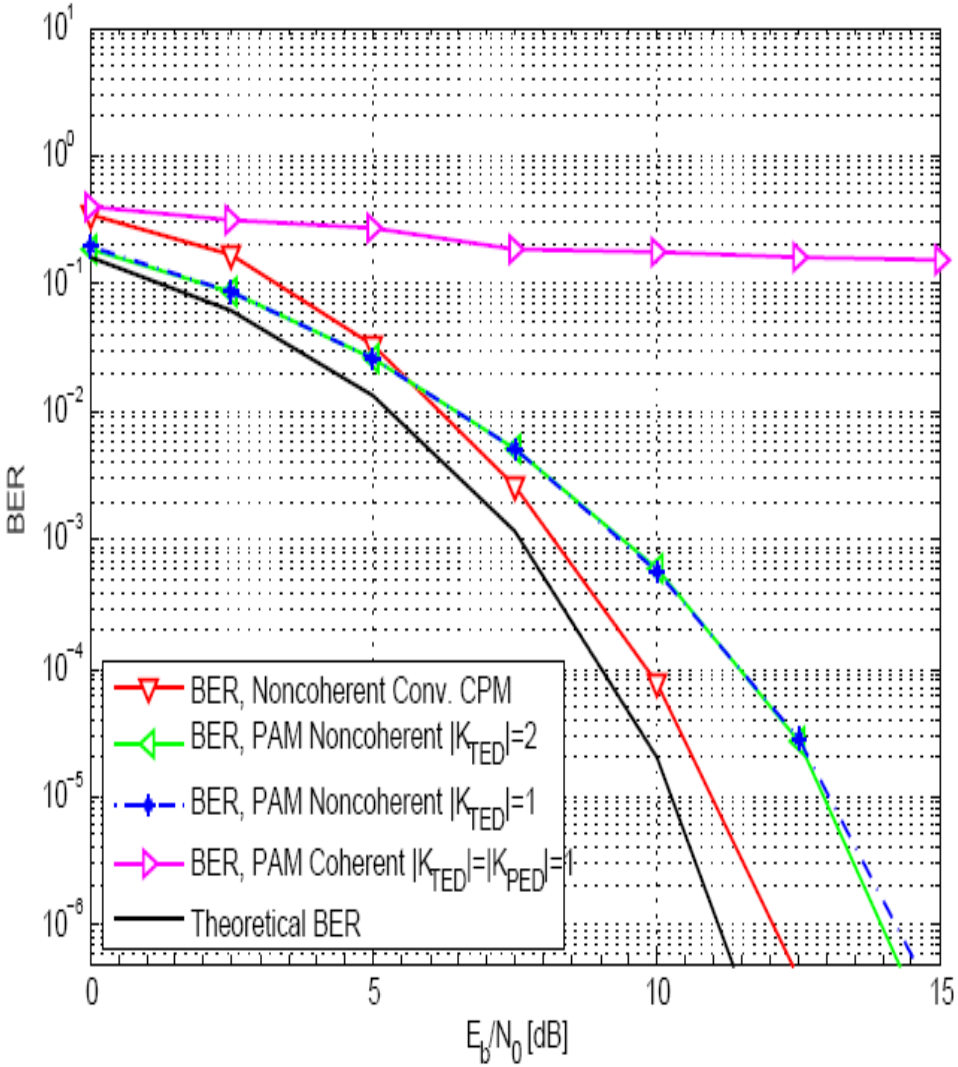
Simulation results (Binary GMSK)

•*BER with no carrier frequency offset*



•*BER with a large carrier frequency offset*

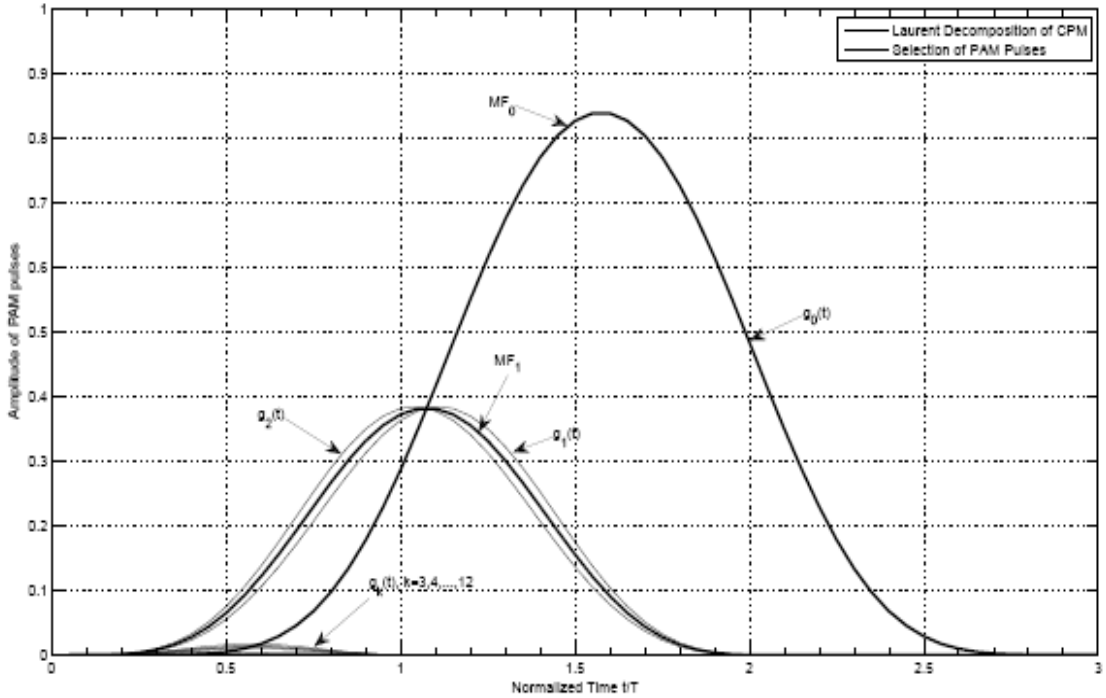


Simulation results

- M-ary CPM system with partial response

M=4, 2RC, h=1/4

- Optimal PAM based detector
- Trellis state = 16
- 12 single-h MFs/Pulses



Reduced complexity detectors chosen for this example

- 4 state detector with L' = 1
- $|\kappa| = |\kappa_{TED}| = |\kappa_{PED}| = 2$ MFs/pulses.
- $|\kappa| = 2$ MFs and $|\kappa_{TED}| = |\kappa_{PED}| = 1$ pulse.

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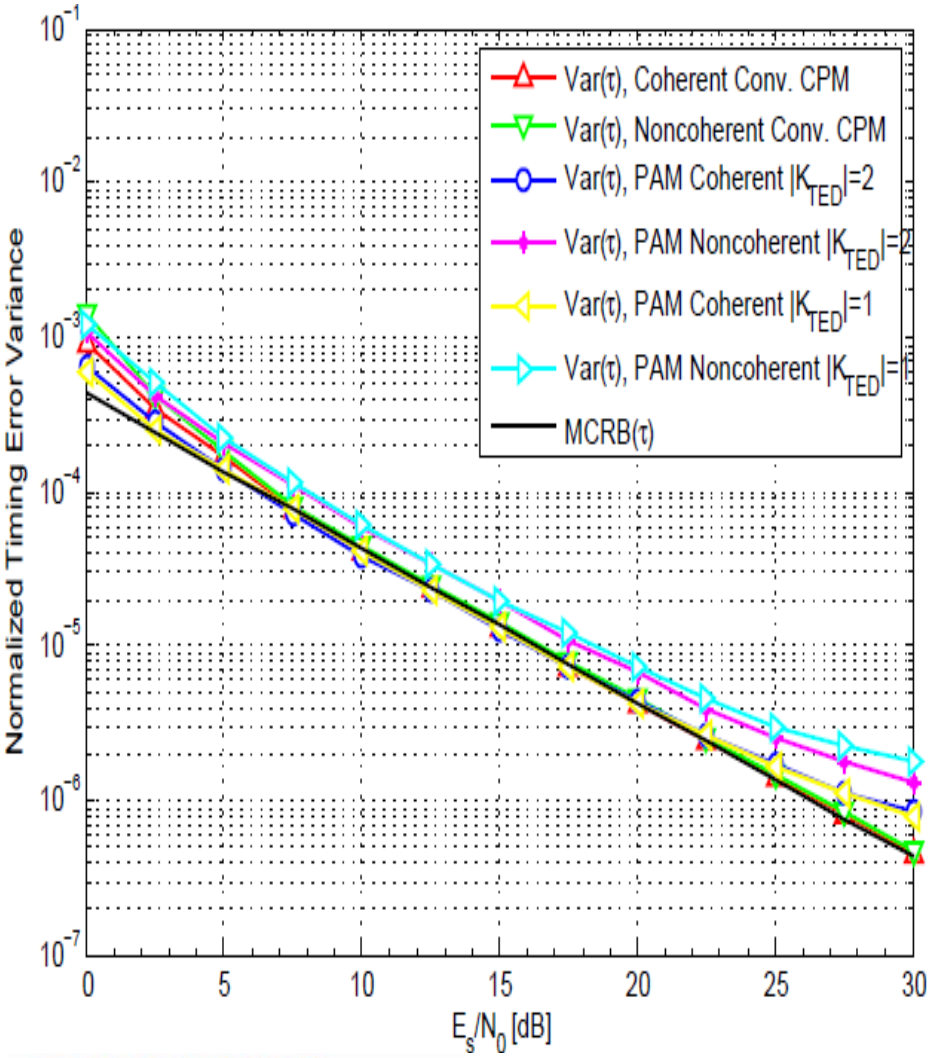
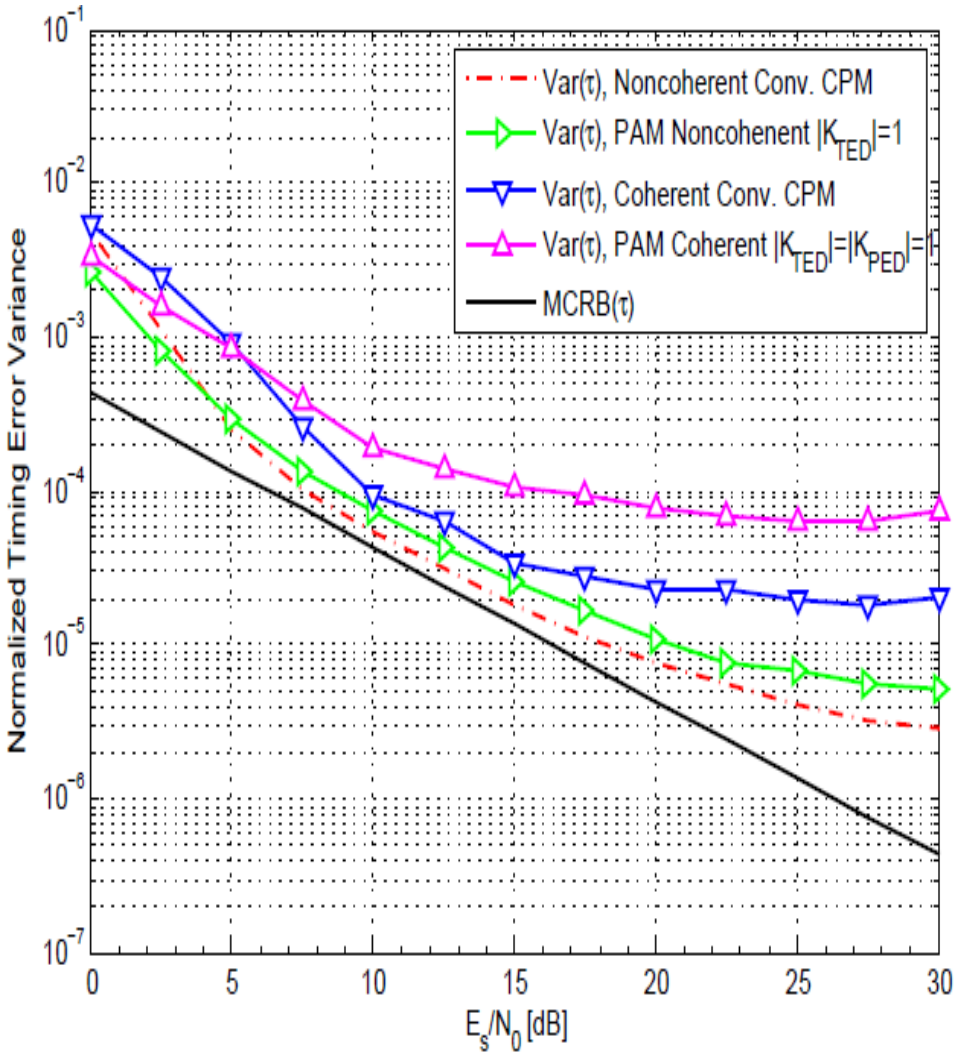
Simulation results (M-ary CPM…)

•*Timing error variance with no carrier frequency offset*

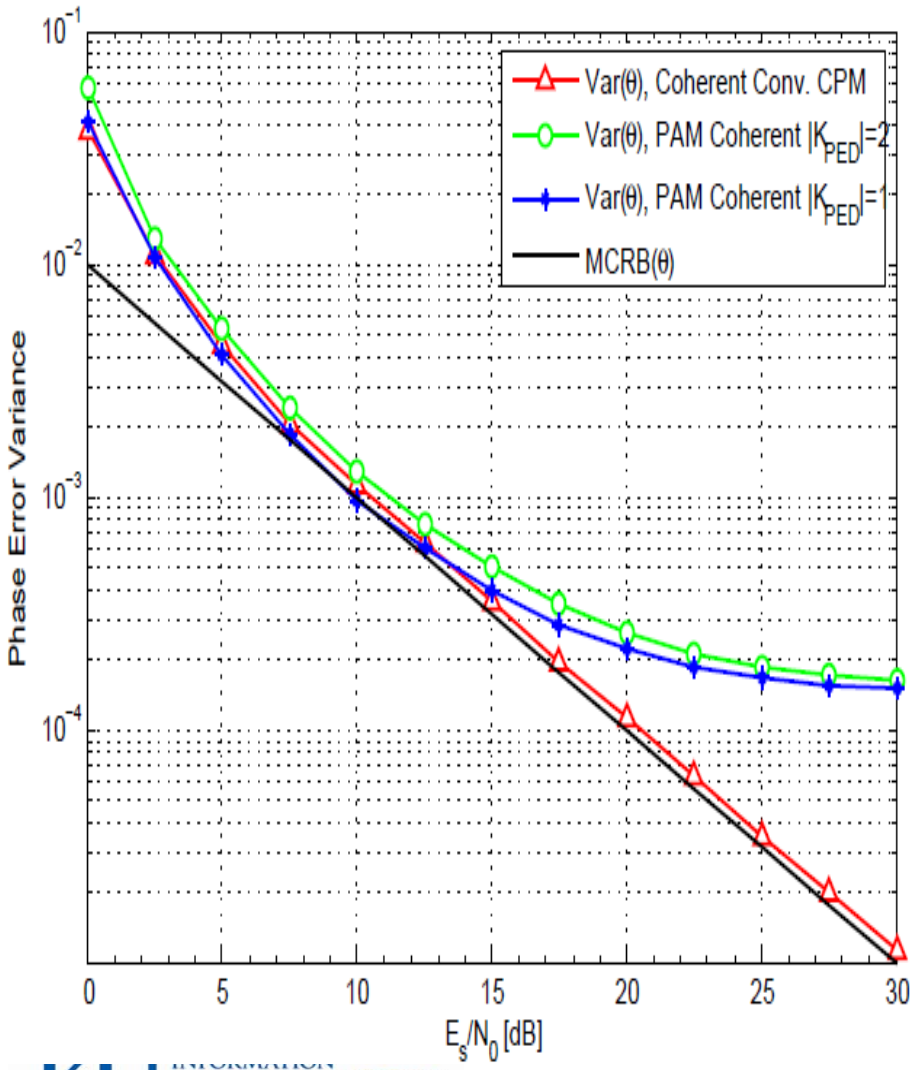


•*Timing error variance with a large carrier frequency offset*

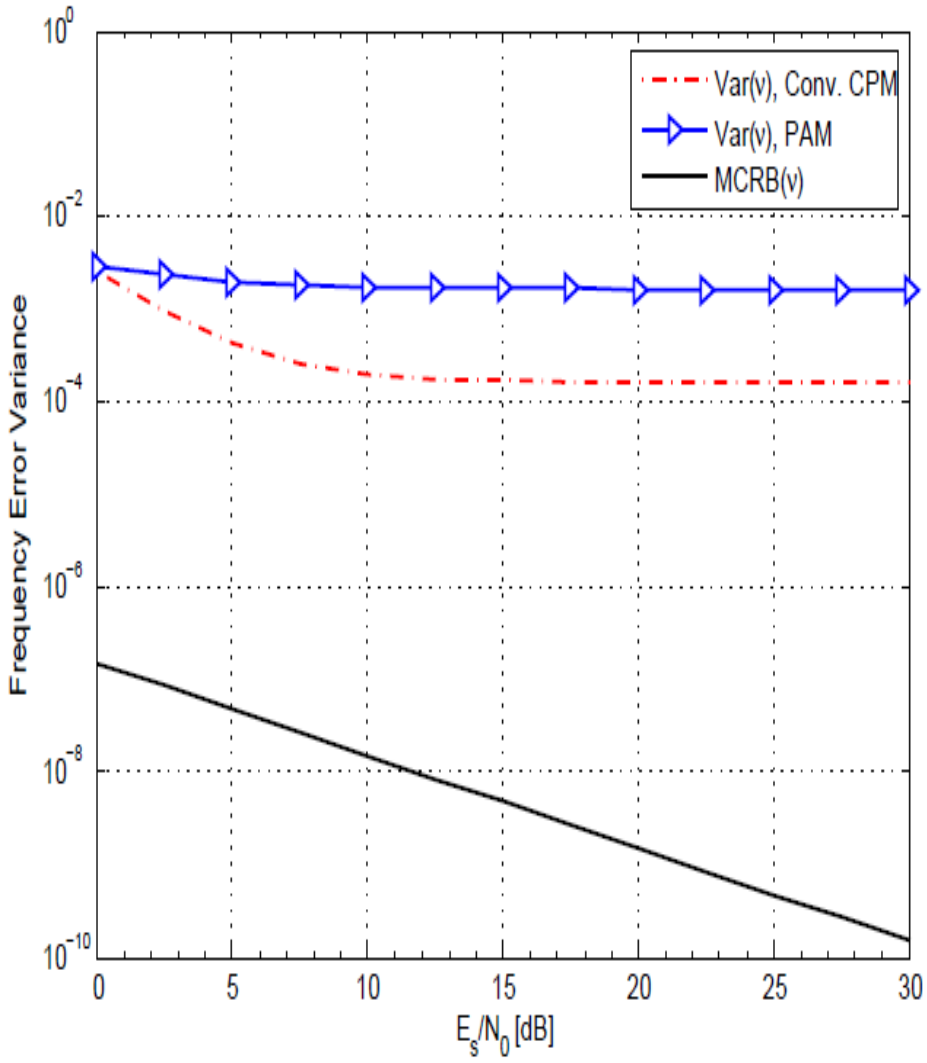


Simulation results (M-ary CPM…)

•*Phase error variance with no carrier frequency offset*



•*Frequency error variance with a large carrier frequency offset*



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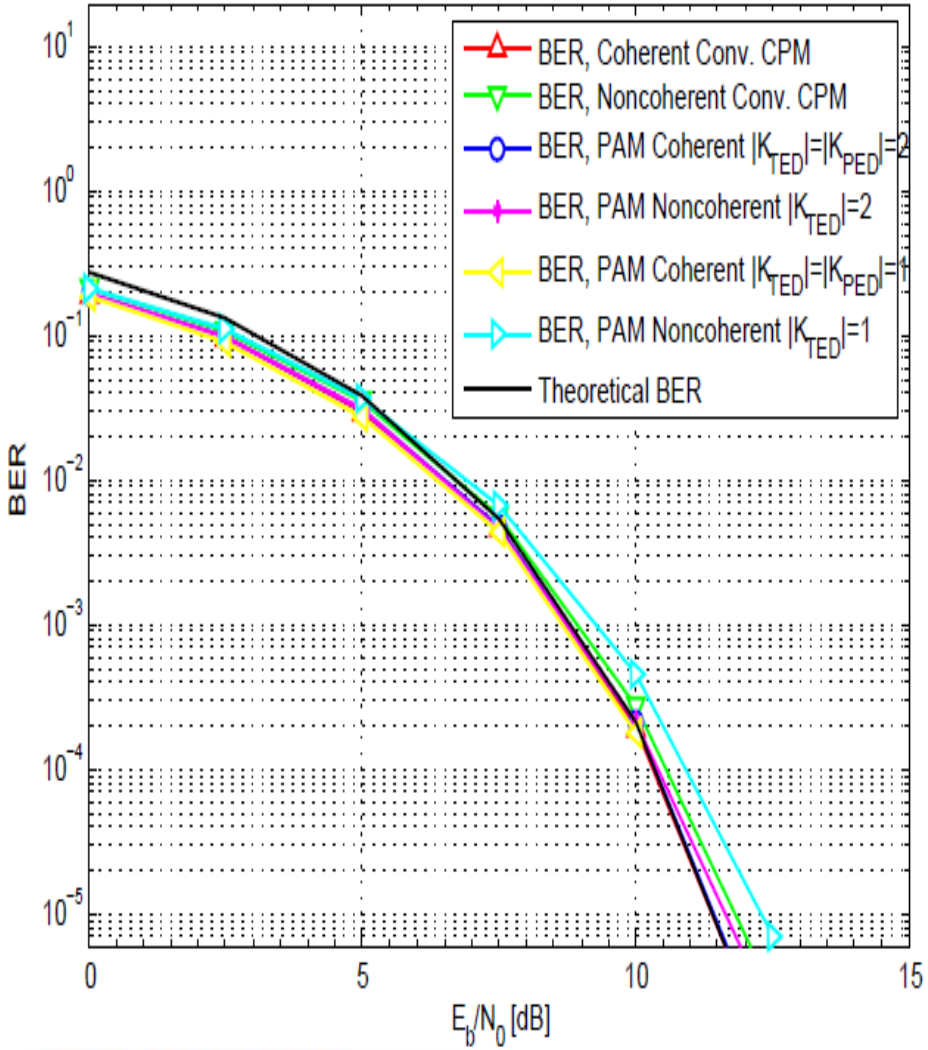
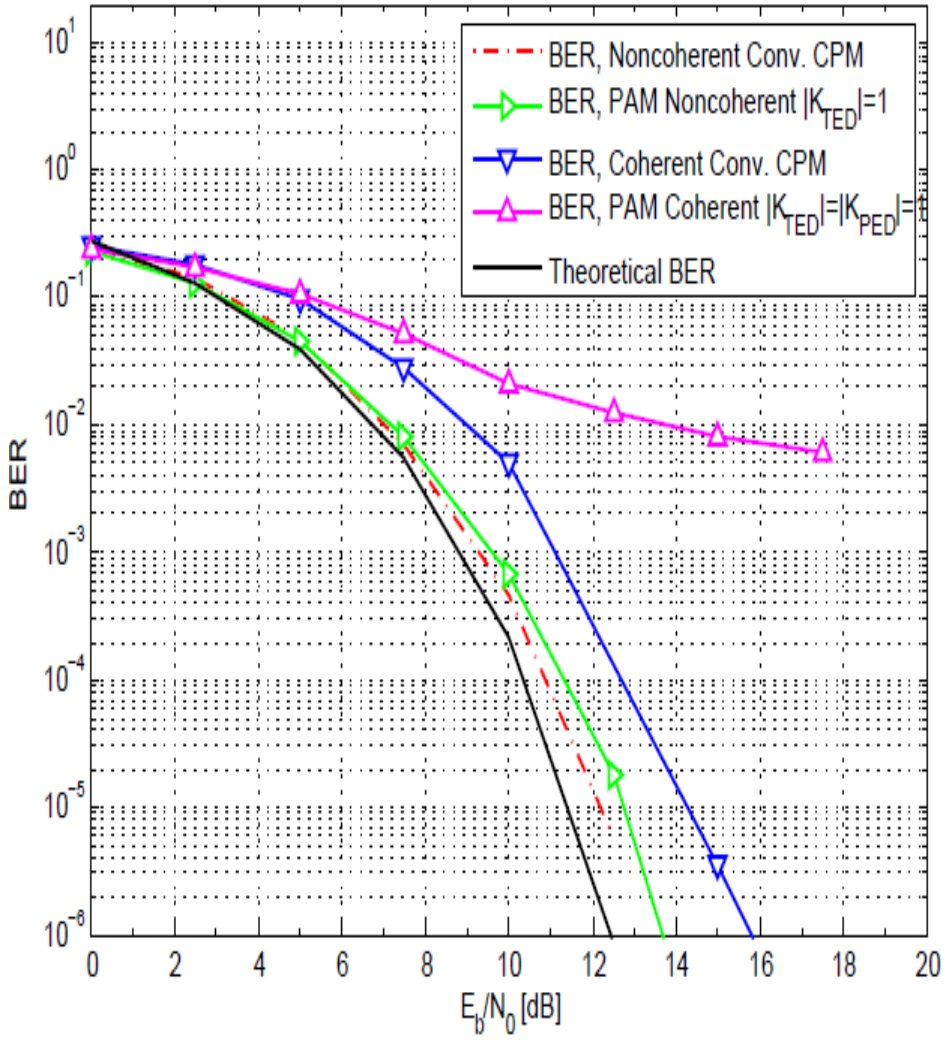
Simulation results (M-ary CPM…)

•*BER with no carrier frequency offset*



•*BER with a large carrier frequency offset*



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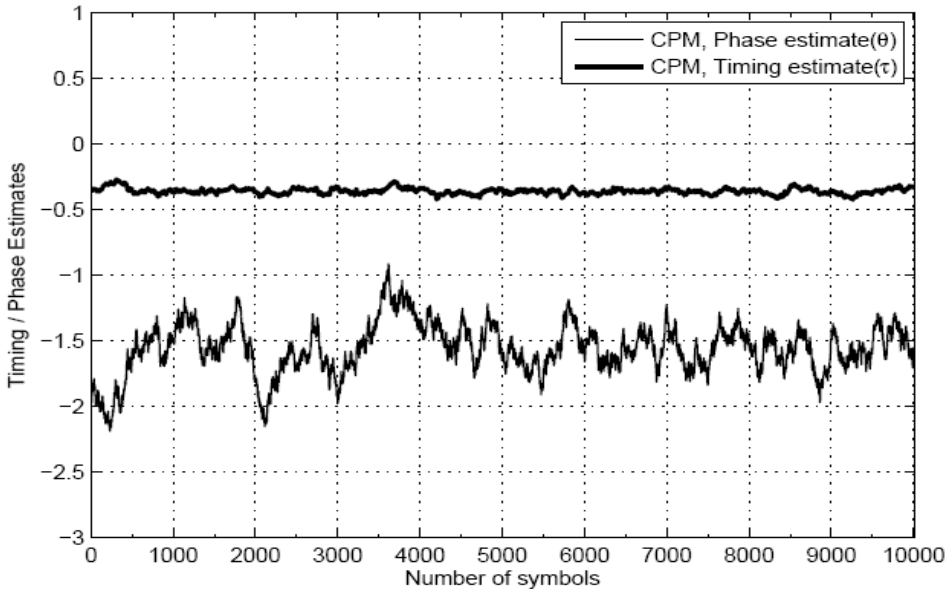
Summary of simulation results

- PAM based reduced-complexity CPM detectors provide very good tracking characteristics under no carrier frequency offset.
- Coherent and noncoherent detection can be done based on PAM based detectors. The noncoherent detectors are worse by about 2-3 dB in BER under all practical requirements and under no frequency offset condition.
- With a frequency offsets on the order of 10^{-4} of the symbol rate, the performance of PAM based detectors does not suffer deterioration in terms of tracking accuracy and BER.
- With the carrier frequency offset on the order of the symbol rate, noncoherent detection outperforms coherent detection in terms of tracking accuracy and BER.
- Noncoherent detection allows further simplification of the receiver structure by alleviating the need for a second stage of frequency recovery.

False lock recovery using reduced complexity detector configurations

- M-ary partial response CPM systems suffer from false lock problems during signal acquisition.
- Under false lock, the synchronizers settle at incorrect timing and phasing instants rendering poor BER, timing and phase error variance.
- NDA auxiliary lock detectors remove false locks but has a longer acquisition time.



CPM (M=4, 3RC, h=1/2)

Metric	Value by Simulation	Theoretical
BER	0.2808	0
Timing Variance	0.5007×10^{-3}	0.0511×10^{-3}
Phase Variance	0.0397	0.0006

Due to the variable lengths of the PAM filter components, PAM based configurations can deal with this problem more effectively than its conventional CPM counterpart.

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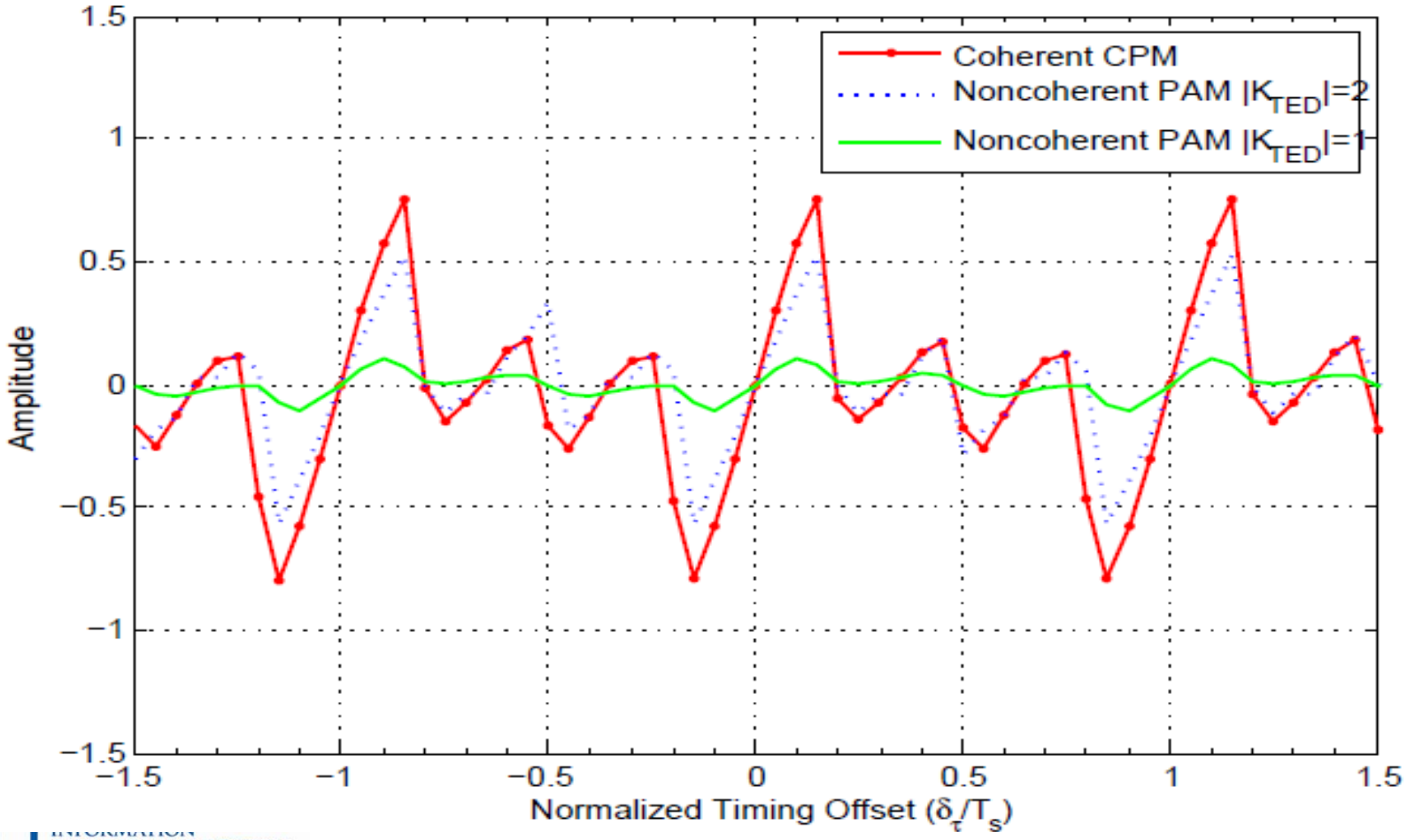
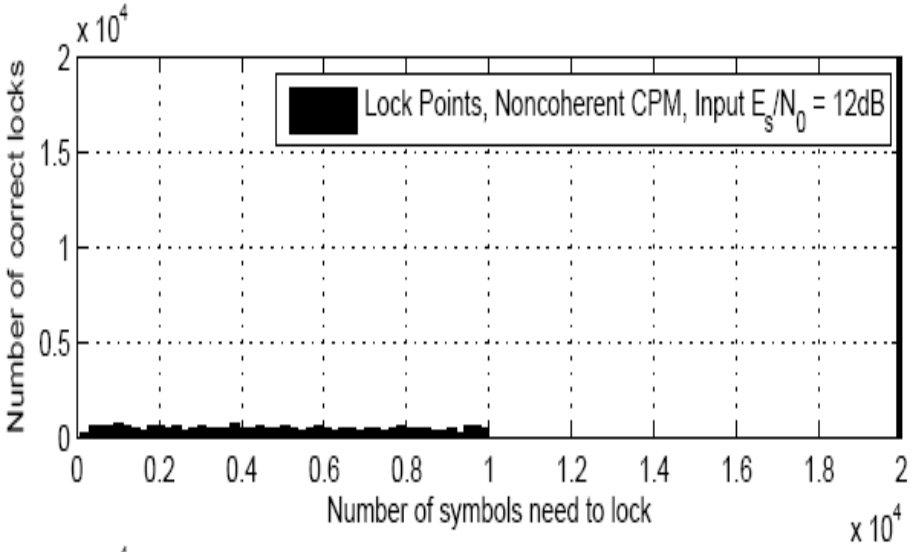
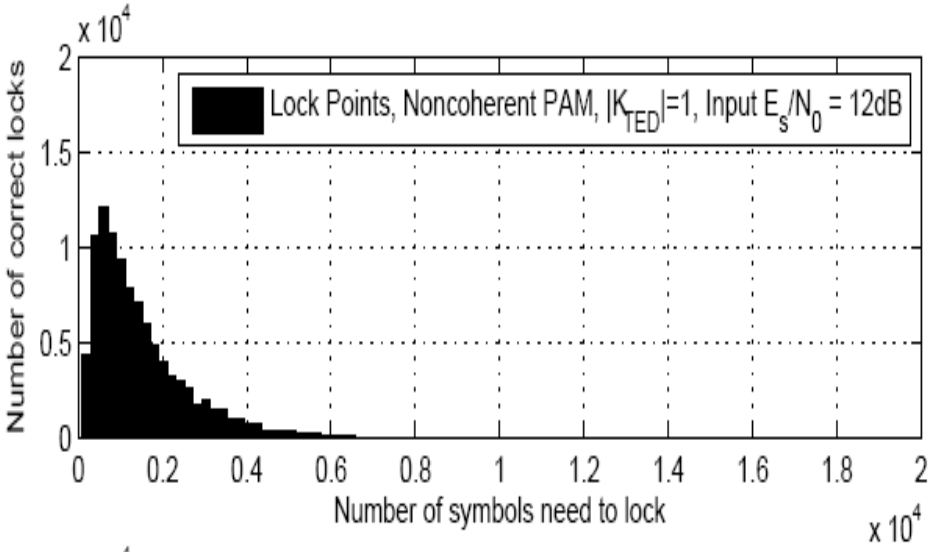
False lock recovery … contd.

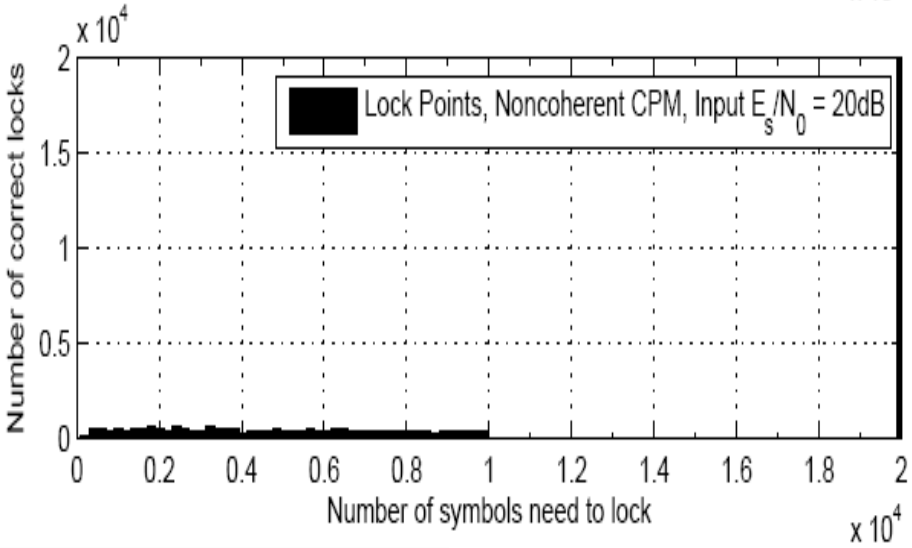
- S-curves for M-ary CPM (M=4, 3RC, h=1/2)

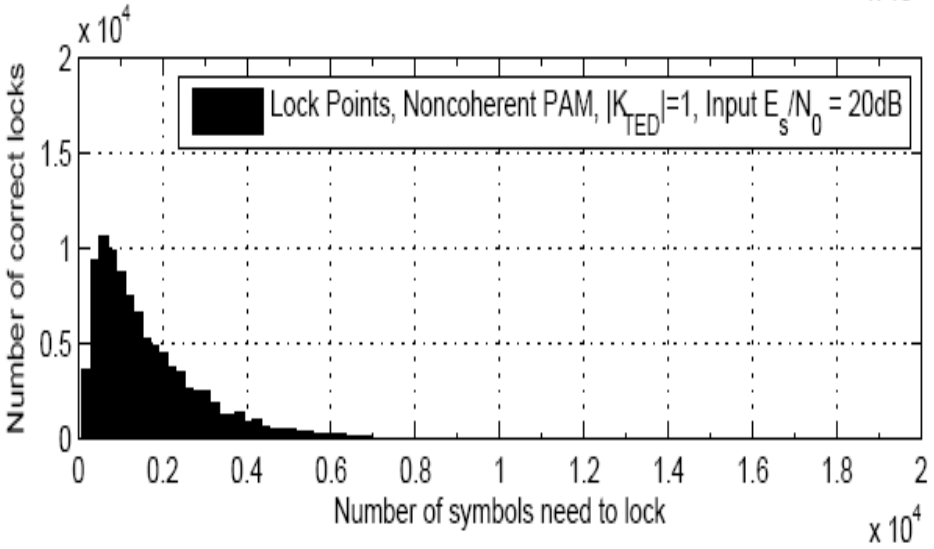


False lock recovery … contd.









False lock recovery … contd.

- Observations

  - CPM (M=4, 3RC, h=1/2), conventional and PAM based with 1 pulse noncoherent TED with $B_\tau T_s = 5\times10^{-3}$ during initial acquisitions.

No Frequency Offset	Noncoherent conventional	Noncoherent PAM $\lvert K_{TED}\rvert = 1$
No. of Simulations	10^5	10^5
No. of False locks	10^5	30
False lock probability	1	0.0003
Lock state	No lock	Consistent over no. of symbols

Conclusions and future work

•**Conclusion**

- Synchronizers provide a comparable performance against conventional CPM receivers.
- Under a large carrier frequency offset, a PAM based receiver in noncoherent mode offer similar performance as its CPM counterpart
- A novel method of decision-directed false lock recovery for PAM based CPM receivers.

• **Future work**

- Joint phase and timing recovery in wireless fading channels.
- Possibility of different PAM based error detector configurations for acquisition and tracking stages.

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Thank you!

Questions??