Optimizing Providers' Profit in Peer Networks Applying Automatic Pricing and Game Theory

by

Sohel Q. Khan
B.S.E.E., The University of Kansas, USA, 1995
M.S.E.E., The University of Kansas, USA, 1998

Presented to the Department of Electrical Engineering and Computer Science and the Faculty of the Graduate School of the University of Kansas in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

Committee:

Prof. David W. Petr, Chair

Prof. Victor Frost

Prof. John Gauch

Prof. Tyrone Duncan

Prof. Bozenna Pasik-Duncan

Prof. Jianbo Zhang

The University of Kansas
October 24, 2005

The Dissertation Committee for Sohel Q. Khan certifies that
this is the approved version of the following dissertation:

Optimizing Providers' Profit in Peer Networks Applying Automatic Pricing and Game Theory

Committee:

Prof. David W. Petr, Chair

Prof. Victor Frost

Prof. John Gauch

Prof. Tyrone Duncan

Prof. Bozenna Pasik-Duncan

Prof. Jianbo Zhang

Acknowledgement

First, thanks to my parents, sisters, and brothers for providing support and care.

Dr. David Petr's support, guidance, and criticism have been valuable in completing the dissertation. Dr. Petr was extremely busy this year; however, he took time reviewing my dissertation a couple of times. Although only five of us attended, he sincerely taught classes on optimization theory and integrated traffic-engineering analysis. Concepts learned from these classes aided me in conducting this research.

Dr. Zhang has helped me to learn microeconomics and game theory. These theories are central to this research. To accommodate my busy work schedule, Dr. Zhang came to his office at night and on weekends to discuss my research and to provide genuine suggestions.

Dr. Victor Frost has shown great enthusiasm and interest in my doctoral research. He was my mentor throughout graduate school. He advised me in my academic and professional lives. I first learned about traffic engineering when I worked for him on ATM ABR during my Master's study.

Dr. Bozenna Pasik-Duncan and Dr. Duncan have provided tremendous help and support in my academic and personal lives. Bozenna's mathematics classes helped me to build a strong foundation in probability theory and statistics. Dr. Gauch provided emotional support during my qualifier and comprehensive examinations.

My late friend Dr. Mamun read my "draft zero" and provided valuable suggestions on game theory, market demand, and cost function. He passed away one month after reviewing my draft. Peace to his departed soul.

Brian, Ann, and Danielle helped me to improve grammar and style; particularly, Brian was enthusiastic in reading this dissertation.

I wrote the major portion of this thesis in La Prima-Tazza Coffee shop, Lawrence, Kansas. The staff of the coffee shop allowed me to sit there for long duration of times. All my friends provided care and support

Sprint Network Services Sabbatical Program provided support for this research. Randy Smischny encouraged me to apply for the sabbatical program. Special thanks to the Sprint sabbatical selection committee and Don Hallacy (Network services President 2000-2002) for awarding me the sabbatical scholarship. Kathy Walker (ND President) and Lori Samazin (Executive Manager) supported me throughout the sabbatical program. Ben Vos (Director) and Manish Mangal (Manager) allowed me to work from the KU library during the crucial stage of this research.

Thanks to all of you.

Abstract

This research exploits the agility of game theory by synthesizing economic theories and Internet traffic engineering techniques to optimize the profit of Internet Service Providers (ISP), and to meet the customer desire of automatic subscription from any provider that offers the lowest price.

We propose a new *Automatic Price Transaction-based One-to-Many Peer Network* architecture that facilitates customers' options for subscribing to services from providers based on the negotiated price. This model is for enterprise-provider IP peer networks or customer-provider wireless networks. In this model, customers and providers perform simultaneous price negotiations by a Sealed-Bid-Reverse auction protocol. We suggest Session Initiation Protocol (SIP) entities and call flow to implement the mechanism. Our model extends the one-to-one IP peering architecture (IP Network-Network-Interface) of the Alliance for Telecommunications and Industry Solutions (ATIS). Our model also extends the one-to-one Online Charging architecture of the Third Generation Partnership Project (3GPP).

Implementation of the architecture causes strategic interaction among the providers; thus, a game theory model is required to compute the service price and to optimize the providers' profit.

We propose a new game theory model—the *Providers Optimized Game in Internet Traffic*—to optimize providers' profit in the proposed architecture subject to constraints of network architecture, traffic pattern, and game strategies. This model determines strategic price using a myopic Markovian-Bayesian game of incomplete information and an extension of previous work based on the Bertrand oligopoly model. Our model is sensitive to the dynamic Internet traffic demand, the congestion in networks, and the service class. Selecting a strategically appropriate price is one of our methods to optimize profit; the others are minimizing the network congestion sensitive cost and optimizing routes. The model associates a congestion indicator—the mean IP packet count in a network queue system—with the service cost. An M/M/1 queuing analysis determines the mean packet count. The model applies two well-known non-linear programming techniques, the Gradient Projection algorithm and the Golden section line search, to minimize the mean packet count and to optimize routes in providers' networks.

This dissertation presents the novel models, validates the models by analyses and simulations, evaluates advantages of the models, determines providers' the best strategies for optimizing their profit, and introduces traffic-engineering applications.

The dissertation concludes that our approach achieves a relative advantage in profit over the classical Bertrand model for both the homogeneous and heterogeneous service-based Internet markets. Our model yields positive profit for all providers and decreases the market price of services relative to customers' budgets while guaranteeing their preferences. The novel model optimizes profit of providers in one or multiple Bayesian-Nash equilibriums and the Paretro-efficient outcomes subject to the network architecture, traffic pattern, service class mix, and strategies available. Providers achieve fair market shares with these equilibriums. In addition to the profit optimization, providers can implement our method to perform least price routing, traffic load balancing, capacity planning, and service provisioning.

TABLE OF CONTENT

1 INTRODUCTION.. 13

1.1 BACKGROUND MICROECONOMIC CONCEPTS 16
1.1.1 Profit 16
1.1.2 Game Theory.. 17
1.1.2.1 Bayesian Static Game of Incomplete Information.................................... 18
1.1.2.2 Bayesian Nash Equilibrium 19
1.1.3 Oligopoly.. 20
1.1.4 Sealed Bid Reverse Auction 20
1.2 BACKGROUND RESEARCH ON NETWORK PRICING 21
1.2.1 Service per Customers' Bids... 21
1.2.2 Static Congestion Game.. 21
1.2.3 Provider's Monopolist Game... 22
1.2.4 Peer Providers in a Series 23
1.2.5 Game of Incomplete Information in Sealed Bid Reverse Auction.................. 23
1.2.6 Transaction-level Pricing Network Architecture.............................. 24
1.3 PROBLEM STATEMENT AND PROPOSED SOLUTION 25
1.3.1 The Proposed Price Transaction Architecture and Protocol 26
1.3.2 Proposed Providers' Game of Oligopoly 28
1.3.3 Proposed method of Optimizing Providers' Profit............................. 29
1.3.4 Proposed Algorithm.. 31
1.3.5 Research Methods.. 31
1.4 DISTINGUISHING CHARACTERISTIC OF OUR APPROACH 33
1.5 SUMMARY OF CONTRIBUTION... 36
1.6 STRUCTURE OF THE DISSERTATION... 37

2 NETWORK ARCHITECTURE AND PROTOCOL 38

2.1 NETWORK ARCHITECTURE.. 38
2.1.1 SIP Entities.. 38
2.1.2 ATIS-PTSC Reference Model.. 42
2.1.3 Our Extension to ATIS Model 43
2.1.4 3GPP IMS Charging Architecture.. 45
2.1.5 Our Extension to 3GPP IMS Charging Architecture 48
2.1.6 Other Protocol-based Networks 50
2.2 PROPOSED AUTOMATIC PRICE TRANSACTION PROTOCOL....................................... 51
2.3 PROPOSED SIP CALL FLOW 53
2.4 CHAPTER SUMMARY.. 54

3 PROVIDERS' GAME OF OLIGOPOLY 55

3.1 MODEL SELECTION 55
3.2 SERVICE CLASS AND ENTERPRISE PREFERENCE 59
3.3 MODEL PARAMETERS 62
3.3.1 Market Capacity and Market Demand Functions 62
3.3.2 Marginal Cost Function 64
3.3.3 Reservation Price of an Enterprise 67
3.3.4 Profit Function 69
3.4 PROPOSED OLIGOPOLY MODEL 71
3.5 THE MOVEMENT OF THE BELIEF FUNCTION 77
3.6 PROVIDERS' STRATEGIES 79
3.7 CHAPTER SUMMARY 84

4 PROVIDERS' PROFIT MAXIMIZATION BY OPTIMUM ROUTING 85

4.1 NETWORK ARCHITECTURE CONSTRAINTS 88
4.2 TRAFFIC PATTERN AND QUEUE SYSTEM CONSTRAINTS 89
4.3 MEAN PACKET COUNT IN THE M/M/1 MODEL 91
4.4 SESSION ARRIVAL DISTRIBUTION 92
4.5 THE DEVELOPMENT OF A NON-LINEAR OPTIMIZATION PROGRAM 92
4.6 CHAPTER SUMMARY 97

5 NETWORK AND TRAFFIC FLOW DESIGN 98

5.1 NETWORK TOPOLOGY 98
5.2 PACKET LENGTH DISTRIBUTION 100
5.3 TRAFFIC ENGINEERING RULE 101
5.4 CAPACITY ASSIGNMENT 107
5.5 SESSION ARRIVAL PATTERN 109
5.6 TRAFFIC FLOW DESIGN 109

6 A SNAPSHOT OF THE ALGORITHM 116

6.1 THE LAYERED VIEW OF THE ALGORITHM 116
6.2 PERFORMANCE MEASUREMENT METRICS 118
6.3 SESSION LEVEL MONTE-CARLO SIMULATION ALGORITHM 119

7 MATHEMATICAL ANALYSES AND VALIDATION 123

7.1 THE RESERVATION PRICE 124
7.2 SERVICE COST COEFFICIENT VALUES IN MARGINAL COST 124
7.2.1 Analytical Marginal Cost Function 125
7.2.2 Simulated Marginal Cost Function 127
7.2.3 Service Cost Coefficient Values 128
7.3 HOMOGENEOUS SERVICE-BASED MARKET 131

7.3.1 Study of the Rejection Neutral Strategy Set ... 132
7.3.2 General Equation of Bid Price for All Strategies ... 140
7.3.3 Study of Identical Strategies ... 141
7.3.4 Study of Non-Identical Strategy Set ... 144
7.3.5 Bayesian-Nash and Pareto-Efficient Strategy ... 148
7.4 HETEROGENEOUS SERVICE-BASED MARKET ... 153
7.4.1 Study of Identical Strategy Set ... 153
7.4.1.1 The Rejection Neutral Strategy Set ... 154
7.4.1.2 Study of Other Strategy Sets ... 156
7.4.2 Non-Identical Strategy Set ... 158
7.5 CHAPTER SUMMARY ... 159

8 SESSION LEVEL MONTE-CARLO SIMULATION, APPLICATIONS, AND ADVANTAGES ... 160

8.1 HOMOGENEOUS SERVICE-BASED MARKET ... 160
8.1.1 Experiment Objectives ... 160
8.1.2 Parameters ... 162
8.1.3 Validation ... 163
8.1.3.1 Functional Validation ... 163
8.1.4 Application ... 167
8.1.4.1 Finding a Safe Strategy ... 167
8.1.4.2 Finding Pareto-Efficient Outcome Strategy Set ... 171
8.1.4.3 The Routing Scheme ... 177
8.1.4.4 Traffic Load Adjustment ... 179
8.1.5 Advantage of the Model ... 182
8.2 HETEROGENEOUS SERVICE-BASED MARKET ... 184
8.2.1 Experiment Objectives ... 184
8.2.2 Parameters ... 185
8.2.3 Validation of the model ... 186
8.2.3.1 Functional Validation ... 186
8.2.3.1.1 Experiment 1 ... 186
8.2.3.1.2 Experiment 2 ... 189
8.2.3.1.3 Experiment 3 ... 192
8.2.4 Application ... 195
8.2.4.1 Finding the Pareto-Efficient Outcome Strategy Set ... 195
8.2.4.2 Preferred Strategy ... 203
8.2.5 Advantage of the Model ... 206
8.3 CHAPTER SUMMARY ... 207

9 CONCLUSION.. 210

9.1 SUMMARY OF CONTRIBUTIONS 210
9.1.1 A Novel Automatic Price Transaction Architecture 210
9.1.2 An Extension of the Current ATIS and 3GPP Architecture.......................... 210
9.1.3 Session Initiation Protocol based Price Transaction Protocol 211
9.1.4 The Providers Optimized Game in Internet Traffic 211
9.1.5 An Analytical Model, a Network Model, and a Session Level Monte-Carlo Simulator212
9.1.6 A Framework to Determine the Best Preferred Strategy.............................. 213
9.2 LIMITATIONS.. 214
9.2.1 Traffic Distribution Pattern 214
9.2.2 The Cost Function... 214
9.2.3 Network Queue Model 215
9.3 ADVANTAGE 215
9.3.1 Improvement on Classical Models... 215
9.3.2 Automation of Pricing and Billing... 216
9.3.3 Synthesis of Game Theory and Traffic Engineering Techniques.................. 216
9.3.4 Implementation of Strategies 217
9.4 PRACTICAL APPLICATIONS... 218
9.4.1 Automatic Price-based Services 218
9.4.2 Profit Optimization and Determining Optimum Throughput 218
9.4.3 Traffic Load Distribution... 218
9.4.4 Least Price Routing.. 219
9.4.5 Forecasting and Capacity Planning 219
9.4.6 Service Provisioning 220
9.4.7 Innovation Disclosure.. 220
9.5 FUTURE WORK 220
9.5.1 Variable Reservation Price.. 220
9.5.2 Experiment on 3GPP Network... 220
9.5.3 Priority based Queue system 221

BIBLIOGRAPHY 222

APPENDIX A: MATHEMATICAL OPTIMIZATION TECHNIQUE 233

THE NECESSARY AND SUFFICIENT CONDITIONS.. 233
THE GRADIENT PROJECTION ALGORITHM.. 234
THE GOLDEN SECTION LINE SEARCH.. 235

APPENDIX B:LIST OF ACRONYMS 237

Figures

1.1: Enterprise-Provider one-to-many peer network topology 26
2.1: Session Initiation Protocol Entities....40
2.2: ATIS/PTSC IP Peering Reference Diagram....42
2.3: Network Architecture of Duopoly Market 44
2.4: 3GPP IMS Architecture....46
2.5: The current 3GPP IMS Online Charging Architecture....46
2.6: 3GPP Online Charging System....47
2.7: Extended 3GPP Charging Architecture in Duopoly Market 48
2.8: Price Transaction Protocol....51
2.9: Session Initiation Protocol (SIP) Control Flow 53
3.1: Demand Function....64
3.2: The Sketch of Steady State Price, Cost, Throughput, and Profit....69
3.3: Change in Belief Function due to the change in Marginal cost....78
3.4: Proposed Strategy Diagram 82
4.1: Change in Mean Packet count in the network. 87
5.1: Simulation topology 99
5.2: VoIP Packet Length....100
5.3: Single Integrated FIFO Queue system....103
5.4: M/G/1 System Delay for Heterogeneous services....105
5.5: Internal Network Topology of Two providers....107
5.6: Each O-D pair has five different routes....109
6.1: Layered View of the Proposed Algorithm....117
6.2: Session Level Monte-Carlo Simulation Flow Algorithm for Duopoly Market....120
7.1: Uniform traffic flow across the network in optimized load....125
7.2: Marginal Cost as a Function of Service Cost Coefficient and Network Load 128
7.3: Strategy set of experiments....132
7.4: Analytical Result for *Rejection Neutral* Strategy (Homogeneous Service) 136
7.5: A.com: Analytical vs. Simulated Results ({A.com RN, B.com RN})....139
7.6: Analytical vs. Simulated Results ({A.com VHR, B.com VHR}) 142
7.7: Analytical vs. Simulated Results (Strategy: {A.com VLR, B.com VLR})....143
7.8: Solving Non-Identical Strategies Bid Price Equations by Numerical Analysis....146
7.9: Comparison of Dissimilar strategies....147
7.10: Probability Density Funciton (pdf) of Market Load....148
7.11: 2D Plot—Analytical Bayesian Nash Equilibrium in Homogeneous Market 150
7.12: 3D Plot—Analytical Bayesian Nash Equilibrium in Homogeneous Market 152
7.13: Heterogeneous based-Market: Analytical and Simulation Results (RN strategy sets)....155
7.14: Heterogeneous based-Market: Analytical and Simulation Results (Other strategy sets)....157
8.1: Strategy set of experiments....162
8.2: Simulation Result: Comparison of *Random Rejection* and *Rejection Neutral* Strategies. 165
8.3: Comparison of all strategies with the *Rejection Neutral* strategy 168
8.4: Very High and Neutral strategy providers' load and marginal cost 169

8.5: Hypothetical Market Load Probability Density Function (pdf)..172
8.6: 2D Plot: Simulated Bayesian Nash Equilibrium in Homogeneous Market (Scenario 1).................173
8.7: 3D Plot: Simulated Bayesian Nash Equilibrium in Homogeneous Market (Scenario 1).................174
8.8: 2D Plot- Simulated Bayesian Nash Equilibrium in Homogeneous Market.....................................175
8.9: 3D Plot-Simulated Bayesian Nash Equilibrium in Homogeneous Market...........................176
8.10: Load balancing by strategy assignment...180
8.11: Analytical Load adjustment by Strategy Assignment...181
8.12: Analytical Network load for adjusting B.com strategy 181
8.13: A Strategy set of heterogeneous service market...185
8.14: Heterogeneous Results: Price and Cost for Rejection Neutral Strategies 187
8.15: Comparison of Profit and Throughput...188
8.16: Heterogeneous Results of strategies: VHR-RN-VLR vs. RN-RN-RN...190
8.17: Heterogeneous Results of strategies: VLR-RN-VHR vs. RN-RN-RN...193
8.18: 3D Plot—Simulated Bayesian Nash Equilibrium in Heterogeneous Market (Scenario 1) 197
8.19: 2D Plot—Simulated #1 Bayesian Nash Equilibrium in Heterogeneous Market (Scenario 1) 198
8.20: 2D Plot—Simulated #2 Bayesian-Nash Equilibrium in Heterogeneous Market (Scenario 1).......199
8.21: 2D Plot—Simulated #3 Bayesian-Nash Equilibrium in Heterogeneous Market (Scenario 1).......200
8.22: 2D Plot—Simulated #1 Bayesian-Nash Equilibrium in Heterogeneous Market (Scenario 2).......201
8.23: 2D Plot—Simulated #2 Bayesian Nash Equilibrium in Heterogeneous Market (Scenario 2) 202
8.24: Example of No Bayesian Nash Equilibrium...203
8.25: Price of Services: VLR-RN-VHR vs. VLR-RN-VHR..204
8.26: Cause of Red Price higher than Green...205

Tables

1.1: Classes of Games ... 17
2.1: 3GPP IMS Functional Components ... 45
2.2: Components of different types of networks ... 50
3.1: Marginal cost equation ... 67
3.2: Proposed Strategies ... 82
3.3: Proposed feasible Strategies of the providers ... 83
5.1: Capacity Assignment ... 108
5.2: O-D pairs and paths ... 110
5.3: O-D Traffic Matrix ... 110
5.4: Capacity Matrix of Each Network ... 111
5.5: Inequality Constraint ... 111
5.6: A portion of G Matrix ... 113
5.7: Equality Constraints ... 114
7.1: The Reservation price of different types of services ... 124
7.2: The Service Cost Coefficient values ... 130
7.3: Parameters for homogeneous service-based network ... 131
7.4: Analytical Result (Homogeneous Service Market) ... 138
7.5: Expected Unit Profit of Providers for different combination of strategies. ... 149
7.6: Summary of Parameter for Heterogeneous services ... 153
8.1: Parameters for simulation and analytical studies ... 162
8.2: Reduced set of providers' feasible strategies ... 171
8.3: Scenario 1—The Normalized Expected Profit in Homogeneous market ... 173
8.4: Scenario 2—The Normalized Expected Profit in homogeneous market ... 175
8.5: Comparison of Results: Minimum-Hop vs. Maximum-Hop ... 178
8.6: Summary of Parameter for Heterogeneous services ... 185
8.7: Heterogeneous strategies for functional validation experiment 1 ... 186
8.8: Heterogeneous strategies for functional validation Experiment 2 ... 189
8.9: Results at a Market Load of 57% ... 191
8.10: Heterogeneous strategies for functional validation experiment 3 ... 192
8.11: Heterogeneous strategies to determine Bayesian-Nash Equilibrium ... 195
8.12: Scenario 1--The normalized Expected profit in Heterogeneous market ... 196
8.13: Scenario 2--The normalized Expected profit in Heterogeneous market ... 201

1 Introduction

Session Initiation Protocol (SIP) supported peer networks have recently ascended to prominence among Internet service providers according to Yankee Group reports [77]-[79]. Automating the price transaction for services and optimizing profit of providers in such peer networks are recent challenges for engineers. There is neither a well-established method, nor an automatic mechanism for computing the service price in peer networks today.

Small providers are wholesale customers of large providers. These customers want options for subscribing to services from large providers in one-to-many peer networks with an automatic price transaction mechanism. They also desire to select a provider instantaneously that offers the lowest price. Today, one-to-many peer customers transport IP traffic through large providers based on the network load. However, in our knowledge, no mechanism exists today for such transport based on the service price.

Analogous to the desire of small providers, individual wireless customers want to peer with multiple wireless providers and automatically subscribe to services from the provider of their choice based on the service price.

We propose the new *Automatic Price Transaction based One-to-Many Peer Network* architecture to meet customers' desire for automatic price negotiations that are concurrent with multiple providers. This architecture for one-to-many peer networks supports a price transaction protocol, SIP entities and a SIP call flow. The architecture allows customers to broadcast their budget and instantaneously subscribe to the provider of their choice based on the competitive service price analogous to the Sealed-Bid-Reverse auction [43][44]. Our model extends the one-to-one IP peering architecture (IP Network-Network-Interface) of the Alliance for Telecommunications and Industry Solutions (ATIS). Our model also extends the one-to-one Online Charging architecture of the Third Generation Partnership Project (3GPP).

Customers' options of subscribing to any provider create strategic interaction of price among the providers. This strategic interaction of the limited number of providers and their attempt to optimize profit are the microeconomic concepts of game theory in an oligopoly market [1][2]. Thus, we employ provider's price computation method using a game of oligopoly. Our game theory model is a function of the peer traffic capacity and demand, the service cost, and a customer's budget.

Although the traffic capacity and a customer budget remain constant for a relatively short duration of time, the traffic demand and the service cost vary due to the dynamic nature of Internet traffic and the network congestion.

Large providers want to optimize their profit by automatic price computation methods synchronized with the dynamic nature of Internet traffic demand in the competitive market. The existing price computation mechanisms of providers are not dynamic; i.e., the price is often asynchronous with the Internet traffic demand. Providers' marketing departments manually compute prices based on the historical network load, market capacities, and traffic demand levels. By the time a marketing department computes and advertises a new price, the network traffic pattern and market demand may have already changed. Most importantly, the Internet traffic demand is still unpredictable. This causes long reactive delays of price computation that create an obstacle to selling services synchronized with the varying market demand in the competitive market. Thus, there is a need for mechanisms that automatically compute price synchronized with the Internet traffic demand and sensitive to the network congestion.

We propose the new *Providers Optimized Game in Internet Traffic* model that synthesizes a game theory, a traffic-engineering technique, and a non-linear optimization method. The model allows providers to determine competitive price synchronized with the dynamic Internet traffic demand and sensitive to the network congestion. In this model, providers optimize profit by selecting strategically sensitive price and by minimizing congestion sensitive network cost. A mathematical non-linear program associated traffic engineering technique minimizes the congestion sensitive network costs.

This dissertation presents the architecture and the model, validates them by analyses and simulations, evaluates their advantages, determines providers best game strategies that optimize their profit, and introduces traffic-engineering applications.

The dissertation concludes that our approach—the implementation of the architecture and the game model—achieves a relative advantage in profit over the classical Bertrand model for both the homogeneous and heterogeneous service-based Internet markets. Our approach yields positive profit for all providers and decreases the market price of services relative to customers' budget while guaranteeing their preferences. The novel approach optimizes profit of providers in one or multiple Bayesian-Nash equilibriums and the Paretro-efficient outcomes subject to the network architecture, traffic pattern, service class mix, and strategies available. Providers achieve fair market shares through these equilibriums. In addition to the profit optimization, providers can implement our approach to perform least price routing, traffic load distribution, capacity planning, and service provisioning.

In the rest of this document, an *enterprise* is a small regional Internet Service Provider (ISP) that has distributed networks across a continent, but does not have national or international backbone networks. A *provider* is a large ISP that has national and international backbone networks. An enterprise supports access networks, sells services directly to consumers, and peers with providers to transport its long distance and international traffic. A *customer* is either an enterprise or a wireless customer. The price transaction protocol is for the customer-provider peer interface to negotiate price.

We organize the rest of this chapter as follows. Section 1.1 briefly presents microeconomic concepts such as optimizing providers' profit and developing game theory models. We study the outline of the related research in Section 1.2 to comprehend the background of the problem. Section 1.3 presents the problem statement, proposed solutions, and research methods. Section 1.4 discusses the distinguishing characteristics of our approach. Section 1.5 provides a summary of our contributions; and Section 1.6 outlines the document format.

1.1 Background Microeconomic Concepts

1.1.1 Profit

Our research concerns providers' profit. A profit function is typically assumed to be monotonic, bounded, and concave. We define unit profit ($u(.)$) as the steady state network throughput ($Y(.)$) multiplied by the difference between the unit price ($p(.)$) and cost ($\omega(.)$). In other words, it is the difference between the net revenue and the net production cost. We define network throughput as the aggregate rate served by a network, where rate is data units per unit time.

$$u(.) = \left[p(.) - \omega(.) \right] Y(.) \tag{1.1}$$

A provider (n) computes profit from a session as a function of the price (p), the marginal cost (ω), the duration (d), and the bandwidth (y) of the session. The price and the marginal cost are values at the session start time. The total profit of the provider is the sum of the profits from all ($\forall k$) the sessions until the end of the game (e.g. a simulation).

$$\text{Cumulative profit}_n = \sum_{\forall k} (p_{s,t,k} - \omega_{n,s,t,k}) d_{n,k} y_{n,s,k} \tag{1.2}$$

1.1.2 *Game Theory*

The mathematical theory pertaining to the strategic interaction of decision makers is Game Theory. We assume that in the Internet game, providers play the role of rational decision makers and each provider knows that the opponents are also rational. A rational provider always attempts to select the best strategy. Table 1.1 presents four fundamental classes of games and their corresponding equilibriums.

Game Class	Equilibrium
Static Game of Complete Information	Nash Equilibrium
Dynamic Game of Complete Information	Subgame-perfect Nash equilibrium
Static Game of Incomplete Information	Bayesian Nash equilibrium
Dynamic Game of Incomplete Information	Perfect Bayesian Equilibrium

Table 1.1: Classes of Games

A game of complete information is the strategic interaction when providers are aware of each other's strategies or payoffs, i.e., all factors are common knowledge. In the game of incomplete information, at least one provider is unaware of the payoffs or strategies of other providers. In a static game, all providers simultaneously interact without the knowledge of past payoffs or strategies. In a dynamic game, a provider performs strategic interactions repeatedly based on the knowledge of the payoffs or strategies of past interactions.

In today's competitive Internet market, providers do not divulge their payoffs or strategies. A provider may have partial knowledge about other providers with some uncertainty; however, it does not have the complete knowledge. In our research, all providers simultaneously compute bid prices without the knowledge of their opponents' payoffs or strategies; thus, we are interested in studying a static game of incomplete information.

An example of a static game of incomplete information is a sealed bid auction. For example, when a government conducts a sealed auction for a license of certain wireless wavebands, no provider knows bids (actions of strategies) of other providers for the license and expected profit (payoff) of others for winning the license. All the providers submit simultaneous sealed bids. Mathematics refers to this strategic interaction as the Bayesian static game of incomplete information because it uses Bayes' conditional probability rule.

1.1.2.1 *Bayesian Static Game of Incomplete Information*

This strategic form game consists of a set of providers (players), their action spaces, type spaces, probability (belief) functions, and their profit (payoffs). In an Internet market of two providers—A.com and B.com—, we denote the Bayesian static game of incomplete information as follows:

$$G_{Bayesian} = [\{A.com, B.com\}, \{Action_A, Action_B\}, \{Type_A, Type_B\}, \{Belief_A(), Belief_B()\}, \{u_A, u_B\}] \quad (1.3)$$

When a provider bids for a service, the bid represents the *Action* space of the provider. A provider computes the bid based on certain private parameters such as the cost of a service as a function of congestion indicator of a network. Each provider may have its distinct cost function. Here, this cost function represents the *Type* of a provider.

The Belief is a conditional probability function. The belief function of A.com implies its uncertainty about B.com's selection of a pure strategy. In a pure strategy, a player selects a particular strategy from a given set of strategies with 100% probability. A.com has some Belief of the strategies of B.com based on its own strategy. A.com takes an *Action* from the belief function based on its perceived *Type* of B.com in comparison to its own *Type*. The following equation presents A.com's belief function about B.com (i.e., A.com holds belief on B.com's type):

$$Belief_A(.) = Prob_A(Type_B \mid Type_A) \quad (1.4)$$

The belief function is also referred to as the mixed strategy profile. A.com develops a set of feasible strategies from the belief function:

$$strategy\ h_{Aj} : Action_A \longleftarrow h_{Aj}(., Belief_A(.)) \quad (1.5)$$

For example, from a service cost function (*Type*$_A$), A.com develops a belief (*Belief*$_A$) function for the possible bids of B.com; then, A.com selects a bid (*Action*) by a strategy (*h*) such that A.com bid is higher than the perceived bids of B.com.

The development of the providers' belief functions and the selection of the best strategy set from the belief function to maximize providers' profit (payoffs) in the dynamic Internet traffic demand are the principal tasks of our research.

1.1.2.2 Bayesian Nash Equilibrium

A Bayesian Nash equilibrium is a feasible strategy set that maximizes providers' expected profit (*u(.)*) in a static game of incomplete information. This equilibrium occurs when A.com and B.com play their best strategies (h_A^*, h_B^*) and results in a set of optimum expected profit ($E[u_A^*], E[u_B^*]$). In the following definition, A.com plays the best strategy in response to the best strategy played by B.com.

Definition: A strategy set $Strategy = (h_1, h_2, ..., h_j)$ constitutes a *Bayesian Nash Equilibrium* of a game $G = [\{A.com, B.com\}, \{Strategy_A, Strategy_B\}, \{u_A, u_B\}]$ for every feasible strategy (j) such that:

$$E[u_A(h_{Aj}^*, h_{Bj}^*)] \geq E[u_A(h_{Aj}^{\forall j}, h_{Bj}^*)] \tag{1.6}$$

Here, when B.com plays the optimal strategy h_{Bj}^*, A.com has nothing to improve its expected profit by changing strategy from h_{Aj}^*. This also implies that when A.com plays the optimal strategy h_{Aj}^*, B.com has nothing to improve its expected profit by changing strategy from h_{Bj}^*.

$$E[u_B(h_{Aj}^*, h_{Bj}^*)] \geq E[u_B(h_{Aj}^*, h_{Bj}^{\forall j})] \tag{1.7}$$

Therefore, neither A.com nor B.com will benefit in expected profit by changing strategies from the Bayesian-Nash equilibrium strategies.

1.1.3 *Oligopoly*

An Internet oligopoly market consists of a small number of providers that strategically interact to optimize their profit. They collectively influence the network capacity of the market and the market price of services; however, no single provider can completely control the market. In this thesis, A.com and B.com constitute a two provider oligopoly; i.e., duopoly.

There are two fundamental models of oligopoly: the Cournot game of capacity and the Bertrand game of price. In today's competitive Internet market, providers first implement network infrastructure at the peering interface and then assign a price. The Bertrand game of price occurs in the short term; but in the long term, the providers reassign capacity engaging in Cournot's game of capacity. Our study focuses on the short-term market when market capacity remains constant and the providers engage in price bidding. Therefore, we develop a novel model based on the Bertrand game of oligopoly (see details in Chapter 3).

1.1.4 *Sealed Bid Reverse Auction*

The sealed bid reverse auction is the foundation of the price transaction protocol of the novel model. In this auction, a buyer has a maximum price it is willing to pay for a service. This price is the reservation price. The buyer informs providers the reservation price of the service and seeks bids. Privately, providers compute the prices of service and report their prices of service in sealed bids to the buyer.

1.2 *Background Research on Network Pricing*

There is a wide range of methods used to find an optimum policy of pricing for Internet services. Summaries of the pricing research can be found in [9],[26],[27],[28],[29],[30],[31],[32]. The following examples are central to our research.

1.2.1 *Service per Customers' Bids*

In a pioneering study of a pricing model where customers send bids to a provider for a service, Kelly [7] addresses the issues of charging, rate control, and routing for a network that carries elastic—variable rate--traffic. He proposes a market where each customer submits a bid to the provider. In Kelly's research, the bid is the willingness to pay per unit of time. The provider accepts these submitted bids and determines the price of each network link. Then the provider assigns the user a data-rate in proportion to his bid. The rate is inversely proportional to the price of the links the customer wishes to use. The study does not employ game theory because customers do not anticipate the effect of their actions on the prices of the links. Nevertheless, the study shows that such a scheme maximizes the profit.

1.2.2 *Static Congestion Game*

Johari and Tsitsiklis [8] explore the properties of a static game where users of a congested resource anticipate the effect of their actions on the price of the resource. In their study, a single network allocates network capacity among a collection of users. Each user applies a profit function depending on their allocated rates. The profit function depends on the total rate obtained from the network. The optimization of max-flow problems yields the rate. The network supports homogeneous traffic, i.e. only one class of service. The market model is similar to Kelly [7] except that users anticipate the effects of their actions simultaneously. Thus, the model becomes a static game. Johari's network game uses individual bids at each link, as opposed to Kelly's game where each user submits a single bid to the network.

Johari's et al.'s study shows that for a single provider, the users receive a Nash equilibrium profit of at least ¾ of the maximum possible aggregate profit. The results also show that the self-interested behavior of the individual user does not create congestion or degrade performance if a pricing mechanism is carefully chosen. In our research, we use congestion as a parameter of network cost.

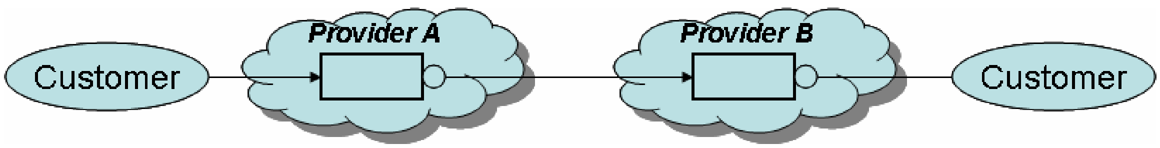
1.2.3 Provider's Monopolist Game

DaSilva [9] espouses a game theory approach when studying static pricing policies for multi-service networks. He conducts the study in ATM[1] networks of priority-based and allocation-based weighted round robin (WRR) scheduling. The study uses a non-cooperative game among a set of users where a provider determines a price in advance. The provider strategy is to optimize the operating point of the network by adjusting the price. A user strategy is to maximize its profit given all other users' service choices. Here, the provider is a monopolist and the users are the players. A provider induces one or more Nash equilibriums according to the network architecture, the available resources, and the pricing policy adopted. The study demonstrates that the adoption of an appropriate pricing policy enables the service provider to offer the necessary incentives for each user to choose the service that best matches its needs, thereby discouraging over-allocation of resources and maximizing customer's profit. Richard La et al. [10] study a similar monopoly market. In contrast, we study an oligopoly market.

[1] Asynchronous Transfer Mode (ATM) network supports cells or fixed sized packets

1.2.4 *Peer Providers in a Series*

Linhai and Walrand [11] present a generic model for pricing Internet services in a multiple provider network. Customers' calls are routed through multiple providers; i.e., all provider networks are connected in series.



The existence of Nash Equilibrium in game theory is used to show the outcome of games between service providers. The result shows that non-cooperative pricing is unfair and may discourage future upgrades of networks. On the other hand, a simple revenue sharing policy is fair, more efficient, and encourages providers to collaborate without cheating. In contrast to the Linhai et al.'s model, the providers in our research do not connect each other. The peering interface of our model is in between an enterprise and multiple providers.

1.2.5 *Game of Incomplete Information in Sealed Bid Reverse Auction*

Bandyopadhyay et al. [13][14] propose an on-line exchange oligopoly model combining the *model of sales* by Varian [1] and a sealed-bid-reverse-auction [1][43][44]. Varian's model associates the Bertrand oligopoly game of incomplete information. Buyers submit their Requests For Purchase (RFP) that describe their requirements for a homogenous product in the online exchange and invite suppliers to view and respond to the RFP. Sellers engage in a static game of incomplete information and attempt to be the lowest bidder. Bandyopadhyay et al. [14] study sellers' behavior by Reinforcement-Learning (RL) simulation. We extend the Bandyopadhyay model to an Internet providers' game of oligopoly in our research.

1.2.6 *Transaction-level Pricing Network Architecture*

Zhangxi Lin et al. [15] propose a transaction level pricing architecture based on a bandwidth broker for a Virtual Private Network (VPN) model. The bandwidth broker schedules data flows with a pricing mechanism for an affiliated VPN gateway. This architecture is a VPN Round Robin (RR) extension of Gupta et al's [16] earlier general equilibrium economic model for priority pricing of network resource allocation. The architecture involves only one provider; therefore, no oligopoly market is involved. The model optimizes the price of service and the provider's profit. The study does not implement any game theory. In our research, we extend this concept of price-based network architecture of one provider to include multiple providers offering similar value-added services and competing for the enterprise customers in an oligopoly market.

1.3 *Problem Statement and Proposed Solution*

In this dissertation, we will solve the following problems:

- Deliver customers' requirement of automatic price-transaction mechanism in one-to-many customer-providers peer networks.
- Develop providers' strategic price computation methods in a competitive market.
- Develop providers' profit optimization method.

Our solutions to the above problems are as follows:

- We propose a new *Automatic Price Transaction-based One-to-Many Peer Network* architecture that includes price transaction mechanisms and protocols to automate price negotiations in one-to-many customer-providers peer network.
- We propose a new game theory model—the *Providers Optimized Game in Internet Traffic*—to optimize providers' profit in our proposed architecture. This model determines strategic price using a myopic Markovian-Bayesian game of incomplete information and an extension of previous work based on the Bertrand oligopoly model. Selecting a strategically appropriate price synchronized with the dynamic Internet traffic demand is one of our methods to optimize profit; the others are minimizing the network congestion sensitive cost and optimizing routes. This model has two distinct parts:
    - The development of providers' oligopoly game.
    - The development of providers' profit optimization method.
- We propose an algorithm to implement the game model. The algorithm synthesizes game theory, internet traffic engineering, and non-linear optimization techniques.

The following sections provide snapshots of these solutions.

1.3.1 *The Proposed Price Transaction Architecture and Protocol*

We briefly describe the *Automatic Price Transaction-based One-to-Many Peer Network* architecture in this section. Chapter 2 presents its detailed description

A Session Initiation Protocol (SIP) session is a voice call or a multi-media connection between two end User-Agents (UAs) in the Internet. In this new price transaction architecture, an enterprise and a provider communicate pricing information and agree on a price for each SIP session; i.e., we assume per call pricing. An enterprise consists of multiple UAs requiring separate SIP sessions. A session originates from one enterprise region and propagates to another region through a provider.

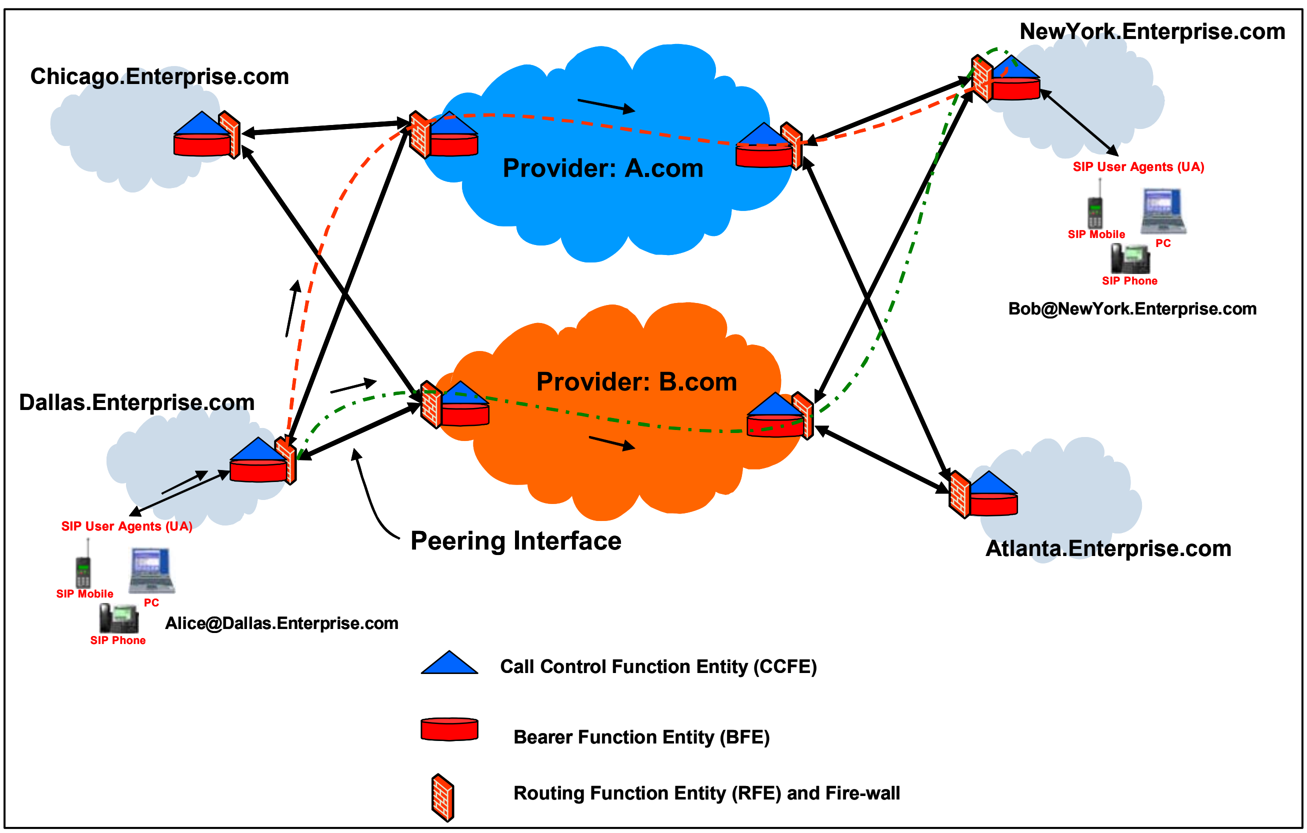


Figure 1.1: Enterprise-Provider one-to-many peer network topology

Figure 1.1 shows two providers (A.com and B.com) in a market providing services to an enterprise that has four regional networks: Chicago, NewYork, Dallas, and Atlanta. Each enterprise peers with A.com and B.com, both physically with optical transport, and logically with distinct Label Switched Paths (LSPs). SIP based control and signaling protocols enable an enterprise to either establish all of its sessions through one provider or distribute its sessions through both the providers. For example, Dallas.Enterprise.com has two choices to initiate a session from Alice@Dallas.Enterprise.com to Bob@NewYork.Enterprise.com. Depending on the price of service bid by A.com and B.com, Dallas.Enterprise.com establishes the call through either the dashed path or dashed-dotted path.

Enterprises have limited budgets and providers privately send bids to enterprises. These two important conditions require that the automatic price transaction protocol implement a pricing negotiation technique analogous to the sealed-bid-reverse-auction theory. In this protocol, an enterprise dynamically requests the price of a session by broadcasting their reservation price by simultaneously sending RFPs to all the providers. Privately, the providers compute and inform the enterprise of their bids. Then, the enterprise selects the lowest bidding provider to setup the session. We define this novel mechanism in the peer network to negotiate price as the automatic sealed-bid-reverse-auction protocol.

We assume that enterprises are rational agents; their reservation prices represent the fair market price of the services and the reservation price of a service is agreed upon prior to implementing the protocol. We also assume that enterprises do not violate agreements by changing the reservation price during the game.

In order to maintain the Quality of Service (QoS) of each session, the networks in this study are appropriately traffic engineered to meet anticipated queuing delays. This is accomplished by implementing capacity constraints through traffic engineering rules as specified in Section 5.3. Each provider supports alternative routes through its network and has a mechanism to perform optimum routing.

1.3.2 *Proposed Providers' Game of Oligopoly*

In this section, we briefly present the providers' oligopoly game of our model. Chapter 3 describes the game in details.

In this proposed model, each provider computes the price of services by a static game of incomplete information in an oligopoly market. The model assumes that there are only a few providers in a market and the billing is asymmetric—providers bill enterprises for establishing sessions and transporting traffic, but enterprises do not bill providers. Providers dominate the market by their strategies to compute price. One provider's action will influence the market price, profit, and traffic flows of all providers. However, one provider alone cannot completely control these parameters.

All the players (enterprises and providers) are assumed to implement technical mechanisms prior to the start of the game. This means providers do not provision or activate any new network component during the game. No network failure occurs during the game. All the players sign business agreements prior to the start of the game; i.e., no new player joins after the game begins and no player leaves until the game ends. A reservation price is agreed during the business agreements. Customers are rational agent: they do not violate their agreements by chaning the reservation price during the game. All providers' access bandwidth is limited at the peering interface. It is assumed that no single provider's capacity alone meets the sum of the bandwidth demand of all the enterprises in a region. In this market, the aggregate peer bandwidth of all providers is assumed to exceed the total market bandwidth demand. The lowest priced provider may sell to maximum bandwidth capacity and the higher priced provider may sell to the residual bandwidth demand. The model takes into account the dynamic nature of Internet traffic demand.

In the Internet terminology, a session is an IP call. The session initiation is performed by the signaling layer and IP packets flow through the media layer. A media session generally consists of many IP packets. We assume that the session arrival distribution is Poisson [74] and the session duration distribution is exponential.

We consider that each session initiation request is an instance of a game. When a session initiation request arrives, each provider develops a belief function based on a *myopic* Markovian-Bayesian game of incomplete information. Then, it determines a service price from this belief function by implementing the specific strategies discussed in Section 3.6.

The parameters of the belief functions are the number of providers in the market, the market capacity, the perceived market demand, the reservation price of service, and the marginal cost of a provider.

Determining providers' belief functions and strategies is the central task of our research.

1.3.3 Proposed method of Optimizing Providers' Profit

In this section, we briefly present the providers' profit optimization method of our model. Chapter 5 describes the method in details.

Provider's profit optimization is central to our research. We propose a new algorithm that synthesizes game theory, traffic engineering, and non-linear programming technique to optimize profit. We state the profit (equation (1.1)) optimization problem as follows:

$$\begin{aligned}&\text{Maximize} \quad u(.)\\ &s.t. \begin{cases}\text{Network Architecture Constraint}\\ \text{Internet Traffic Pattern and Queue System Constraint}\\ \text{Game Strategy Constraint}\end{cases}\end{aligned} \tag{1.8}$$

$$Max\ u(.) = Max\ (p-\omega)Y \tag{1.9}$$

$$(Max\ pY + Max\ (-\omega Y)) \Rightarrow Max(p-\omega)Y \tag{1.10}$$

$$(Maximize\ pY + Minimize\ \omega Y) \Rightarrow Maximize\ u(.) \tag{1.11}$$

Therefore, our intention is to perform the following two objectives to optimize profit ($u(.)$), although we may not be able to simultaneously achieve the both:

- Maximize revenue (pY).
- Minimize service cost (ωY).

Selecting a strategically appropriate price is our method to optimize revenue. We will provide a best strategy selection method that determines appropriate price from the belief function of the providers' oligopoly game.

Change in traffic pattern varies the degree of congestion in the network. A key indicator of network congestion is the mean packet count in the network's queue systems. An increase in the packet count in the system increases the mean delay in packet transmission. Consequently, it degrades the service quality. The degradation of service is detrimental to revenue. Thus, our model associates the network congestion with the service cost.

The mean packet count in the queue system of each provider varies with the change in the traffic load of its network and the routing pattern of traffic inside the network. Enforcing optimal routing [85] to minimize network congestion—the mean packet count in the queue system—is our method of minimizing service cost. We apply two well-known non-linear programming techniques, the Gradient Projection and the Golden Section Line search methods [46][48][49] [50], to minimize the mean packet count in the system.

Each network node of this research is equipped with an infinite memory single integrated output queue per link using the First-In-First-Out (FIFO) scheduling scheme. We assume that the IP packet arrival process and the packet size distributions, respectfully, are Poisson and Exponential. When traffic aggregates into a queue, the aggregate traffic arrival process and packet length distributions are Poisson and Hyper-Exponential. Thus, we assume the well-known classical Markovian (M) General model (M/G/1)[74][75] of queuing theory. Thus, we perform M/G/1 queuing analysis [74] to develop traffic-engineering rules. However, we approximate the mean packet count in the queue system using

M/M/1 theory so that we may use results from the theory of M/M/1 network queue systems.

1.3.4 *Proposed Algorithm*

Our algorithm for a session or a game instance to optimize provider profit consists of the following steps:

i) Enforce traffic engineering rules based on M/G/1;

ii) Perform optimum traffic routing;

iii) Approximate the optimum congestion indicator (mean packet count[2] in the network based on M/M/1);

iv) Develop instantaneous congestion-sensitive service cost;

v) Develop the belief function by the proposed game of oligopoly;

vi) Select the best strategy to determine strategically appropriate price;

vii) Conduct game: simulation of session initiations-terminations and emulate customer price negotiation by sealed bid reverse auction protocol.

1.3.5 *Research Methods*

We conduct mathematical analyses and simulation to evaluate the performance of the *Automatic Price Transaction-based One-to-Many Peer Network* architecture that implements the *Providers' Optimized Game in Internet Traffic* model.

Our research methods consist of the followings:

- Develop the *Automatic Price Transaction-based One-to-Many Peer Network* architecture and associated protocols for a two providers SIP based network.

---

[2] The literature [85] develops optimum routing as a function of optimum mean delay. On the other hand, we develop optimum routing as a function of optimum mean packet count because majority of the vendor routers keep the record of mean packet count instead of mean delay. We want to stress that there is no difference in the mean delay method and our mean packet count method because they are directly related through Little's Law [59],[60].

- Develop the *Providers' Optimized Game in Internet Traffic* model:
  - Develop a duopoly market, define parameters of the belief function, develop analytical model of the belief function, and identify a set of strategies.
  - Develop the non-linear program to perform optimal routing [85].
  - Design a network, develop traffic engineer rules, and assign traffic paths.
- Develop a simulation model in the MATLAB[3] tool.

We verify analytical models by simulation results. By maintaining the simulated market demand equal to the mathematical desired demand, we compare the simulated market price and the simulated provider profit with corresponding values from analysis. We determine the best strategy (the Bayesian-Nash equilibrium and Pareto-efficient outcome) to optimize provider market shares of profit in all market demand for the homogenous and heterogeneous classes of service. Chapter 7 and 8 describe details of these methods.

[3] MATLAB) is an integrated technical and mathematical computing tool and is a product of MathWorks (www.mathworks.com).

1.4 *Distinguishing Characteristic of our approach*

In our approach, customers have options for subscribing to services from a provider of choice based on the price using the new *Automatic Price Transaction-based One-to-Many Peer Network* architecture. In addition, we propose a method for providers to optimize profit using the new game model, the *Providers Optimized Game in Internet Traffic*. This game model is sensitive to the dynamic Internet traffic demand, the congestion in networks and the service class.

The Third Generation Partnership Project (3GPP) develops wireless standards that refer to pricing as charging. The recent work [69]-[73] in 3GPP on charging uses a wireless consumer to provider (one-to-one) model. However, it does not provide options for customers to negotiate price with providers in one-to-many peer architecture similar to our architecture.

SIP based peering among multiple providers is a new phenomenon. The ATIS-PTSC[4] is developing SIP based IP peering standards between two providers for one-to-one peer network [68]. However, the ATIS initiative lacks automatic pricing mechanism and one-to-many peer features.

The Internet Engineering Task Force (IETF) is an Internet professional community that develops Internet protocol specifications known as Request For Comment (RFC). The IETF RFC 3455 [67] specifies SIP header fields to transport price information; however, it does not provide any example of SIP flow to implement price transaction. We provide an example of SIP flow to illustrate the price transaction method.

Lin et al.' [15] research is an example of a transaction-based pricing, which can be viewed as the automatic pricing between an enterprise and a provider. However, they do not provide solutions for enterprise-provider one-to-many peer networks.

[4] The Alliance for Telecommunications Industry Solutions (ATIS) is a North American standard organization. Packet Technologies and Systems Committee (PTSC) is an ATIS committee that develops standards related to Internet services, architectures, and signaling.

Significant Internet services pricing research [9][10][11][17][18][21][23][26] relates monopoly markets where consumers strategically interacts to get services from a single provider The study of an oligopoly market where providers are competing for enterprises is the main distinguishing characteristic of our research.

The majority of the literature on pricing [9][26][27][28][29][30[[31][32] does not provide any price transaction protocol or algorithm to compute price. In this dissertation, we suggest an automatic price transaction protocol, a SIP flow, and an algorithm to compute price.

Although academics conducted significant research on dynamic pricing in the 1990s, critics pointed out that the computational complexity would make the dynamic pricing expensive and hard to implement [9]. The recent significant technological advance in microprocessors and memory enables networks to perform complex computations on per session and per packet basis. Therefore, dynamic pricing schemes will not be hard to implement. In addition, the fall in the price of microprocessors will also make it inexpensive. Criticism against the dynamic pricing is no longer valid as the technology advances and becomes affordable. It is particularly true for the Voice over IP (VoIP). More importantly, our dynamic pricing scheme is not between a consumer and a provider; rather, it is at the peering interface between provider and enterprise to transport aggregate traffic.

Another common criticism [9] of dynamic pricing is that the customers may have to pay more than their budget if the price fluctuates; as a result, a dynamic pricing scheme will encounter adverse reaction from them. Our proposed dynamic pricing mechanism deploys a sealed bid reverse auction. In this mechanism, enterprises send their fixed budget value as a reservation price to the providers and the providers always bid less than the customers' budgeted amount.

While we propose a dynamic pricing mechanism, we implement a static game. As mentioned earlier, our model stems from the Bandyopadhyay et al. [13][14] and Varian's [1] static game of incomplete information. In our model, the commodity is the internet bandwidth rate per class of service whereas in

Bandyopadhyay et al.'s model the commodities are goods (e.g. auto-parts) sold in an on-line exchange. The Bandyopadhyay et al. oligopoly model assumes a symmetric market—the market demand and marginal cost do not change during the game. Internet traffic demand and network congestion dynamically change depending upon the time of the day, day of the week, and special days of the year. Thus, static market demand and static marginal cost do not map well with the provider game of oligopoly. We take into account the dynamic nature of Internet traffic demand and congestion in the network; thus, we study an asymmetric market.

The Bandyopadhyay et al. model is a two-step static game. A firm sells its total capacity at once, and then another firm sells the total residual demand. In our model, each SIP-based session setup is an event of a game and the bandwidth for each session is much less than the market capacity. The sessions are established as well as deactivated according to the arrival load. One of the parameters of the game uses a one-step near-sighted history for each session arrival game. Thus, our model is a "myopic" Markovian game. In addition, a market consists of regional markets that have capacity restrictions. We study both the homogeneous and the heterogeneous service-based networks.

In [14], the Reinforcement Learning (RL) procedure by simulation is proposed for determining the best strategy from the mixed strategy equilibrium. The RL is suitable when marginal cost is constant. Due to the dynamic nature of the Internet, converging to a best strategy with RL will be difficult to achieve. The implementation of the RL mechanism in the network device may also add extra cost. Therefore, we simplify the implementation by defining a set of feasible strategies from the mixed strategy equilibrium. Then, we identify the best strategy from this set by analytical and simulation methods.

1.5 *Summary of Contribution*

The major contributions of our research are as follows:

- We proposed the *Automatic Price Transaction-based One-to-Many Peer Network* architecture allows providers and customers to automatically negotiate price. It facilitates customers' options for subscribing services from a provider that offers the lowest price. This proposed architecture introduces a new service in the Internet and the wireless market.
- The proposed architecture extends the ATIS one-to-one peer and the 3GPP charging architectures to support one-to-many peer model.
- We propose a price transaction protocol and a SIP flow for the proposed architecture.
- Proposed *Providers Optimized Game in Internet Traffic* model allows providers to offer competitive service price within the budget of the customers. The model eliminates the reactive time of price computation. The model is sensitive to the dynamic internet traffic demand, the network congestion cost, and the service class.
- We propose an algorithm to implement the game model synthesizing game theory, traffic engineering technique and non-linear programming.
- We develop a simulation tool implementing the proposed algorithm.
- Our method determines the dominant, the Bayesian-Nash equilibrium, and the Pareto-efficient outcome strategies from a set of feasible strategies. These strategies maximize providers' expected profit.
- Our method achieves relative advantage over the classical Bertrand model of price, which is commonly used in the short-term market.
- Our method decreases the market price of services relative to the customers' budgets while guaranteeing customers' preferences.
- Our method optimizes profit in fair market share and in fair market throughput.

- In addition to the profit optimization, providers can implement our method to perform least price routing, traffic load distribution, capacity planning, and service provisioning.

1.6 Structure of the Dissertation

In Chapter 2, we present the *Automatic Price Transaction-based One-to-Many Peer Network* architecture and associated price-transaction protocol, and the SIP call flow. Chapter 3 develops providers' game of oligopoly by defining parameters and stating assumptions. A method of defining a feasible strategy set is presented. We develop a non-linear program in Chapter 4 to optimize traffic flow in the network to minimize the mean packet count in the network queue system. This traffic flow optimization minimizes the marginal cost of service and maximizes provider profit. In Chapter 5, we present the research design of a duopoly network architecture, assigning the capacity of links and describing traffic flow through the network. Chapter 6 presents the algorithm of the *Providers Optimized Game in Internet Traffic* model and the simulation algorithm. In Chapter 7, we perform mathematical analyses and validation. In Chapter 8, we present simulation results and model applications for homogeneous and heterogeneous service-based networks. We conclude with lessons learned and possible future directions of this research in Chapter 9. We provide two appendices: In Appendix A, we outline mathematical optimization techniques; in Appendix B, we present acronyms.

2 Network Architecture and Protocol

This chapter describes the new *Automatic Price Transaction-based One-to-Many Peer Network* architecture where customers peer with providers by Session Initiation Protocol (SIP) based intelligent entities at the interconnect interfaces. These SIP entities automatically perform price negotiations, session management, policy and security enforcements, and service delivery assurance. This chapter focuses on the price-based network architectures, price negotiation techniques, and the SIP protocol.

2.1 Network Architecture

In this section, we first present outlines of SIP entities. Second, we briefly describe the general Internet Protocol (IP) peering network architecture of Alliance for Telecommunications Industry Standards (ATIS)[5] and 3GPP charging architecture. Then, we propose our price-based network architecture and protocol. Finally, we present a SIP flow.

2.1.1 SIP Entities

SIP is a signaling protocol to create, modify, and terminate multimedia sessions in the Internet. IETF Request For Comment (RFC) 3261 [66] describes the foundation of SIP. Other RFCs define SIP extensions to deliver signals for IP based multimedia applications. SIP is a nascent protocol and continued development of SIP standards and applications are underway. A detailed description of SIP can found in SIP related IETF RFCs[6] and literatures [61]-[65]. The main entities of SIP are User Agents (UA), registrars, proxy servers, location server, redirect servers, and presence servers.

---

[5] ATIS standards can be viewed at http://www.atis.org

[6] SIP RFCs can be viewed at SIP, SIPPING, SIMPLE, and MMUSIC working groups of IETF (www.ietf.org).

UAs reside in users' applications such as phones, computers, video equipment, Personal Digital Assistants (PDAs). This equipment can be either mobile or fixed. A UA initiates and establishes voice or multi-media sessions with another UA. When a UA is connected to the network, it first registers its location with the SIP network entity called a registrar.

Proxy servers are SIP routers. Generally, a proxy and a registrar are located in the same physical box. The function of a registrar is to keep the location addresses of the users. A proxy learns the location address of the destination from the nearest registrar and routes a SIP message towards the destination addresses. In case a registrar does not reside in the same box as a proxy, the proxy seeks the destination address from a location server, which contains a database of current locations of each user.

A proxy server can forward a SIP message to either a single destination or multiple destinations. A proxy server capable of forwarding SIP messages to multiple destinations is called a forking proxy. A redirect server does not route a SIP message but provides the potential address of the destination to the UA that sends the SIP message. Note that we do not show many other SIP messages in this example.

A Back-to-Back User Agent (B2BUA) is the combination of two user agents or proxies into the same entity. It breaks an end-to-end session to multiple call legs. It terminates a session then reformulates and re-originates the session. This enforces security and policy to a SIP session.

A presence server provides information about reachability, availability, consent, and user profiles. The ongoing projects at IETF and in the research community are adding innovative features in the presence server.

We illustrate a hypothetical scenario in Figure 2-1. A high school buddy from Crawford, Texas wishes to speak to President Bush. When he dials Bush's phone number, a SIP INVITE message is sent from the UA of his phone to the proxy and the registrar in Texas.com, which cannot locate Bush. Therefore, it forwards the

INVITE to the redirect server in Crawford.com, which advises the UA to try in Bush@WashingtonDC.com.

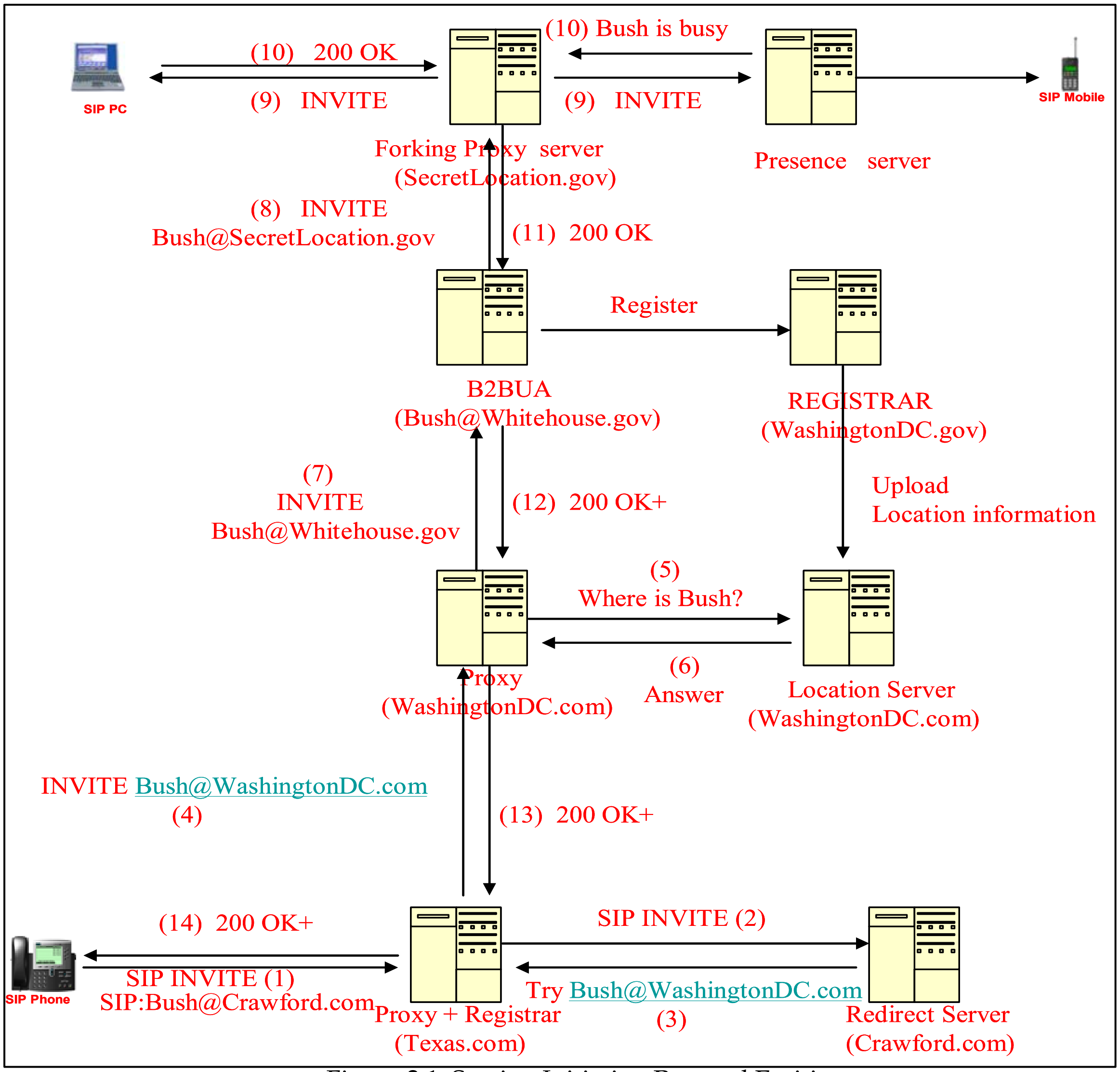


Figure 2.1: Session Initiation Protocol Entities

When the INVITE message arrives at the WashingtonDC.com, it queries the nearest location server for the destination address of Bush. Upon receiving the address Bush@whitehouse.gov, WashingtonDC.com forwards the INVITE message to whitehouse.gov. In this example, a B2BUA protects the whitehouse.gov network. It hides topology, address, location, and other secured information of whitehouse.gov. The B2BUA retranslates and reformulates both the incoming and outgoing SIP messages. The B2BUA reformulates address Bush@whitehouse.gov to

Bush@SecretLocation.gov and forwards the INVITE to the forking proxy server of SecretLocation.gov. This proxy forks the message to the multiple UAs of Bush. A presence server, which monitors the availability of Bush, tells the forking proxy that Bush is very busy; therefore, should not be disturbed. On the other hand, Bush's computer sends a 200-OK signal saying it is ready to accept the call. The 200-OK message returns to the phone of the buddy in Texas after going through reformulation and translation in the B2BUA at Whitehoue.gov. Then a media session is established between the buddy and the PC of Bush.

2.1.2 *ATIS-PTSC Reference Model*

At present, the Packet Technologies and Systems Committee (PTSC) of ATIS is developing a standard for one-to-one IP peering between two providers[7]. Figure 2-2 depicts the reference diagram of the standard.

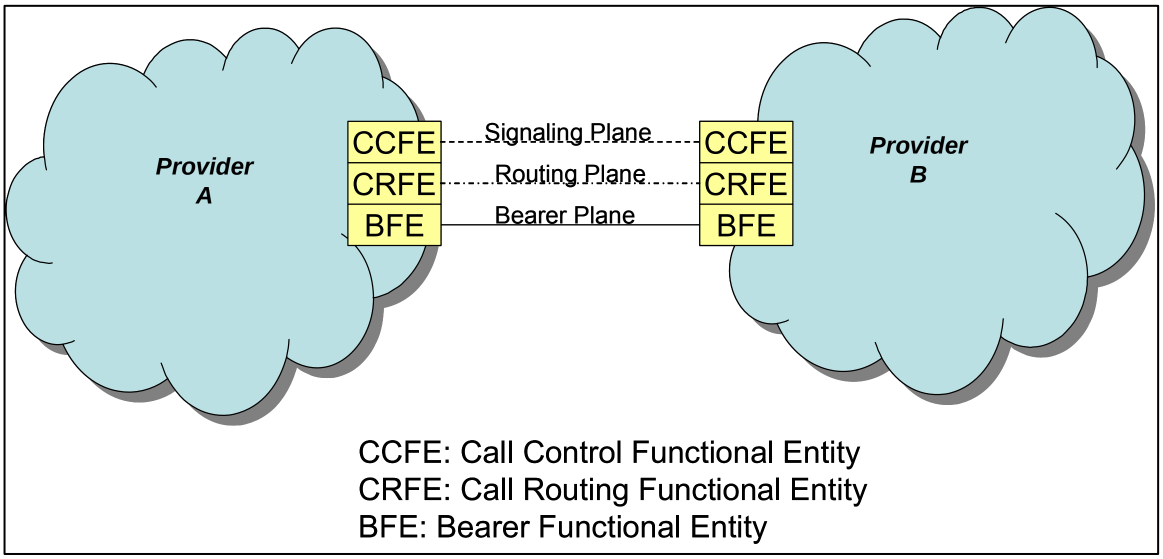


Figure 2.2: ATIS/PTSC IP Peering Reference Diagram

The ATIS viewes a peer interface in three planes: signal, route, and media (bearer). Call Control Functional Entities (CCFEs) interconnect the signaling planes of the peer providers. Call Routing Functional Entities (CRFEs) and Bearer Functional Entities (BFEs) interconnect the routing and the media planes of two peer providers, respectively. The CCFE performs signaling and control operations, enforces signaling security and policies, and conducts other intelligent tasks. For example, tasks of a SIP proxy or a SIP B2BUA are the functions of a CCFE.

A CRFE and a BFE can jointly perform the following operations: traffic routing, media transcoding, security and policy, address and topology security, and other media functions. For example, a Media Relay (MR) or an Edge-Label Switched Router (E-LSR) performs the functions of CRFE and BFE.

[7] The name of the standard is *IP-IP NNI Interconnect*

2.1.3 *Our Extension to ATIS Model*

In the ATIS-PTSC one-to-one peer architecture, an enterprise interconnects with only one provider. We propose a one-to-many peer architecture that allows an enterprise to peer with multiple providers. Each enterprise can maintain physical connections to all providers in the market. The enterprise configures separate and parallel Label Switch Paths (LSPs) to all the providers. The LSPs are elastic, i.e. the bandwidth of the data path through the providers may vary. This enables each enterprise to either transmit all of its traffic through one provider or distribute its traffic to all providers. LSPs are configured through the BFEs of the enterprise and providers. Note that the providers are not connected with each other.

We propose two new modules—a price *broker* and a price *analyst*—as a part of the peering mechanism between an enterprise and providers. An enterprise price broker computes the reservation price of a service and develops a Request For Purchase (RFP) data element. An analyst of a provider computes the price of service based on the provider's game strategies as proposed in Chapter 3.

We also propose a forking proxy server at the CCFE of the enterprise and a combined module of a presence server and B2BUA at the CCFE of each provider.

The automatic transaction protocol of Section 2.1.6 illustrates price negotiation between an enterprise broker and a provider analyst. An enterprise provides services to the consumers—SIP user agents—requiring separate multi-media sessions through the provider's network. In the enterprise network, when a UA requests a connection, the price broker sends the RFP to the forking proxy. This proxy transmits the RFP to all the peer providers. In a provider network, the presence server receives the RFP from the enterprise and passes it to the price analyst. Then, the analyst informs the presence server of the price of service. The provider's presence server passes the price as a bid to the enterprise proxy, which in turn forwards it to the broker of the enterprise. After receiving all the bids from all the providers, the broker selects the lowest priced provider and instructs the enterprise proxy server to initiate the session to the destination through this provider. Note that the enterprise assumes that all providers deliver identical QoS for each service class. The proxy instructs the BFE to create a media path between the enterprise and the provider to transport media over IP packets.

In a provider network, an analyst is either a central entity or distributed entities located with the CCFEs. We assume that an analyst is a central entity in each provider's network. The analyst can either compute the price of a service periodically or upon a session request. The granularity of the period will be implementation specific and will be determined by the network designers. We assume that the analyst computes the price of a session for each session request.

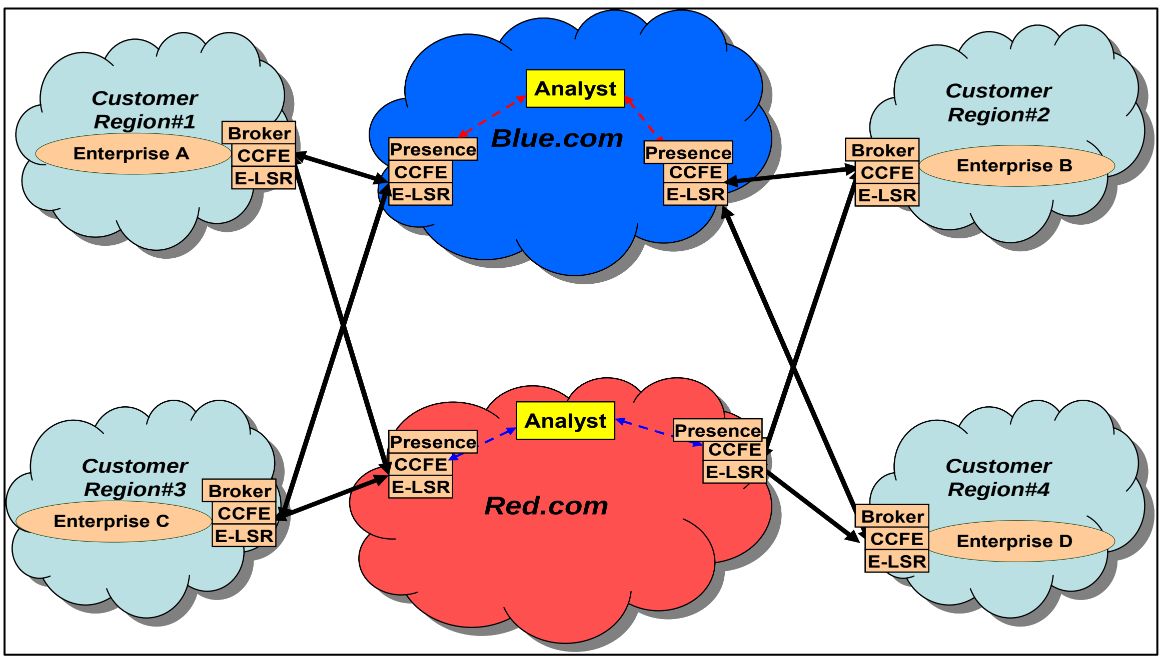


Figure 2.3: Network Architecture of Duopoly Market

Figure 2-3 depicts the proposed network architecture in a duopoly market. There are two providers (Blue.com and Red.com) and four regions in this market. There are multiple enterprises in each region. Each enterprise peers with both Blue.com and Red.com. Each provider implements a centralized analyst. The price broker resides with the CCFE of each enterprise network. E-LSRs perform the functions of CRFEs and BFEs.

2.1.4 3GPP IMS Charging Architecture

The 3GPP standards[8] and [61][62] describe the IMS architecture in details. Six category groups represent the 3GPP IMS entities: session management and routing, databases, interworking, services, support, and charging. Table 2.1 summarizes these categories and their associated functional components.

Table 2.1: 3GPP IMS Functional Components

Categories	**Functional components**
Session Management and Routing	Proxy-Call Session Control Function (P-CSCF) Interrogating-Call Session Control Function (I-CSCF) Serving-Call Session Control Function (S-CSCF)
Databases	Home Subscriber Server (HSS) Subscription Location Function (SLF)
Interworking	Breakout Gateway Control Function (BGCF) Media Gateway Control Function (MGCF) Media Gateway Function (MGWF) Signaling Gateway (SGW) Border Control Function (BCF) Border Gateway Function (BGF)
Services	Application Server (AS) Multimedia Resource Function Controller (MRFC) Multimedia Resource Function Processor (MRFP)
Support	Policy Distribution Function (PDF) Security Gateway (SEG) Topology Hiding Inter-network Gateway (THIG)
Charging	Online and Offline Charging

There are two types of IMS charging functions: online and offline. The online charging function pertains to our research. This allows a provider to automate charging of wireless customers in a one-to-one relationship with customers. However, customers do not have price negotiation options with multiple wireless providers. Figure 2.4 depicts the charging functions related to all other IMS functions in the 3GPP model.

[8] 3GPP IMS standards can be downloaded for free from http://www.3gpp.org/specs/specs.htm

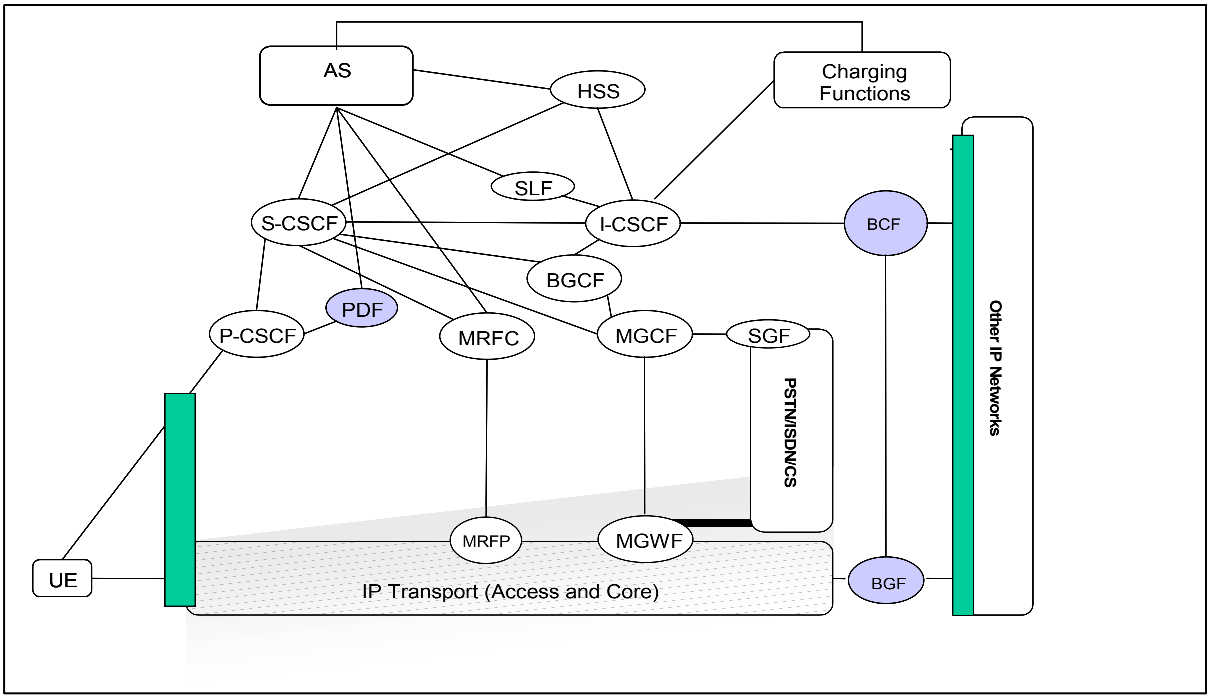


Figure 2.4: 3GPP IMS Architecture

Figure 2.5 depicts a network model of the current 3GPP IMS online charging architecture. It shows that the wireless customers can automatically subscribe from only one provider (one-to-one peer).

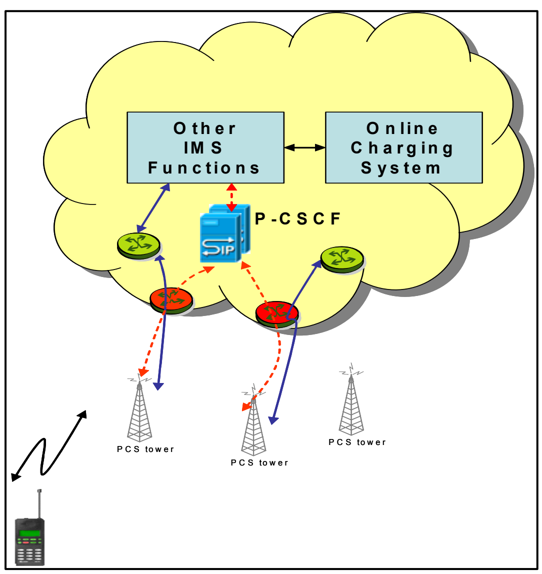


Figure 2.5: The current 3GPP IMS Online Charging Architecture

The current 3GPP IMS online charging system consists of the Event Charging Function (ECF), the Session Charging Function (SCF), the Bearer Charging Function (BCF), the Rating Function, and the Correlation Function as illustrated in Figure 2.6.

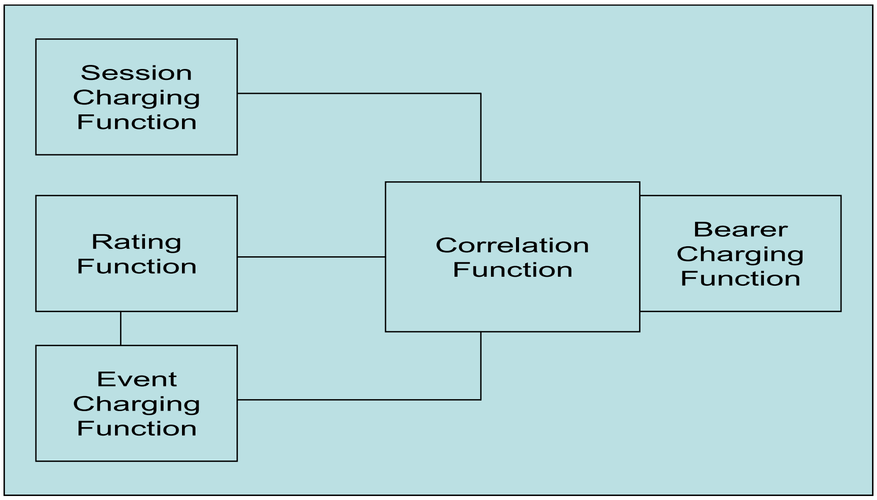


Figure 2.6: 3GPP Online Charging System

One application of the ECF is to enforce event related pricing policies such as purchasing on-demand movies. The SCF charges based on session resource usage, allows or denies a session based on a customer's credit limit, and terminates a session when a customer exceeds the credit limit. The Rating function meters unit usage, (for example, the number of movies purchased), the data volume transported, or the duration of a session). It also computes the total price of a service based on the unit bearer usage price (e.g. $20 per Mbps). The BCF controls bearer usage such as duration or the traffic volume of a session. As the name implies, the Correlation function correlates different information coming from the ECF, BCF, and SCF to produce unique charging identifiers. The standard development for the charging architecture is currently ongoing.

2.1.5 *Our Extension to 3GPP IMS Charging Architecture*

We propose an extension to the 3GPP IMS online charging system to allow customers to negotiate price with multiple providers and select the wireless provider that offers the lowest price.

Figure 2.7 depicts our proposed extensions to the 3GPP online charging architecture in a duopoly market. For simplicity, we illustrate identical networks of two wireless providers: Blue.com and Red.com.

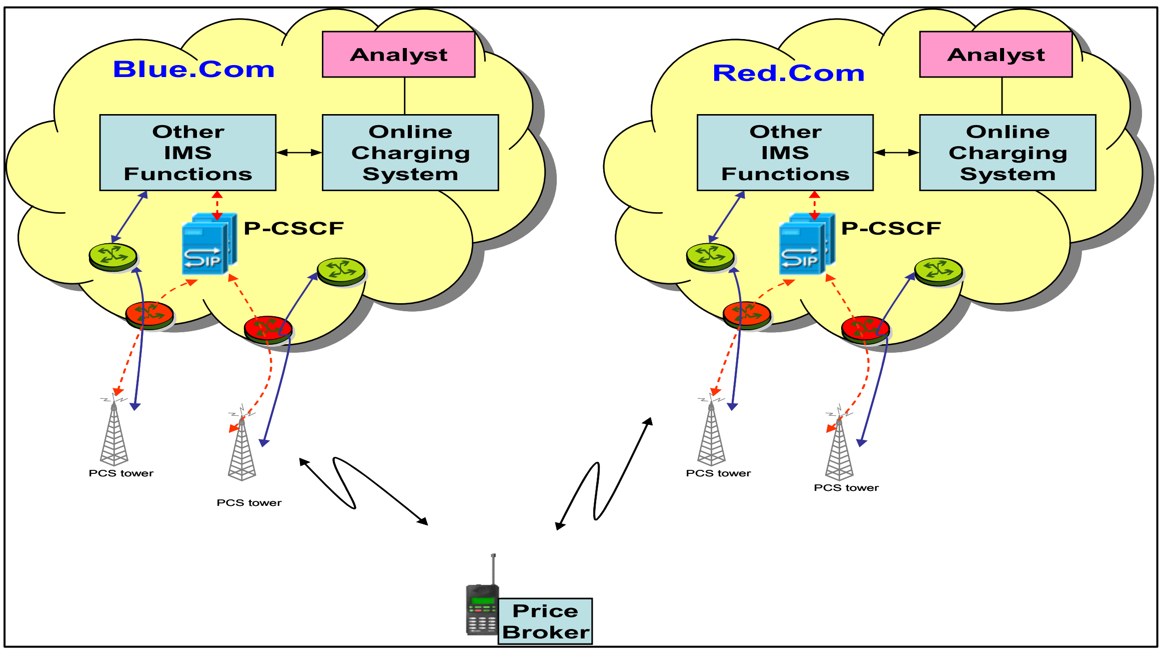


Figure 2.7: Extended 3GPP Charging Architecture in Duopoly Market

In each network, wireless traffic from PCS towers aggregates to IP routers (media layer) and P-CSCF (signaling layer). Each P-CSCF connects with other IMS functional components. We do not show all the IMS function in this figure to reduce complexity. The IMS functions are connect to the IMS online charging functions.

We propose two new modules in the 3GPP Charging architecture—a price *Broker* and a price *Analyst*—allowing a wireless customer to automatically shop for services from multiple providers. In Figure 2.7, the User Equipment's (UE) price broker computes the reservation price of service and develops an RFP data element. Prior to the session initiation process, the UE transmits the RFP to the on line charging module of a provider via P-CSCF and other IMS modules. The analyst connects with the online charging module. An analyst of a wireless provider computes the unit bearer usage price of service based on the provider's game strategies as proposed in Chapter 3.

Price negotiation between a UE's broker and a provider's analyst is analogous to the automatic transaction protocol of Section 2.1.6. The UE transmits the RFP to all the wireless providers. In each network, the P-CSCF receives the RFP and passes it to the price analyst. Then, the analyst feeds the unit bearer usage price of service to the Rating function. The online charging system computes the appropriate price. Using the provider's wireless equipment and SIP, the provider's P-CSCF passes the price as a bid to the customer's broker. After receiving all the bids from the providers, the broker selects the lowest priced provider and instructs the SIP user agent of the wireless UE to initiate the session with this provider.

Our pricing model for both ATIS and 3GPP are analogous. Therefore, in the rest of this thesis, we will concentrate on only one: the proposed one-to-many enterprise-provider peer architecture (extension of the ATIS model) as described in Section 2.1.3.

2.1.6 *Other Protocol-based Networks*

This research illustrates the SIP based IP network; however, our price transaction mechanism is protocol agnostic. In Table 2.2, we illustrate examples of peer modules for different protocols.

To safeguard networks from outside attack and to enforce policies, the network providers are recently deploying Session Border Controllers (SBCs) at the entrance points of their voice and multi-media networks. In this type of network, a price border element may reside in the SBC. Note that our model also extends 3GPP IMS architecture to allow wireless customers options to shop from multiple providers.

Table 2.2: Components of different types of networks

Type of connection	Network Type	Standard	Protocol	CCFE	BFE
VoIP or multi-media session	Distributed IP network	IETF	SIP	Presence and proxy servers	Media Relay or E-LSR
VoIP or multi-media session	Distributed IP network	ITU-T	H.323	Gatekeepers	Media Relay or E-LSR
VoIP or multi-media session	IP or Asynchronous Transfer Mode (ATM)	ITU-T	Bearer Independent Call Control (BICC)	Media Gateway Controller	Media Gateway, Media Relay, or Edge-Switch
VoIP session	Soft-Switch	NA	SIP/H.323	Media Gateway Controller (MGC)	Media Gateway or Media Relay
Multi-media wireless over IP session	IMS	3GPP2	SIP	P/I-CSCF and SIP presence servers	Border Gateway Function (BGF)
Cable providers' multi-media sessions.	Cable Multi-media	CableLabs	Cable Management Server Signaling (CMSS)	Cable Management server (CMS)	Media Relay or E-LSR
ATM VP/VPC	ATM	ATM Forum	PNNI	Edge Switch	Edge Switch
Data layer LSP setup	IP/MPLS	IETF	RSVP-TE or CR-LDP, BGP	E-LSR or multi service edge router	E-LSR
Optical connection	Multi-protocol Lambda Switch (MPλS)	IETF/ITU	GMPLS	Optical controller	Photonic switch

2.2 *Proposed Automatic Price Transaction Protocol*

This section summarizes the proposed protocol. The automatic price transaction protocol is analogous to sealed bid reverse auction in microeconomics. As shown in Figure 2.8, the protocol performs price negotiation, price computation, and price election automatically; i.e., no human intervention is required.

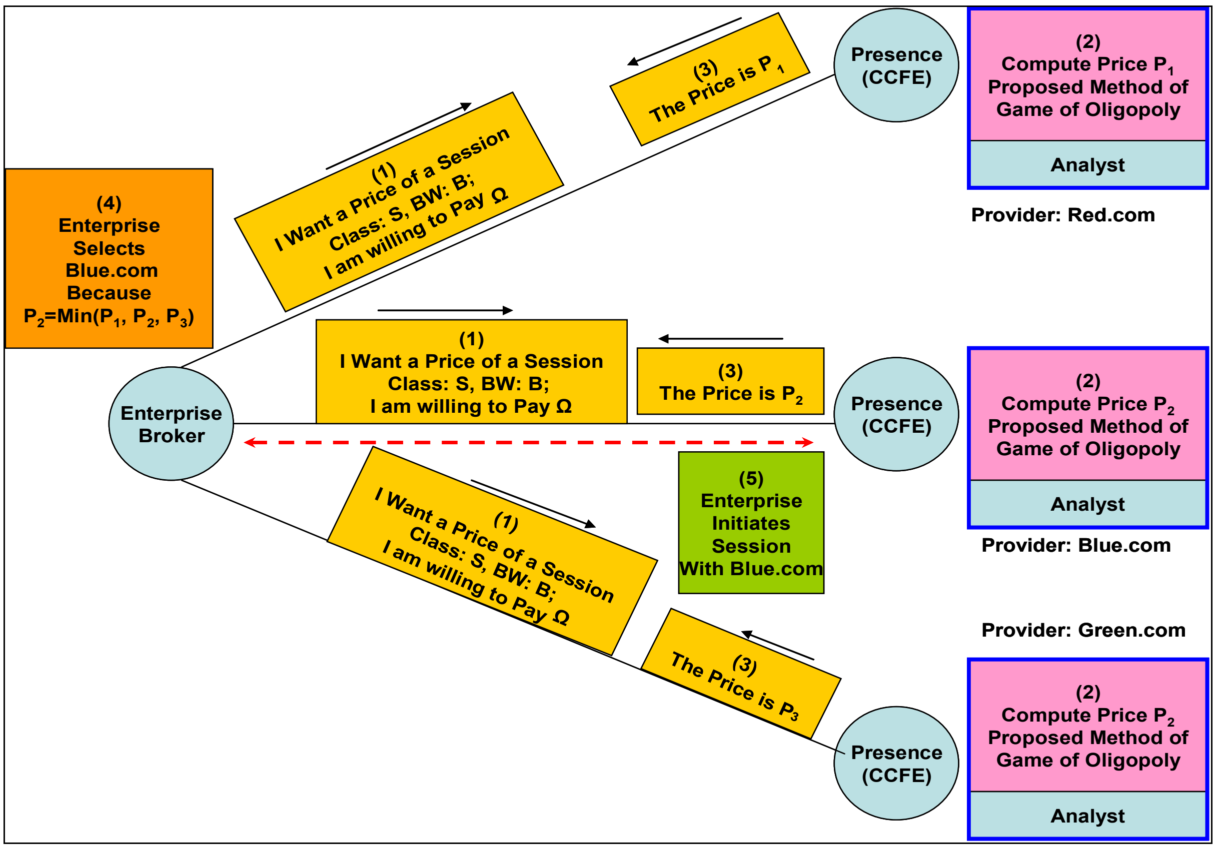


Figure 2.8: Price Transaction Protocol

This price transaction occurs prior to the initiation of each session. The following are the steps of price transaction protocol:

1. When a SIP user agent of an enterprise requests to establish a session, the enterprise broker—aided by the forking proxy—simultaneously sends RFPs to all the participating providers' presence servers in the vicinity. The RFP contains the description of the destination, the service class ($s \in S$), the

enterprise index (i), the session bandwidth (B), and the service reservation price (Ω_{si}).

2. The providers' presence servers query their respective analysts to learn the price of the requested service. Analysts of all the providers compute the price of the service based on their own game strategies.
3. Presence servers of providers notify the enterprise broker of their bids for the session.
4. After receiving all the bids ($p \in P$), the broker of the enterprise selects the lowest priced provider and instructs its peer element to initiate the session.
5. The enterprise peer element sends a SIP INVITE message to the proxy of the winning provider.

2.3 *Proposed SIP Call Flow*

We propose the Figure 2.9 example of SIP call flow to perform price negotiations and session initiations between two SIP User Agents (UAs) in two enterprise regions: Jayhawk and Wildcat.

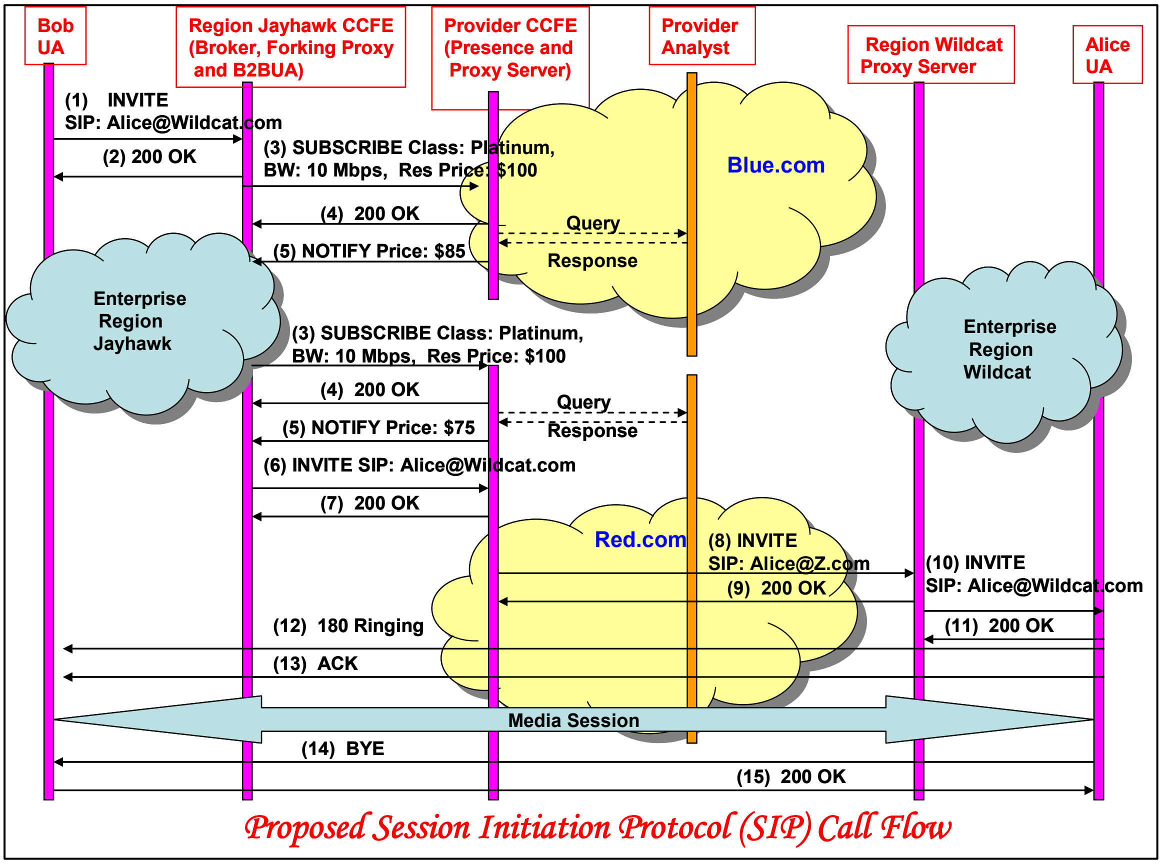


Figure 2.9: Session Initiation Protocol (SIP) Control Flow

Bob from enterprise Jayhawk.com wants to initiate a media session with Alice of the enterprise Wildcat.com. Bob's UA sends a SIP INVITE to the Jayhawk.com CCFE that contains a price broker, a proxy server, a B2BUA, and a forking proxy server. The broker prepares an RFP. The B2BUA writes RFP elements (class, bandwidth, reservation price) in the SIP message. In Figure 2.9, the class, bandwidth, and reservation price of the service are Blue, 10 Mbps, and $100, respectively. The forking proxy sends a SIP SUBSCRIBE message to the CCFEs of participating providers: Blue.com and Red.com.

The CCFE of each provider contains presence, proxy, and B2BUA servers. Upon receiving the INVITE, both the providers' proxies return 200-OK signals to Jayhawk.com's CCFE. The presence servers of the providers query their respective analysts about the price of the session. The analyst informs the computed price of service to the presence servers by using SIP presence extensions.

The providers' presence servers notify the enterprise broker of the price. In Figure 2.9, Blue.com sends a NOTIFY signal indicating that the price of the session is $85 to Jayhawk.com. Similarly, Red.com's NOTIFY signal contains the price of $75. In Jayhawk.com, the broker selects Red.com and the B2BUA transmits INVITE sip: Alice@Wildcat.com to Red.com. The proxy servers of Red.com then transmit the signals to the Wildcat.com proxy server. The session is established using the basic SIP call flows. Note that when a provider cannot meet the session request it denies the session by sending a SIP 4xx error message, e.g., SIP 406 NOT ACCEPTABLE.

2.4 *Chapter Summary*

In this chapter, we described two *Automatic Price Transaction-based One-to-Many Peer Network* architectures: enterprise-provider IP interconnect and wireless customer-provider interconnect. We depicted the protocol to negotiate the price of service in between an enterprise and providers. We illustrated an example of a SIP flow that implemented the protocol.

In this peer network architecture, to establish a session, an enterprise selects a provider that charges the lowest price of service. As a result, providers strategically compete. In the next chapter, we propose a game of oligopoly that suits providers' strategic competition for this network architecture.

3 Providers' Game of Oligopoly

The *Automatic Price Transaction-based One-to-Many Peer Network* architecture creates a market of strategic interaction among providers. The providers compete to maximize their profit. In this chapter, we develop the providers' oligopoly game of our model for our *Automatic Price Transaction-based One-to-Many Peer Network* architecture.

In Section 3.1, we select the oligopoly model. In Section 3.2, we define the classes of service and describe the preference of an enterprise. In Section 3.3, we discuss the parameters and assumptions of this research. In Section 3.4, we develop the proposed oligopoly model. In Section 3.5, we present the provider strategies.

3.1 Model Selection

In microeconomics, there are two well-established models of oligopoly: the Bertrand model of price competition and the Cournot model of capacity competition. These two models are the foundation of all oligopoly models. We first ascertain which of these models suits the proposed price-based peer architecture.

In an Internet market, when the number of providers increases, or the existing providers' deploy additional network capacities, the market capacity increases. As a result, the Cournot strategic interaction occurs and the market power decreases. Recent advances in Wavelength Division Multiplexing (WDM) optical and Third-Generation (3-G) wireless technologies have enabled providers to add enhanced features and greater capacities in the competitive market. Therefore, the Cournot model is a natural fit to analyze the Internet market for long-term competition. Conversely, within the scope of short-term competition, providers first implement a capacity, then assign prices of services for that capacity and engage in "price wars" in fixed capacity rather than through "bandwidth wars." Ultimately, the Cournot model is not a good fit for short-term strategic interaction where price is the strategic variable.

Assumption: Total market capacity or the individual market capacity of a provider does not change during the lifetime of the game.

We consider the Bertrand strategic interaction of price competition applicable to the novel model. The fundamental assumption of our model pertains to short run strategic market interaction. During the time span of the game, no new providers join the market, and no new network device is activated. A further assumption is that there is no breakdown in the network during the time span of the study. These assumptions imply that the market capacity remains fixed during the game. Each provider lowers the price of their services to win over enterprises from their opponent providers. Enterprises subscribe to services from the lowest priced provider.

We briefly describe the classical Bertrand Model [1]-[5] as a function of provider price of services. Assume that a competitive Internet market consists of two providers {A.com, B.com} who provide identical classes of service. Assume also that the cost to produce the service is zero. Denote $\Delta(p_n)$ as the market demand function of bandwidth in Mbps for provider n, where p_n is the price per Mbps of the service. Therefore, the profit of the A.com is as follows:

$$U_A(p_A, p_B) = \begin{cases} p_A \Delta(p_A) & \text{if } p_A < p_B \\ p_A \Delta(p_A)/2 & \text{if } p_A = p_B \\ 0 & \text{if } p_A > p_B \end{cases} \tag{3.1}$$

Equation (3.1) shows that the lower priced provider sells to market demand Δ(p) when its price of service is less than its competitor. When the providers' prices of services are the same, each provider wins half of the market share. If providers engage in a price war by reducing the price of service, they sacrifice profit. If both providers have the same marginal cost (ω > 0) to provide the service, the unique

Nash equilibrium (p_1^*, p_2^*) occurs when their price equates marginal cost [1]-[5]: $p_1^* = p_2^* = \omega$.

Providers' payoffs and strategies are private. Therefore, the corresponding strategic interaction among providers is a game of incomplete information. In this game, providers develop a mixed strategy function to determine the price of a given service. Therefore, we need a game of incomplete information that performs the Bertrand strategic interaction of price and develops a mixed strategy function.

Varian [1] depicts the development of a mixed-strategy function of a duopoly market in an example of a Bertand strategic interaction. This example is called "A Model of Sale", where informed and uninformed customers purchase from two providers. The strategic variable in this market is price. In relation to our research, the informed consumers purchase from the provider that offers the lowest price. There is a fixed cost and zero marginal cost to produce each unit. Consumers have the same reservation price to pay for each unit. By applying the game of incomplete information and considering symmetric equilibrium, providers develop a belief function (*F(p)*) based on probability of either success or fail in offering the lowest bid price. Each provider determines bid prices from this profile. This belief function relates the customers' reservation price, the fixed market demand, and the service price.

The Varian example above does not take into account the limitations of market capacity, variable market demand, or marginal cost associated with each product. Therefore, we need another model that considers these parameters.

Bandyopadhyay et al.'s On-Line-Exchange research [13, 14] proposed an extension to Varian's "A Model of Sale" example, which considers the limitation of market capacity, demand, and marginal cost to produce the product. The model concerns a market of a homogeneous product with symmetric equilibrium. Providers' combined capacity exceeds the total capacity demanded by buyers. Sellers individually cannot supply the entire market. The seller with lowest price sells to capacity and the others only sell to residual demand. In this game, the competition between sellers is not as extreme as the Bertrand model. The competition corresponds to a two-stage static game. Bandyopadhyay et al. develop a mixed strategy profile as a function of the market capacity, the market demand, the marginal cost to produce a product, and the reservation price of services. These parameters are constant values.

There are some differences between Bandyopadhyay's et al. model and the proposed model of this research. Their model is symmetric due to fixed parameters. Our model accounts for the dynamic nature of Internet traffic demand. The change in traffic patterns in the Internet changes the level of congestion in the network. The congestion in the network adds to the cost of providing Internet services. As a result, the marginal cost of service varies over time. Thus, the fixed demand and the fixed marginal cost assumptions of the Bandyopadhyay et al.'s model is not an exact fit to our proposed price-based peer network architecture.

We extend Varian's and Bandyopadhyay et al.'s models to allow for the varying nature of marginal cost and perceived market demand of the Internet. The mixed strategy profile of our model is a function of fixed market capacity, the perceived time varying market demand function, the time varying marginal cost function, and the customers' fixed reservation price of services. This makes our model asymmetric.

3.2 *Service Class and Enterprise Preference*

The commodity of this market is bandwidth. All the sessions in this research have equal bandwidth. The type of value that a provider adds to a session identifies a class of service. To meet the diverse application needs of an enterprise, providers have to furnish different types of services based on the technology and network intelligence used. For example, different applications offered by enterprises may require different levels of security guarantees, types of addressing schemes (e.g. IPv4 vs. IPv6), and types of digital signal processing.

We develop service class based on customers' preference, i.e., how customers value each service. However, the distinguishing value does *not* relate QoS parameters such as delay performance. All class of service must adhere to a required delay performance. Customers value services based on their preferences such as security treatement, encryption, protection from packet dropping, etc. Exactly what the value is not important—all that matters is that customers are willing to pay different amounts for the different services.

Assumption: Enterprises request three groups of services based upon their required security levels: *High, Medium, and Low*.

Assumption: Each provider in a heterogeneous service-based market offers three classes of service called Blue, Green, and Red. These three classes guarantee customers' required security levels of High, Medium, and Low, respectively.

Assumption: Each provider in a homogeneous service-based market offers a single class of service (Green) that guarantees Medium level security.

Assumption: Enterprises prefer that providers guarantee security levels.

We assume that the commodity space (S) is represented by the Internet service bandwidth with the desired security levels {High, Medium, Low} and every enterprise prefers that providers guarantee these security levels. The consumption bundles ***b***, ***g***, and ***r*** in the commodity space (*S*) are mapped, respectively, to Blue, Green, and Red.

$$b, g, r \in S \tag{3.2}$$

We expand the preference definitions of [1] for our proposed class of services. By denoting the enterprise strict preference as "$\succ$", the term $b, g \in S, b \succ g$ implies that an enterprise strictly prefers the security level guaranteed by the Blue service to that of the Green service. Enterprises in this market satisfy the following important properties:

Complete: For b, g $\in$ S, either $b \succ g$ or $g \succ b$. In our model, the security level of the Blue service is strictly preferred over that of Green service; however, the reverse is not true.

Reflexive: For g $\in$ S, $g \succ g$. For the homogeneous based-service (i.e. when only the Green service is available in the market), the security level of all sessions should be Green (i.e. medium).

Transitive: For b, g, r $\in$ S, if $b \succ g$ and $g \succ r$, then $b \succ r$. Blue security level is strictly preferred over Green, and Green security level is strictly preferred over Red. It is also true that the security level of the Blue service is strictly preferred to that of the Red service. This property is important because the market price for Blue service (p_b) will be higher than the market price for Green service (p_g) and the market price of Green service (p_g) will be higher than that of Red(p_r).

$$p_b > p_g > p_r \tag{3.3}$$

Enterprises will also be willing to a pay higher price for Blue over Green and Green over Red. Due to the transitive property, the relation among their reservation prices will be as follows:

$$\Omega_b > \Omega_g > \Omega_r \tag{3.4}$$

Continuity: For b, g $\in$ S, the sets $\{s : b \succ= g\}$ and $\{s : g \succ= b\}$ are closed sets and $\{s : b \succ g\}$ and $\{s : g \succ b\}$ are open sets. In our model, since higher security levels are strictly preferred to the lower security levels, they constitute an open set. In this research, this open set or strict preference property is important because the cost of providing two different levels of security is not the same. In Section 3.3.2, we develop a cost function of providers based on the service cost coefficient for different classes.

Assumption: Different levels of security require a different cost to provide a service. Thus, the costs of producing different classes of service are different.

In this research, service class does not depend on the performance parameter because all classes of traffic share integrated queues in each link and FIFO non-preemptive priority scheduling serves the link. Note also that provider networks in this study implement Call Admission Control (CAC) and enforce traffic-engineering rules to guarantee Quality of Service (See Section 5.3).

As per [1], the profit function (U: X $\rightarrow$ R) quantifies the preference comparison of an enterprise. The fact that the profit enjoyed by the highly secured Blue service is greater than the profit enjoyed by the moderately secured Green services implies that enterprises strictly desire the Blue service over the Green service. The following represents this relation between enterprise's profit and preference:

$$U(b) \succ U(g) \Leftrightarrow b \succ g \tag{3.5}$$

We do not study the profit of the enterprise. Nevertheless, we reflect customer profit by three levels of reservation price as presented in Section 3.3.3.

3.3 *Model Parameters*

3.3.1 *Market Capacity and Market Demand Functions*

In this section, we present definitions and assumptions concerning the market capacity and market demand of our proposed model.

Definition: Market capacity (*Γ*) is the aggregate traffic engineered access bandwidth capacities of all providers in a market. Market capacity is a fixed quantity measured in bandwidth rate per unit time (e.g. Mbps). It is the sum of the capacity of all the access ports of Edge-Label Switched Routers (E-LSRs) or media-relays of all the providers in a market multiplied by the Maximum Traffic Engineered Link Load[9] (ρ_{TE}). By denoting K_n as the total capacity of all the access ports of a provider (n), the following equation represents the market capacity.

$$\Gamma = \sum_{n=1}^{N} K_n \rho_{TE} = \rho_{TE} \sum_{n=1}^{N} K_n \tag{3.6}$$

Assumption: A provider market capacity is finite.

Definition: Market demand (*Δ*) is the aggregate bandwidth requested by all the enterprises in a market. Market demand is a variable quantity measured in bandwidth rate per unit (e.g. Mbps). The maximum market demand is denoted by Δmax.

$$\Delta \le \Delta_{Max} \tag{3.7}$$

Assumption: Maximum Market Demand (Δ_{Max}) is less than the market capacity (*Γ*).

$$\Delta_{Max} < \Gamma \tag{3.8}$$

Assumption: Every provider's market capacity is less than the market demand.

$$\rho_{TE} K_n < \Delta \qquad \forall n \tag{3.9}$$

---

[9] Providers limit load of a network below a maximum limit during capacity planning and traffic engineering to maintain delay jitter level in the node. We define this limit as Traffic Engineered bandwidth capacity and the load as Maximum Traffic Engineered Load.

Assumption: Market demand is greater than (*N-1*) times a provider's market capacity if there are *N* numbers of providers in the market. The relation between market demand and capacity can be written as follows:

$$\rho_{TE}(N-1)K_n < \Delta \qquad \forall n \tag{3.10}$$

To understand equation (3.10), let us assume that there are four providers in a market. Here, market demand is such that if three providers sell to their market capacity, then the remaining provider sells to a fraction of its market capacity when it sells to the residual demand. Note, equation (3.10) subsumes equation (3.9).

The equations (3.7)-(3.10) can be written as follows:

$$\rho_{TE}(N-1)K_n < \Delta \le \Delta_{Max} < \Gamma \qquad \forall n \tag{3.11}$$

Definition: The throughput (Y_n) of a provider is the total outgoing traffic from the provider in all regions.

In our study, market demand is a variable quantity. Each provider has knowledge of its throughput level ($Y_{n,t}$) at time *t*. From this knowledge, each provider develops its perceived market demand function.

Assumption: The provider networks are lossless (no packet drop or session drop occurs).

Assumption: Each provider perceives that the market maintains fair shares of bandwidth among the providers.

Definition: The *perceived* market demand ($\tilde{\Delta}$) of a provider is the multiplication of its production level with the number of providers in the market. We express the *perceived* market demand function by the following equation.

$$\tilde{\Delta}(Y_{n,t}) = NY_{n,t} \tag{3.12}$$

Based on the above assumptions and definitions, we define the market demand as a function of $Y_{n,t}$.

Definition: If perceived market demand is less than the market capacity of a provider, the market demand is the lower bound of equation (3.11); otherwise, the market demand is equivalent to the perceived demand. The market demand function is depicted by Figure 3-1 and represented by the following equation:

$$\Delta(Y_{n,t}) = \begin{cases} \rho_{TE}(N-1)K + \varepsilon & NY_{n,t} \le \rho_{TE}K, \varepsilon > 0 \\ NY_{n,t} & \rho_{TE}K < NY_{n,t} \le \Delta_{Max} \end{cases} \tag{3.13}$$

The top portion of equation (3.13) is to satisfy equation (3.10). The bottom portion of equation (3.13) implies the provider's perception that a fair market share is achieved at the steady state operating load.

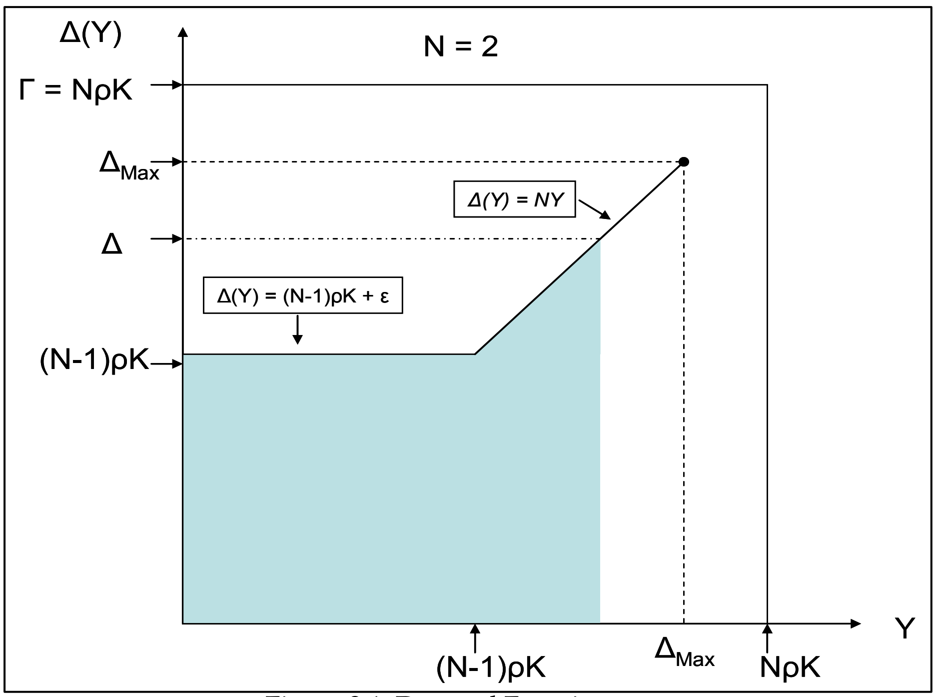


Figure 3.1: Demand Function

3.3.2 *Marginal Cost Function*

In this section, we define cost and marginal cost functions. Cost to provide a service depends on many parameters and network situations.

Assumption: There is no cost associated with the distance.

Rationale: Prior to the explosive growth of the Internet, the expensive Public Switched Telephone Network's (PSTN) price of service was a function of the distance traveled (e.g. long-distance or international) by a call. Massive deployment of fiber-optic cables around the globe reduced the distance specific cost for Internet services. At present, ISPs do not charge based on distance.

We assume the following four influences on the service cost:

1. Congestion in the network
2. Protocol used to provide a service (service class discriminator)
3. Amount of service (commodity)
4. Providers' fixed cost to produce the service

Assumption: The service cost is a function of the congestion indicator in the network, i.e. the optimum mean packet count in the queue system in the network.

Rationale: Congestion in the network increases the delay in packet transmission. The delay degrades the service quality. The degradation of service is detrimental to the revenue because it will be reasonable for providers to pay the enterprise a penalty for delay violation. The mean packet count in the queue system is a congestion indicator of the network. Congestion in the network varies with time. By performing optimum routing, the congestion in a network can be well distributed across the network; as a result, the network can support more traffic compared to a non-optimized network. The efficient routing yields the optimum mean packet count in the network queue system.

Assumption: The service cost is a function of the service cost coefficient.

Rationale: The class is differentiated by the service cost coefficient parameter. The service cost coefficient parameter depends on the protocol and intelligence applied to provide the service. For example, to guarantee levels of security requires different network costs. As mentioned earlier, the service cost coefficient of service class is not differentiated by the performance parameter because all classes of traffic share integrated queues in each link and is serviced by FIFO non-preemptive priority scheduling.

Assumption: Each provider maintains the identical QoS for all class.

Assumption: The service cost coefficient of a class is the same for all the providers.

Rationale: The service cost coefficient of a class will either be the same for all the providers or be different for different providers. In reality, providers purchase equipment and software from the same set of vendors. Therefore, the service cost coefficient of an identical class for different provider is generally the same.

Assumption: The cost of a service is a function of a provider's fixed cost.

Rationale: Different providers assign different costs to deploy and maintain the service.

From the above assumptions, we can assert the following:

Assertion: The service cost is a function of the service cost-coefficient (δ_s), the mean packet count in the queue system ($\hat{M}_n$), throughput (Y_n), and the provider fixed cost coefficient (θ_n).

$$Cost_{n,s,t}(Y_{n,t}) = g(Y_{n,t}) = \delta_s \hat{M}_{n,t} Y_{n,t} + \theta_n Y_{n,t} \tag{3.14}$$

Note that the service cost is computed for bandwidth per unit of time. Therefore, call duration is not considered in equation (3.14). The mean packet count in a network varies with the change in the throughput of the network, i.e. $M_{n,t}$ is a function of $Y_{n,t}$.

$$f(Y_{n,t}) \rightarrow M_{n,t} \tag{3.15}$$

In microeconomics, the marginal cost is defined as the change in cost ($\partial Cost(.)$) due to the change in production or output (∂Y).

$$\text{The marginal cost} = \frac{\partial Cost(.)}{\partial Y} \tag{3.16}$$

Definition: Marginal service cost is the increase in cost for adding another unit amount of bandwidth in the network.

Based on the above definition and equations (3.14)-(3.16), the marginal cost function of a service class for a provider is represented as follows. (Note, cost is a continuous function of Y).

$$\omega_{n,s,t}(\hat{M}_{n,t}) = \frac{\partial g(Y_{n,t})}{\partial Y_{n,t}} = \delta_s (Y_{n,t} \frac{\partial \hat{M}_{n,t}}{\partial Y_{n,t}} + \hat{M}_{n,t}) + \theta_n \tag{3.17}$$

The above equation denotes the marginal cost of a service as a function of the service cost coefficient, provider fixed cost coefficient, and change in the mean packet count of each provider with respect to the change in throughput. The service cost coefficient and the provider fixed cost jointly enforce a differentiated price per class and per provider.

Since at each instant of time, each provider has a distinct mean packet count, the marginal cost of two providers may not be identical at any instant of time. Besides, fixed cost coefficient (θ_n) of each provider is unique. Therefore, marginal cost will be different for different providers even if the mean packet count is the same.

Table 3.1 illustrates a sample representation of marginal cost equations for different providers in heterogeneous service networks supporting Blue, Green, and Red classes of service. Section 7.2 presents the rationale for selecting the following service cost coefficient values.

Table 3.1: Marginal cost equation

	Blue	Green	Red
Provider 1	$1.00(Y_{1,t}\frac{\partial \hat{M}^*_{1,t}}{\partial Y_{1,t}} + \hat{M}^*_{1,t}) + 10$	$0.10(Y_{1,t}\frac{\partial \hat{M}^*_{1,t}}{\partial Y_{1,t}} + \hat{M}^*_{1,t}) + 10$	$0.01(Y_{1,t}\frac{\partial \hat{M}^*_{1,t}}{\partial Y_{1,t}} + \hat{M}^*_{1,t}) + 10$
Provider 2	$1.00(Y_{2,t}\frac{\partial \hat{M}^*_{2,t}}{\partial Y_{2,t}} + \hat{M}^*_{2,t}) + 10$	$0.10(Y_{2,t}\frac{\partial \hat{M}^*_{2,t}}{\partial Y_{2,t}} + \hat{M}^*_{2,t}) + 10$	$0.01(Y_{2,t}\frac{\partial \hat{M}^*_{2,t}}{\partial Y_{2,t}} + \hat{M}^*_{2,t}) + 10$

3.3.3 *Reservation Price of an Enterprise*

Not only is the reservation price of a service (i.e., the maximum price) that an enterprise is willing to pay for a unit of each service, it is also the upper bound of the enterprise budget. Reservation prices are determined during the business agreement and remain constant throughout the lifetime of the game. Enterprises are rational agents and they do not violate the agreement by changing the reservation price. For homogeneous services, the reservation price is a fixed value for all enterprises. For the heterogeneous services (Blue, Green, and Red), there are three fixed reservation prices.

We explained the following relation between the enterprise profit and preference in Section 3.2:

$$U(b) \succ U(g) \Leftrightarrow b \succ g \tag{3.18}$$

Due to the relation of equation (3.18), an enterprise will be willing to pay a higher price for Blue service over Green service and Green service over Red service. Because of this relation and the transitive property of preference, the relation among their reservation prices will be as follows:

$$\Omega_b > \Omega_g > \Omega_r \tag{3.19}$$

How does an enterprise broker compute its maximum bid price or the reservation price (Ω_s) for a service? Enterprises may adopt many different methods to compute the maximum reservation price for their services. Providers may not dictate the method of computing reservation prices to enterprises. However, we suggest that enterprises assume monopoly market while determining the price of service. Here, we do not study the method of determining the reservation price.

Assumption: The bandwidth required for each session is the same.

Assumption: The reservation prices do not change during the game.

Assumption: The reservation prices are always greater than all providers' marginal costs, i.e. $\Omega_s > \omega_s$

Computation of a reservation price can be a future research topic. For example, a study on the influence of varied reservation price on providers' profit would be useful to observe whether enterprises can control the market power of the providers.

3.3.4 *Profit Function*

Profit obtained from time T_0 to T_{end}, while maintaining upper bound of throughput (Y), can be described as follows.

$$U(p(.)) = \int_{T_0}^{T_{end}} \int_0^Y (p(t,y) - \omega(t,y))dydt \tag{3.20}$$

In the proposed network architecture, a session occurs at each instance of a game. A session initiation event can be either a Set-Up Request or Tear-Down Request. For each session set-up request, if the CAC of a provider can admit the call, the provider computes the bid price and the enterprise activates the session through the winning provider network. Since the bandwidth of each session is much smaller than a providers' market capacity, and since calls activate and de-activate, the winning provider continues to take part in the game for subsequent session initiation requests. The steady state operating point (network throughput) of a provider is achieved when the provider's price stabilizes with the competitive market price. If two providers are competing in a market and they adopt different cost functions, their steady state operating point (network throughput) can be different although both of them will operate at the same competitive market price. We will examine this by session level Monte-Carlo simulation in Chapter 8.

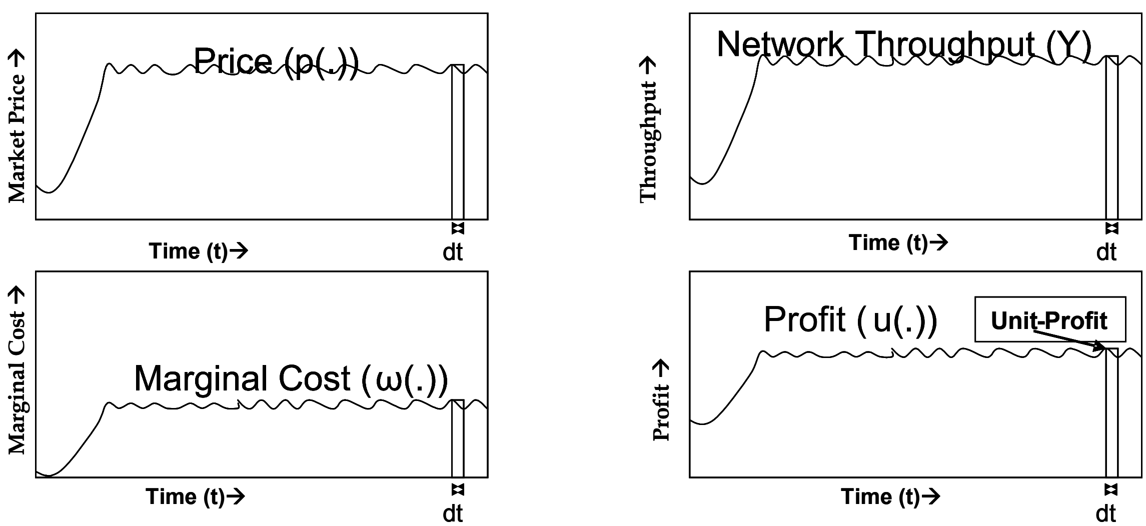


Figure 3.2: The Sketch of Steady State Price, Cost, Throughput, and Profit

Figure 3.2 provides a sketch of the market price, network throughput, marginal cost, and providers' profit to explain our unit profit function. The figure illustrates that in a steady state, marginal cost and network throughput do not significantly vary for a unit duration of time (*dt*). As a result, the price of service also remains stable. The provider's profit per unit time will be the product of the steady state throughput and the difference between steady state unit price and marginal cost. For example, in a steady state, if a bid price is $90 per Mbps, a marginal cost is $50 per Mbps, and a steady state operating point throughput is 300 Mbps, then the unit profit obtained is $12,000.

Definition: The *unit profit* of a provider is the profit per unit duration (e.g. one second) measured at an instant of the steady state throughput ($\hat{Y}$) when the bid price and the marginal cost of the provider converge to $\hat{p}$ and $\hat{\omega}$.

$$u(p) = (\hat{p} - \hat{\omega})\hat{Y} \tag{3.21}$$

A provider (*n*) computes profit from a session as a function of the price (*p*), the marginal cost (*ω*), the duration (*d*), and the bandwidth (*y*) of the session. The price and the marginal cost are values at the session start time. The total profit of the provider is the sum of the profits from all ($\forall k$) the sessions until the end of the game (e.g. a session-level Monte-Carlo simulation).

$$Cumulative\ profit_n = \sum_{\forall k} (\tilde{p}_{s,t,k} - \tilde{\omega}_{n,s,t,k}) d_{n,k} y_{n,s,k} \tag{3.22}$$

3.4 *Proposed Oligopoly Model*

This section derives our game of oligopoly model based on Varian's and Bandyopadhyay et al.'s models described in section 3.1. These two models extend the Bertrand oligopoly model to the static game of incomplete information. The static Bayesian game [1]-[5] represents the static game of incomplete information.

As described in Chapter 1, a static Bayesian game consists of Action space, Type space, Strategy space (mixed strategy profile or belief function), and Payoff space. In this research, we propose that the static Bayesian game consists of the following elements:

- Strategic players: providers (N).
- Action space: the bids of the providers (p_{bid}).
- Type space: the marginal cost function of the providers ($\omega(.)$).
- Strategy space: the set of functions over mixed strategy profile or belief function $F(.)$. This is a price randomizing cumulative probability distribution function.
- Payoff space: the expected unit profit ($u(.)$) of the providers at the steady state.

The commodity of the market is bandwidth (y) and the strategic variable is the price (p) of Internet services (s). In this game, the strategy of a provider is to maximize expected profit.

By applying Varian's and Bandyopadhyay et al.'s methods of developing $F(p)$ and our assumptions in this chapter, we develop $F(p)$ for an Internet duopoly market.

Assumption: The price randomizing cumulative probability distributive function $F(p)$ is a continuous function and the associated probability distribution function is $f(p)$. Each provider implements the same method to develop $F(p)$.

Assumption: The providers compete in an asymmetric equilibrium market, i.e. each provider determines a price from the different *F(p).*

Definition: The minimum price (p_{Min}) is a price that allows a provider to win a bid with 100% probability.

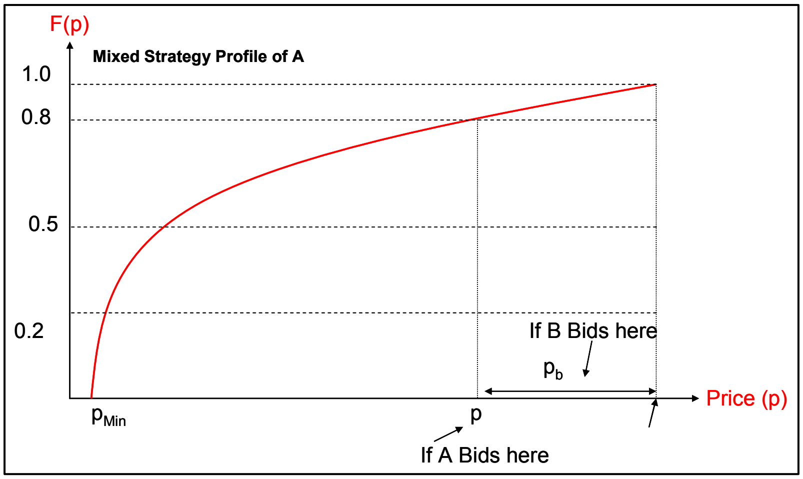
Definition: The mixed strategy profile of a provider, $F(p) = Prob(\mathrm{p_b} \le p)$, represents the opponents' probability of selecting bid (p_b) less than or equal to a price p, where $p_b \in [p_{Min}, p]$. This definition also implies that if a provider bids a price p, then its opponents will win the bid with a probability of $Prob(\mathrm{p_b} \le p)$.

Assumption: If both providers bid the same price (a tie), enterprises select a provider at random (uniform distribution); i.e. enterprises select with 50% probability. However, the probability of a tie is negligible.

We define a bid as the price per rate per class of service. Consider in a market with two providers: A.com and B.com. Denote *F(p)* as the mixed strategy profile of A.com. If A.com bids with a price p from *F(p)* and B.com bids with any other price p_b, then two possible scenarios occur.

- Scenario 1: A's price is lower than B's price: $p_b > p$.
- Scenario 2: A's price is higher than or equal to B's price: $p_b \le p$.

Scenario 1: A.com's price (p) is lower than B.com's price ($p_{B\,bid}$): $p_{B\,bid} > p$



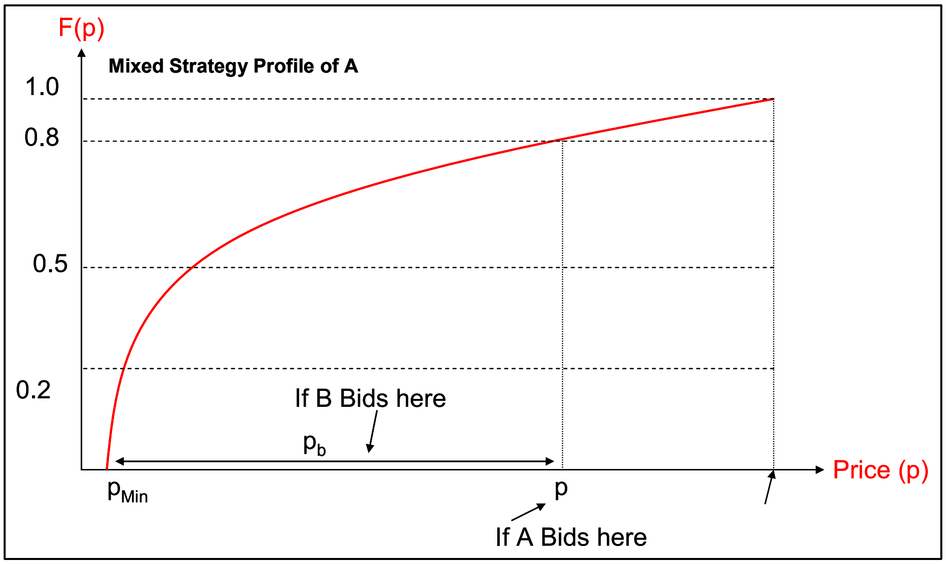
- This event (Scenario 1) occurs with a probability equal to $1 - F(p) = Prob(p_b > p)$ which is 0.2 in the above figure.
- Since p is the lower bid, A.com wins. Consequently, p becomes the market price at that instant.
- Denote the unit profit obtained at this price by A.com as $u_L(p)$ and the profit obtained in a long duration of time is (from time T_1 to T_2) as $U_L(p)$
- At the steady state operating point, if the bid price of A.com is lower, it will sell to its market capacity. Thus, it will operate on maximum market capacity throughput.
- At the steady state, price and marginal cost do not significantly vary. Since this price is lower than that of B.com, A.com sells to the market capacity at this steady state ($\hat{Y} = \rho_{TE} K$). If the game occurs for a unit duration of time, the unit profit obtained at this price by A.com can be represented by the following equation:

$$u_L(p) = (p - \omega(.))\hat{Y} = (p - \omega(.))\rho_{TE} K \tag{3.23}$$

- By selecting the lowest price p_{Min}, A.com can sell to its market capacity with 100% probability; thus, it can obtain unit profit as follows:

$$u_L(p_{Min}) = (p_{Min} - \omega(.))\rho_{TE} K \tag{3.24}$$

Scenario 2: A.com's price is higher (p) than B.com's price (p_B): $p_B \leq p$



- This event (Scenario 2) occurs with a probability of $F(p) = Prob(p_b \leq p)$ which is 0.8 in the above figure.
- Since p_b is the lower price, B.com wins.
- At the steady state operating point, if the bid price of A.com is higher, it will sell to the residual market demand. Thus, at a steady state, it will operate on a throughput of the residual market demand ($\Delta(.) - \rho_{TE} K$).
- Denote the profit obtained at this price by A.com as $u_H(p)$.
- The unit profit obtained at this price by A.com in the steady state is represented by the following equation:

$$u_H(p) = (p - \omega(.))(\Delta(.) - \rho_{TE} K) \tag{3.25}$$

- If a provider's strategy were to maximize its expected unit profit by having a high price strategy, it would rather select a reservation price. In this case, the unit profit function of A.com appears as follows:

$$u_H(\Omega) = (\Omega - \omega(.))(\Delta(.) - \rho_{TE} K) \tag{3.26}$$

From the above two scenarios, the expected unit profit ($\bar{u}(p)$) of A.com at the steady state can be expressed as follows:

$$\bar{u}(p) = u_L(p)(1 - F(p)) + u_H(p)F(p) \tag{3.27}$$

From equation (3.27), we can derive the mixed strategy profile:

$$F(p) = \frac{u_L(p) - \bar{u}(p)}{u_L(p) - u_H(p)} \tag{3.28}$$

Assumption: The probability that a provider bids a price less than or equal to the reservation price is 1; i.e. *F(Ω) = 1*.

Since *F(p) = 1* at price *p = Ω*, equation (3.27) yields the following:

$$\bar{u}(p) = u_H(\Omega) \tag{3.29}$$

By substituting equations (3.24)-(3.29) in (3.27), the mixed strategy profile of A.com can be developed as follows:

$$F(p) = \frac{(p - \omega(.))\rho_{TE}K - (\Omega - \omega(.))(\Delta(.) - \rho_{TE}K)}{(p - \omega(.))(2\rho_{TE}K - \Delta(.))} \tag{3.30}$$

Following the above approach, we can derive the mixed strategy profile for *N* providers when a provider plans to bid either highest or lowest. Now A.com is competing with *N-1* providers. The probability of A.com's winning the bid is $(F(p))^{N-1}$ if its bid is the highest and the probability of winning the bid is $(1-(F(p))^{N-1}$ if its bid is the lowest. The steady state expected unit profit is as follows:

$$\bar{u}(p) = u_L(p)(1 - (F(p))^{N-1}) + u_H(p)(F(p))^{N-1}$$
$$\Rightarrow F(p) = \left[\frac{u_L(p) - \bar{u}(p)}{u_L(p) - u_H(p)}\right]^{N-1} \tag{3.31}$$

By having the lowest bid among the providers, A.com can sell to its market capacity obtaining the following steady state unit profit:

$$u_L(p) = (p - \omega(.))\rho_{TE}K \tag{3.32}$$

If A.com's bid is the highest, it obtains the following steady unit profit by selling the residual bandwidth ($\Delta(.) - \rho_{TE}(N-1)K$).

$$u_H(p) = (p - \omega(.))(\Delta(.) - (N-1)\rho_{TE}K)) \tag{3.33}$$

In this case, to maximize profit, A.com's strategy will be to select the highest price (i.e., Ω).

$$u_H(\Omega) = (\Omega - \omega(.))(\Delta(.) - (N-1)\rho_{TE}K)) \tag{3.34}$$

From equations (3.29), (3.31)-(3.34), the following belief function of A.com is derived:

$$F(p) = \left[\frac{(p - \omega(.))\rho_{TE}K - (\Omega - \omega(.))(\Delta(.) - (N-1)\rho_{TE}K)}{(p - \omega(.))(N\rho_{TE}K - \Delta(.))} \right]^{\frac{1}{N-1}} \tag{3.35}$$

In Section 3.3, we defined the marginal cost as a function of optimized mean packet count in the network queue system and the market demand as a function of network throughput. Based on these definitions, we postulate the following equation from equation (3.35) for the game time (*t*):

$$F_{n,s,t}(p_{n,s,t}) = \left[\frac{(p_{n,s,t} - \omega_{n,s,t}(M^*_{n,t}))\rho_{TE}K - (\Omega_s - \omega_{n,s,t}(M^*_{n,t})(\Delta(Y^*_{n,t}) - (N-1)\rho_{TE}K))}{(p_{n,s,t} - \omega_{n,s,t}(M^*_{n,t}))(N\rho_{TE}K - \Delta(Y^*_{n,t}))} \right]^{\frac{1}{N-1}} \tag{3.36}$$

If A.com plans to bid such that its bid price is higher than N_1 providers and less than N_2 providers in an N providers' market, A.com can *approximately* develop the belief function from the following equation using our above method:

$$\bar{u}(p) = \frac{N_2}{N-1} u_L (1 - (F(p))^{N-1} + \frac{N_1}{N-1} u_H (F(p))^{N-1} \tag{3.37}$$

Since our research focuses on a duopoly market, where a provider will bid either highest or lowest. Thus, we will not develop the belief function using equation (3.37).

Equation (3.36) is a cumulative distribution function of a provider's probability of losing a bid because its opponents' bids are lower. In other words, the function $F_{n,s,t}(p_{n,s,t})$ represents the probability that the other providers will win the bid if A.com selects a price (*p*) at a game time (*t*) for the service class (*s*). This also implies that the bid prices ($p_{other,s,t}$) selected by the other providers are lower than A.com price $p_{A,s,t}$ with a probability of $F(p_{A,s,t}) = Prob(p_{other,s,t} \le p_{A,s,t})$.

The function $F_{n,s,t}(p)$ is the belief function of this game of incomplete information because it addresses the conditional probability that the other providers will not bid a price lower than (*p*) with a probability (1 – $F_{n,s,t}(p)$) if a provider (n) is to win the bid.

Our game is a static game—it does not keep or rely on the total history. In each game instance, the game computes the change in cost from one game instance to the next game instance. Since the game looks into a *one-step* history and forgets all other history, the strategic interaction corresponds to a *"myopic"* Markovian-Bayesian [4] static game of incomplete information. The dynamic game—that relies on the total history is outside the scope of this research and is a future research topic.

3.5 The Movement of the Belief Function

For each session initiation request, providers compute a new belief function, $F_{n,s,t}(p)$, to determine a bid. As discussed earlier, the belief function's parameters are market capacity and demand, marginal cost, and reservation price. The marginal cost of service is a function of the optimized mean packet count in the network, and the cost coefficients of a provider. The service cost coefficient values are unique for each class and for different providers. The demand function varies with an increase in throughput. The belief function $F_{n,s,t}(p)$ of a service varies with the marginal cost of a provider and the market demand. For all these reasons, the $F_{n,s,t}(p)$ per service is not identical for all the providers at a certain instant of time. For three services, each provider supports three belief functions. Moreover, for two provider networks, there are total of six belief functions.

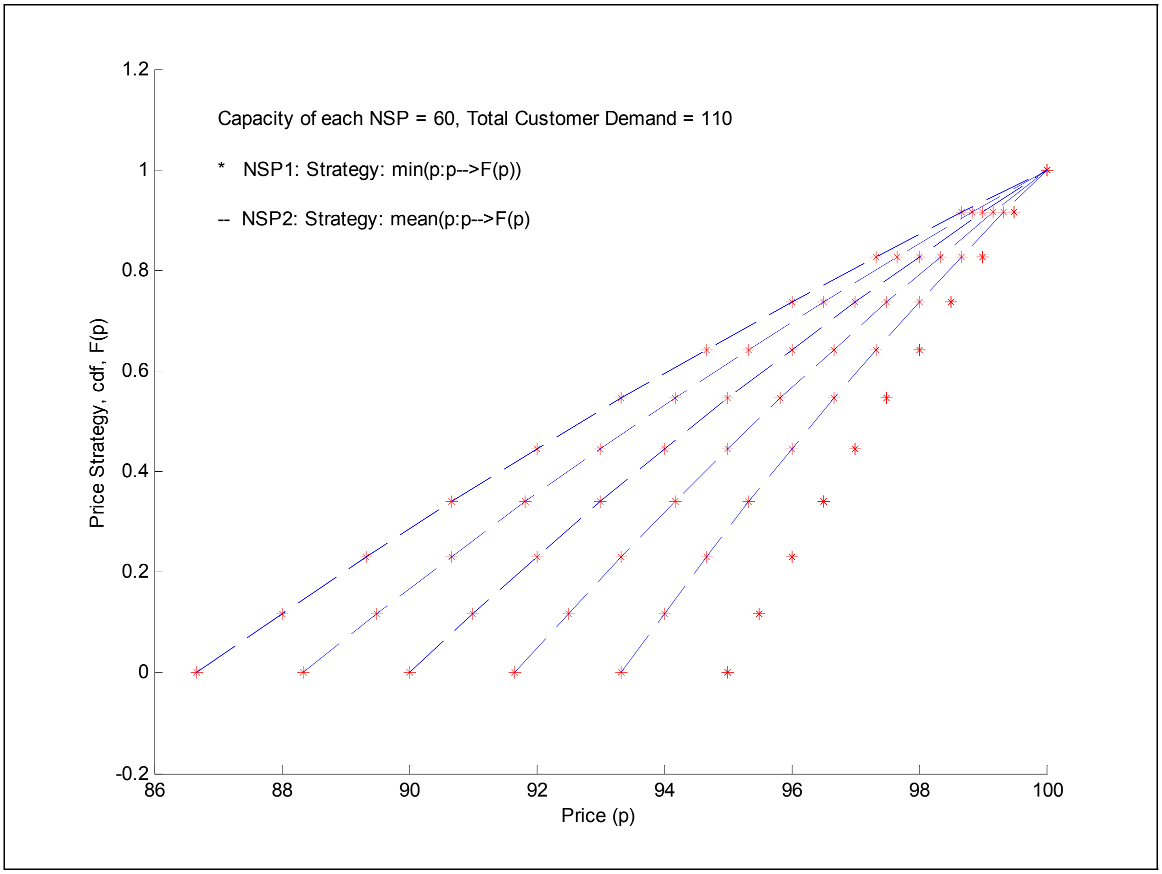


Figure 3.3: Change in Belief Function due to the change in Marginal cost

Figure 3.3 illustrates how a change in marginal cost shifts the belief function in a homogeneous service duopoly market. Both the providers have identical networks and use identical cost functions. The belief functions, $F_{n,s,t}(p)$, are drawn for different marginal costs. The increase in marginal cost shifts a belief function to the right; thus, increases the price of a product. This figure is a hypothetical representation for illustration purpose and does not represent a result of the study.

3.6 *Providers' Strategies*

A provider's strategy is to maximize its expected profit by selecting a price using the belief function. The strategy space is the set of functions over the belief function $F_{n,s,t}(p)$. This section specifies a strategy space over the belief function to select bids in the game. In our study, each provider adopts its own strategies to determine bid ($p_{n,s,t}$) using the belief function $F_{n,s,t}(p)$.

In Section 1.1.2.1, we described the following equation to represent A.com's a set of feasible strategies from the belief function in a Bayesian Game of Incomplete Information:

$$strategy\ h_{Aj} : Action_A \longleftarrow h_{Aj}(., Belief_B(Type_A)) \tag{3.38}$$

Since in our study, the action space is the bid ($p_{n,s,t}$) of a provider, we can rewrite the above equation as follows:

$$strategy\ h_{Aj} : p^{bid}_{A,s,t} \longleftarrow h_{Aj}(F_{A,s,t}(.)) \tag{3.39}$$

A provider computes a bid from $F_{n,s,t}(p)$ using certain rejection probability of winning the bid. We map this rejection probability with provider's strategy.

Definition: The *Rejection Coefficient* (γ) of a provider is the probability of having its bid rejected. The *rejection probability* of selecting a price $p_{my\ bid}$ is $F(p_{my\ bid}) = Prob(p_{others\,bid} \leq p_{my\ bid}) = \gamma$.

Definition: A *Winning coefficient* (ξ) is the probability of winning a bid.

Definition: A *No Rejection Strategy* of a provider is the strategy when the probability (γ) of having the bid rejected is zero. In other words, in this strategy the probability of winning a bid is 100%. Here, the *winning coefficient* is $\xi = 1.0$. In this case, the following equation is true.

$$F_{n,s,t}(p_{Min,s,t}) = \gamma = 0 \tag{3.40}$$

By substituting Equation (3.40) in Equation (3.36) and performing algebra, the following *No Rejection Strategy* price is developed:

$$p_{Min,n,s,t} = \frac{(\Omega_s - \omega_{n,s,t}(M^*_{n,t}))(\Delta(Y_{n,t}) - (N-1)C)}{\rho_{TE} K} + \omega_{n,s,t}(M^*_{n,t}) \quad (3.41)$$

Definition: An *Absolute Rejection Strategy* of a provider is the strategy when the probability (γ) of losing the bid to the opponents is almost 100%. In other words, in this strategy having the bid rejected is almost 100%. Here, the *winning coefficient* is $\xi = 0.0$.

$$F_{n,s,t}(p_{absolute\_risk,n,s,t}) = \gamma = 1.0 \quad (3.42)$$

By selecting a bid equal to the enterprise's reservation price, a provider increases the probability of rejection to 100%. Therefore, the reservation price of enterprises is the *Absolute Rejection Strategy* price of a provider.

$$p_{absoute\_risk,n,s,t} = \Omega_s \quad (3.43)$$

$$F_{n,s,t}(p_{absolute\_risk,n,s,t}) = F_{n,s,t}(\Omega_s) = \gamma = 1.0 \quad (3.44)$$

In between the *No Rejection Strategy* price and *Absolute Rejection Strategy* price, a provider can select a bid with a certain probability of having the bid (i.e. the session) rejected. For example, if A.com wishes to win a session at time (*t*) for a Blue (*b*) class of service with a probability ξ, A.com needs to select a bid-price:

$$p = p^{bid}_{A,b,t} \text{ s.t. } 1 - Prob_{A,b,t}(p_{B,b,t} \le p^{bid}_{A,b,t}) = 1 - F_{A,b,t}(p) = \xi,$$

$$\text{where } p_{B,b,t} \in [p_{Min,B,b,t}, (p_{A,b,t} - \varepsilon)], \varepsilon > 0.$$

In other words, A.com needs to select a bid price ($p^{bid}_{A,b,t}$) with a rejection probability $F_{A,b,t}(p^{bid}_{A,b,t}) = \gamma$. In Figure 3.3, if A.com's strategy is to select a bid price with 20% rejection probability ($p^{bid}_{A,b,t} : F_{A,b,t}(p \le p^{bid}_{A,b,t}) = 0.2$), then the bid prices of A.com were 89.5, 90.5, 92.0, 93.5, 94.7, and 96.0 for game instants of 1 through 6, respectively.

We define the following strategies by partitioning the probability of winning into four ranges: *Very High Rejection, High Rejection, Low Rejection,* and *Very Low Rejection*.

Definition: A *Very High Rejection Strategy* is the strategy when the probability of rejection is more than 80% but less than 100%. Here, the winning coefficient is $0.0 < \xi < 0.2$ and the rejection coefficient is $0.8 < \gamma < 1.0$.

Definition: A *High Rejection Strategy* is the strategy when the probability of rejection is more than 50% but at most 80%. Here, the winning coefficient is $0.2 \le \xi < 0.5$ and the rejection coefficient is $0.5 < \gamma \le 0.8$.

Definition: A *Low Rejection Strategy* is the strategy when the probability of rejection is more than 20% but less than 50%. Here, the winning coefficient is $0.5 < \xi < 0.8$ and the rejection coefficient is $0.2 < \gamma < 0.5$.

Definition: A *Very Low Rejection Strategy* is the strategy when the probability of rejection is more than 0% but at most 20%. Here, the winning coefficient is $0.8 \le \xi < 1.0$ and the rejection coefficient is $0.0 < \gamma \le 0.2$.

The mixed strategy profile $F_{n,s,t}(p)$ is a price randomization cumulative distribution function within an interval of $[p_{Min,n,s,t}, \Omega_s]$. According to [14] and the definition of the Nash equilibrium, providers often attempt to maximize their expected profit by a well known strategy of selecting bids at random within the interval $[p_{Min,n,s,t}, \Omega_s]$ with a probability of $F_{n,s,t}(p)$. We define this price randomization as a *Random Rejection* strategy.

Since $F_{n,s,t}(p)$ is a continuous function, price randomization requires an infinite number of points in the price interval. A continuous function can be quantized into a discrete function for implementation. We have illustrated an example algorithm to implement a discrete *Random Rejection* strategy in Section 8.1.3.

Stochastically, a mean price should yield the same expected outcome of the random price. The mean price corresponds to the *Rejection Neutral* strategy in our model.

Definition: A *Rejection Neutral Strategy* is the strategy that yields the mean price of service from a strategy profile $F_{n,s,t}(p)$ at each game instant (t) since it provides equal reject probability of wining (accepted) or losing (rejected) the session.

Figure 3-4 and Table 3.2 illustrate these strategies. Note, rejection implies that an enterprise rejects the bid of a provider because some other provider's bid is lower.

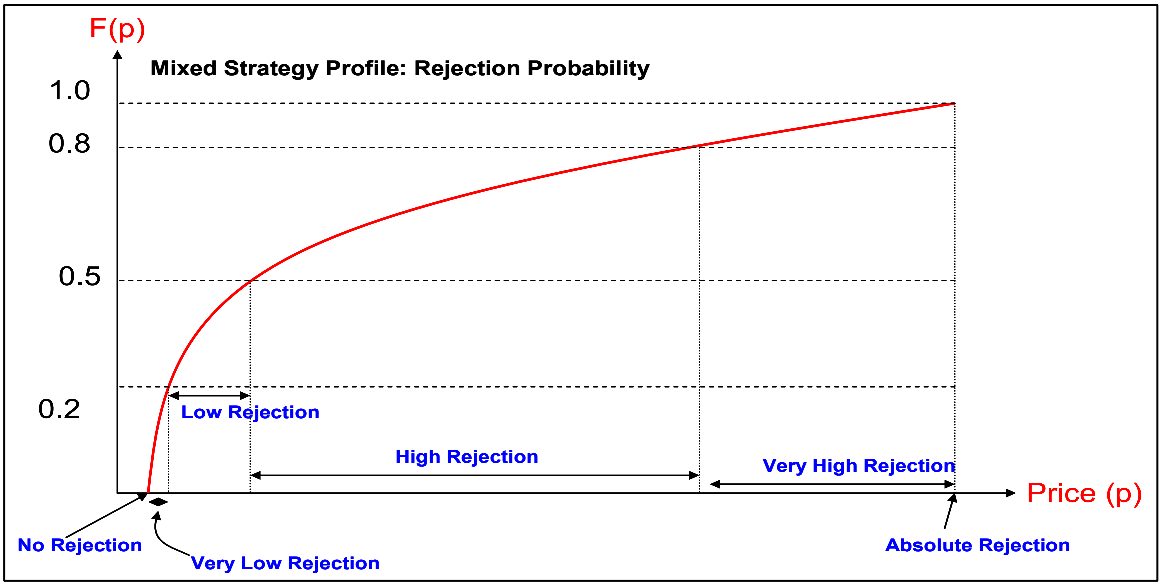


Figure 3.4: Proposed Strategy Diagram

Table 3.2: Proposed Strategies

Strategy	Winning coefficient	Rejection Probability	Example
No Rejection	ξ =1.0	$\gamma = 0.0$	$p_{n,s,t}^{bid} : F_{n,s,t}(p_{n,s,t}^{bid}) = Prob(p \le p_{n,s,t}^{bid}) = \gamma = 0.00$
Very Low Rejection	$0.8 \le \xi < 1.0$	$0 < \gamma \le 0.2$	$p_{n,s,t}^{bid} : F_{n,s,t}(p_{n,s,t}^{bid}) = Prob(p \le p_{n,s,t}^{bid}) = \gamma = 0.05$
Low Rejection	$0.5 < \xi < 0.8$	$0.2 < \gamma < 0.5$	$p_{n,s,t}^{bid} : F_{n,s,t}(p_{n,s,t}^{bid}) = Prob(p \le p_{n,s,t}^{bid}) = \gamma = 0.30$
Rejection Neutral	ξ =0.5	$\gamma = 0.5$	$p_{n,s,t}^{bid} = Mean(F_{n,s,t}(p))$
High Rejection	$0.2 \le \xi < 0.5$	$0.5 < \gamma \le 0.8$	$p_{n,s,t}^{bid} : F_{n,s,t}(p_{n,s,t}^{bid}) = Prob(p \le p_{n,s,t}^{bid}) = \gamma = 0.62$
Very High Rejection	$0 < \xi < 0.2$	$0.8 < \gamma < 1.0$	$p_{n,s,t}^{bid} : F_{n,s,t}(p_{n,s,t}^{bid}) = Prob(p \le p_{n,s,t}^{bid}) = \gamma = 0.97$
Absolute Rejection	ξ =0.0	$\gamma = 1.0$	$p_{n,s,t}^{bid} : F_{n,s,t}(p_{n,s,t}^{bid}) = Prob(p \le p_{n,s,t}^{bid}) = \gamma = 1.0$

To reduce implementation cost of equipment and software, providers' may not implement all strategies of Table 3.2 in a network. We define the strategy set of Table 3.3 as the feasible strategy and conduct session level Monte-Carlo simulations to find the best strategy of the game.

Table 3.3: Proposed feasible Strategies of the providers

Strategy	Feasible strategies
Very Low Rejection	$p_{n,s,t}^{bid} : F_{n,s,t}(p_{n,s,t}^{bid}) = Prob(p \leq p_{n,s,t}^{bid}) = \gamma = 0.05$
Low Rejection	$p_{n,s,t}^{bid} : F_{n,s,t}(p_{n,s,t}^{bid}) = Prob(p \leq p_{n,s,t}^{bid}) = \gamma = 0.35$
Rejection Neutral	$p_{n,s,t}^{bid} = Mean(F_{n,s,t}(p))$
High Rejection	$p_{n,s,t}^{bid} : F_{n,s,t}(p_{n,s,t}^{bid}) = Prob(p \leq p_{n,s,t}^{bid}) = \gamma = 0.65$
Very High Rejection	$p_{n,s,t}^{bid} : F_{n,s,t}(p_{n,s,t}^{bid}) = Prob(p \leq p_{n,s,t}^{bid}) = \gamma = 0.95$

Providers need to adopt strategies such that the market price for Blue service (p_b) is higher than the market price of Green service (p_g) and the market price of Green service (p_g) is higher than that of Red service (p_r).

$$p_b > p_g > p_r \tag{3.45}$$

In Chapter 8, we will present a method to adopt strategies to satisfy equation (3.45).

3.7 *Chapter Summary*

In this chapter, we proposed an oligopoly model to determine the price of services in our peer network architecture. Our oligopoly model is based upon the Bertrand oligopoly model of price, Varian's oligopoly example *A Model of Sale*, and Bandyopadhyay et al.'s online exchange research.

In Bertand's model, the strategy of each seller is to determine a price of a product that it wishes to sell. Sellers display their prices simultaneously. The customers purchase from the seller with the lowest price. The Varian example provides insight into the development of the foundation of a Bertrand mixed strategy equilibrium for a duopoly market. Bandyopadhyay et al. extend Varian's mixed strategy equilibrium to develop a sealed bid reverse auction-based online exchange oligopoly model. In this model, the market demand and the marginal cost of production are fixed values in addition to market capacity and reservation price.

We extended the above static mixed strategy equilibrium to meet the requirements of the dynamic nature of Internet traffic. In our model, the market demand and the marginal cost of providing services are variable parameters.

The mixed strategy equilibrium function provides an infinite set of strategies to select a price with a certain Rejection probability of winning or losing an enterprise. Due to the limitation of technology, we need to assign only a few strategies from the mixed strategy profile. Therefore, we reduce the mixed equilibrium strategies to a feasible set.

The best strategy should allow a provider to maximize its expected profit by selecting an optimum price of service. This is possible by minimizing the marginal costs of services. In the next chapter, we will describe a mathematical non-linear technique for minimizing the service cost by optimizing the traffic flow of the network.

4 *Providers' Profit Maximization by Optimum Routing*

This chapter develops a mathematical optimization method to implement optimal routing [85] in the network. The objective of this optimization is to maximize the profit of a provider. As defined in Section 3.3.4, unit profit (*u(p)*) is a function of price (*p*), marginal cost (*ω*), and network throughput (*Y*) as follows:

$$u(p) = (p - \omega)Y \tag{3.46}$$

The following states our optimization problem:

$$\begin{aligned} &\textit{Maximize} \quad u(p) \\ &s.t. \begin{cases} \textit{Network Architecture Constraint} \\ \textit{Internet Traffic Pattern and Queue System Constraint} \\ \textit{Game Strategy Constraint} \end{cases} \end{aligned} \tag{3.47}$$

Let us first look into maximizing *u(p)*:

$$\begin{aligned} &\textit{Maximize} \quad u(p) \\ &= \textit{Max}\ (p - \omega)Y \end{aligned} \tag{3.48}$$

$$\textit{Max}\ pY + \textit{Max}\ (-\omega Y) \Rightarrow \textit{Max}\ (p - \omega)Y \tag{3.49}$$

$$(\textit{Maximize}\ pY + \textit{Minimize}\ \omega Y) \Rightarrow \textit{Maximize}\ u(p) \tag{3.50}$$

To optimize profit, we need to optimize price and minimize a provider's marginal cost of services. Game theory techniques of Chapter 3 and 7 perform surplus (pY) optimization subjected to the game strategy constraints.

According to microeconomics, there is a strong correlation between the profit maximization and the cost minimization [1][2]. If a provider reduces the cost of producing services, it can increase profit.

Theorem: *Cost minimization is a necessary condition for the profit maximization.*

The proof of the above theorem is presented in [1].

In Section 3.3.2, we presented marginal cost as function of the mean packet count in the network. In addition, we described the rationale for the relationship between a provider's service cost and the mean packet count ($\hat{M}$) of the provider's network queue system. This marginal cost equation (3.17) is a function of the service cost-coefficient (δ_s), the mean IP packet counts in the network queue system ($\hat{M}_n$), throughput (Y_n), and provider fixed cost coefficient (θ_n):

$$\omega_{n,s,t}(\hat{M}_{n,t}) = \delta_s(Y_{n,t}\frac{\partial \hat{M}_{n,t}}{\partial Y_{n,t}} + \hat{M}_{n,t}) + \theta_n \tag{3.51}$$

This chapter's focus is the minimization of marginal cost; therefore, the optimization problem can be stated by:

$$Minimize \quad \{\delta_s(Y_{n,t}\frac{\partial \hat{M}_{n,t}}{\partial Y_{n,t}} + \hat{M}_{n,t}) + \theta_n\} \tag{3.52}$$

The cost-coefficient (δ_s) and the provider fixed cost coefficient (θ_n) are fixed values. Since in equation (3.51) the marginal cost is a linear function of the mean packet count, minimization of the mean packet count will minimize the marginal cost; consequently, the providers' profit will be maximized. Thus, if we ignore for now the $Y_{n,t}\frac{\partial \hat{M}_{n,t}}{\partial Y_{n,t}}$ term, the following equation approximates the optimizing profit problem:

$$Minimize \quad \tilde{M} \tag{3.53}$$

The minimization of the mean packet count can be accomplished by implementing optimal routing to equally distribute traffic flows across the network. The literature [85] develops optimum routing as a function of optimum mean delay. On the other hand, we develop optimum routing as a function of optimum mean packet count because majority of the vendor routers keep the record of mean packet count instead of mean delay. We want to stress that there is no difference in the mean delay method and our mean packet count method because they are directly related through Little's Law [59],[60]. This method is often referred to as "load balancing" in terms of Internet traffic engineering.

The optimal routing should distribute traffic across the network to minimize the change in the mean packet count in the network for the addition of each new session. This optimized load balancing is expected to perform the following minimization in low load as shown in Figure 4.1:

$$Minimize \quad \left(Y_{n,t} \frac{\partial \hat{M}_{n,t}}{\partial Y_{n,t}} \right) \tag{3.54}$$

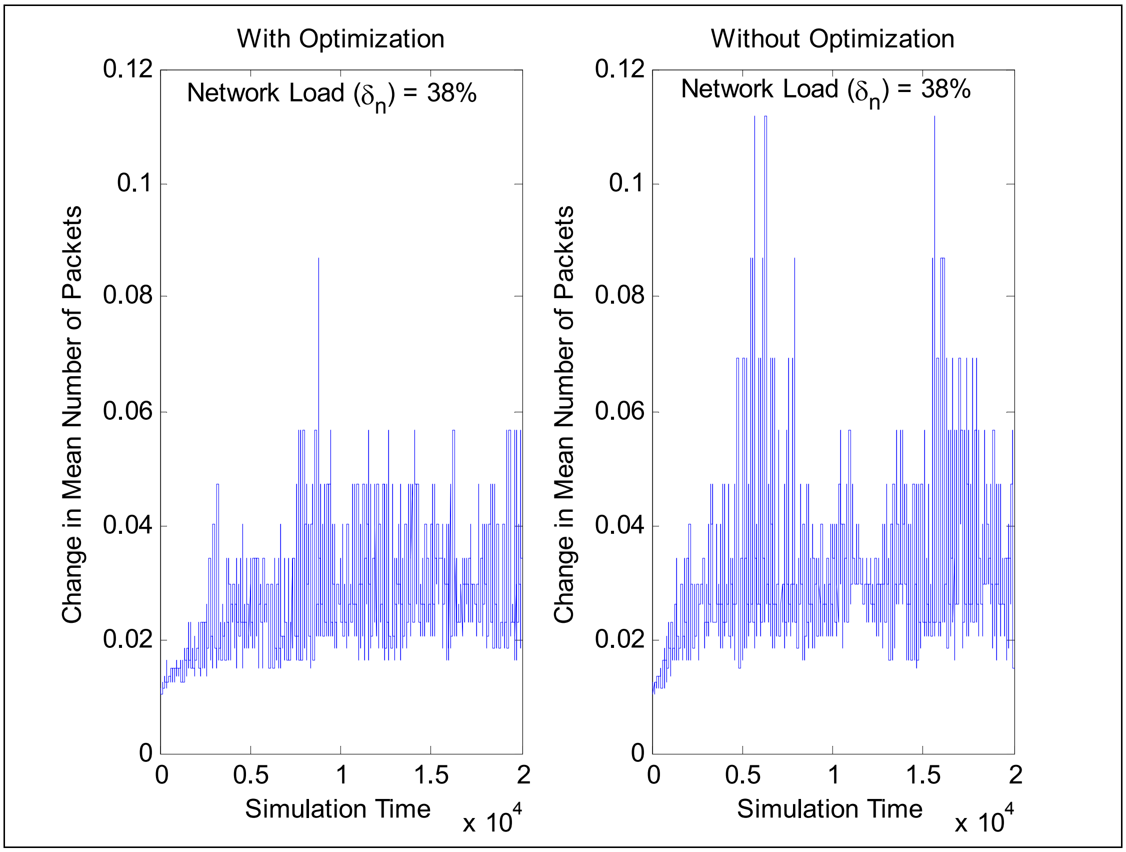


Figure 4.1: Change in Mean Packet count in the network.

Figure 4.1 shows session level Monte-Carlo simulation results of the change in the mean packet count (mean number of packets) in a network when a new session is added for a network load of 38%. The simulation uses the parameters for the homogenous service-based network presented in Table 7.3 and {*Rejection Neutral, Rejection Neutral*} strategy set. The optimum load balancing caused the reduced change in mean packet count (left plot) compared to that (right plot) of the non-optimized load balancing method. Note that the figure demonstrates the improvement but not the optimization; therefore, it is a weak evidence of minimization at best

We implement a mathematical non-linear programming technique (Gradient Projection method) to perform optimal routing of [85] to minimize delay.

4.1 Network Architecture Constraints

We will now discuss the network architecture constraints. Each provider network supports elastic LSPs. Traffic flows from the origin E-LSR or Media-Relay to the Destination E-LSR or Media-Relay through uni-directional LSPs. Each network link is bi-directional, i.e. each link supports two uni-directional LSPs paired in opposite directions for each Origin-Destination (O-D) path. Each O-D pair is connected with alternate LSPs. Traffic between an O-D pair is allowed to take different routes; consequently, total traffic flow for each O-D pair may be split among several paths. A path is an ordered set of links. As per the assumption of Chapter 3, the physical capacity of a network does not change during the lifetime of the game. A similar assumption can be made for network architecture.

Assumption: Network architecture does not change during the lifetime of the game.

This assumption specifies that the network is already built; i.e., the network architecture cannot be changed and the physical links are already provisioned. Nevertheless, the peak rates of the LSPs vary with the optimized flow rates. Chapter 5 describes the network architecture constraints in details: the topology of the network, traffic flow for each O-D pair, and capacity constraint for each link.

4.2 *Traffic Pattern and Queue System Constraints*

The queue system of each link consists of a queue and a server. The mean packet count in a queue depends on the type of the queue and the scheduling system employed by the server. We assume the following concerning the queue system of this study.

Assumption: Each outbound link of a provider supports a single integrated queue with FIFO scheduling.

The following properties describe the method of service differentiation through the FIFO scheduling of this research.

- The class of service is differentiated by the cost of guaranteeing three security levels {*Blue, Green, and Red*} (see Sections 3.2 and 3.3).
- The class of service is not differentiated by the performance (e.g. QoS) parameters.
- Traffic of all classes of service must adhere to the same upper bounds of the QoS matrix (see Section 5.3).

In addition to the type of queue system and scheduling algorithm, traffic patterns such as packet arrival distribution and packet length distribution influence the packet count in the network queue system. Therefore, it is critical to understand the traffic pattern of the network.

Floyd and Paxon [52] explain that it is often difficult to develop a simulation model for an IP network since network and IP traffic patterns are continuously changing. According to [54][55], internet traffic is self-similar, which is modeled often by Fractional Brownian motion [56]. The self-similar nature of Internet traffic was observed prior to the introduction of Voice-over IP (VoIP) and Internet Multimedia Sub-system (IMS) applications. To our knowledge, no established model for emerging internet traffic exists to date. However, a few recent studies, e.g.

[57],[58], show that the Internet traffic tends to mimic independent Poisson distribution as the load in the network increases.

Recently, VoIP, IMS, and video applications are coming into vogue for internet applications. As such, traffic arrival distribution and packet length distribution of traffic types are continuously changing. Since it is difficult to ascertain empirical values, we perform our study based upon the following assumed IP packet arrival and length distributions.

Assumption: IP packet arrival distribution is Poisson

Assumption: IP packet lengths are exponentially distributed

Our objective is to synthesize the game theory with the well-established queuing theory to optimize provider's profit and profit. The M/M/1 system [59] is a well-established traffic analysis method for a FIFO based queuing and scheduling system in academic fields that allows for Poisson distributed packet arrival and exponentially distributed packet length. When traffic with Poisson distributed arrival rate aggregates into an integrated FIFO queue, the aggregate arrival distribution continues to be Poisson. When traffic with Exponential distributed packet lengths merges into an integrated queue, the aggregate packet distribution is hyper-exponential. We should thus adopt the M/G/1 model for computing the mean packet count in the queue system. However, in order to use results from the theory of networks of queues, we approximate with M/M/1 model. This is one of our limitations of this research.

4.3 *Mean Packet count in the M/M/1 Model*

An M/M/1 system consists of a single server queue. It assumes a Poisson arrival process and a negative exponential distributed service time. If the mean arrival rate is λ packets per second and the mean service rate is μ packets per second, the mean packet count in the *M/M/1* system (queue + server) can be attributed as per classical queuing theory [59]:

$$\bar{M} = E[packets] = \frac{\frac{\lambda}{\mu}}{1-\frac{\lambda}{\mu}} = \frac{\lambda}{\mu-\lambda} \tag{3.55}$$

Assuming the mean length of IP packets is $\bar{L}$ bits, l is the link index, and C_l is the capacity of the link in bits per second, the mean service rate μ_l packets per second can be represented by:

$$\mu_l = \frac{C_l}{\bar{L}} \tag{3.56}$$

Equation (3.55) can be expressed as follows, where x_j bits per second is the traffic flow of each LSP and j is the LSP index.

$$\bar{M}_l = E[packets] = \frac{\lambda\bar{L}}{C_l - \lambda\bar{L}} = \frac{\sum_{j:l\in j} x_j}{C_l - \sum_{j:l\in j} x_j} \tag{3.57}$$

The above equation represents the mean packet count in one queue, i.e. for one unidirectional link (l) of a node. The sum of the mean packet count at each queue system for the whole network is the sum of all the uni-directional links in the network as follows:

$$\hat{M} = \sum E[packets] = \sum_l \frac{\sum_{j:l\in j} x_j}{C_l - \sum_{j:l\in j} x_j} = f(\mathbf{x}) \tag{3.58}$$

4.4 *Session Arrival Distribution*

In the simulation study, we increase or decrease the market demand load by changing the arrival rate of sessions. Thus, we need to determine the session arrival distribution. The PSTN established model for the connection arrival probability distribution is Poisson and the connection duration distribution is Exponential. No well established models for the session arrival and session length distributions for IP, video, and wireless centric applications are yet developed. Often in simulation studies (e.g. in [81]-[84]) it is assumed that the call arrival distribution is Poisson and call length distribution is Exponential based on PSTN assumption. Similarly, we conduct this research based on PSTN assumption.

Assumption: Session arrival distribution is Poisson.

Assumption: Session length distribution is Exponential.

4.5 *The Development of a Non-linear Optimization Program*

In constrained non-linear programming, an optimal point must satisfy the First Order Necessary Condition (FONC), which is also known as Kuhn-Karush-Tucker condition. In addition, finding an optimum point requires satisfying the Second-Order Necessary and Second-order Sufficiency conditions (SONC/SOSC). Simpler non-linear programming problems can be solved by the analytical FONC and SONC/SOSC approach. The complicated non-linear programming problems need to be solved by well-established non-linear programming approach such as the Gradient Project algorithm. We apply the Gradient Projection algorithm to find optimum mean packet count in the network.

The Gradient Projection algorithm requires a line search function. Various line search algorithms can be implemented to locate the minimum of an objective function. We apply the Golden Section line search algorithm. The Gradient

Projection algorithm and the Golden Section Line Search are briefly described in the Appendix A and in [46].

In this section, we develop the non-linear optimization program based on [50] by using the Gradient Projection algorithm and the Golden Section line search method.

We denote traffic between an origin and destination at an instant of time as R_w where w is the O-D pair index and the set of LSPs associated with O-D pair w is $J(w)$. Since we assume that no packet drop occurs in the network of this study, the traffic between an O-D pair must be equal to its associated set of LSPs. Therefore, in the non-linear programming model, the following equality constraint must be satisfied:

$$\sum_{j \in J(w)} x_j = R_w \tag{3.59}$$

A physical link should not support more than its traffic-engineered capacity; i.e. the sum of LSP traffic flow in a link must be less than or equal to the traffic-engineered capacity of the link. Therefore, the following non-linear constraints must be satisfied:

$$\sum_{j: l \in J} x_j \le \rho_{TE} C_l \tag{3.60}$$

Section 5.3 describes the Maximum Traffic Engineered Link Load (ρ_{TE}). The following is the non-negativity constraint of the traffic flow through each LSP.

$$x_j \ge 0 \tag{3.61}$$

Our optimization problem is to minimize equation (3.58), which describes the sum of the mean packet count at each queue system for the whole network, while satisfying the constraints from (3.59)-(3.61). We represent this optimization model by the following non-linear programming problem.

$$
\begin{aligned}
&Minimize: \hat{M} = \sum_{l} \frac{\sum_{j:l\in j} x_j}{C_l - \sum_{j:l\in j} x_j} \\
&Subject\ to: \sum_{j:l\in J} x_j \le \rho_{TE} C_l \\
&\qquad \sum_{j\in J(w)} x_j = R_w \\
&\qquad x_j \ge 0
\end{aligned}
\tag{3.62}
$$

The non-linear programming problem of equation (3.62) can be represented as the following standard general form:

$$
\begin{aligned}
&Minimize: \quad f(\mathbf{x}) \\
&subject\ to: \quad \mathbf{h(x)} = 0 \\
&\qquad \mathbf{g(x)} \le 0.
\end{aligned}
\tag{3.63}
$$

Where *f(x)*, *h(x)*, and *g(x)* functions are as follows:

$$
f(\mathbf{x}) = \sum_{l} \frac{\sum_{j:l\in j} x_j}{C_l - \sum_{j:l\in j} x_j} \tag{3.64}
$$

$$
h(\mathbf{x}) = \sum_{j\in J(w)} x_j - R_w = 0 \tag{3.65}
$$

$$
g_1(\mathbf{x}) = \sum x_j - \rho_{TE} C_l \le 0 \tag{3.66}
$$

$$
g_2(\mathbf{x}) = -\mathrm{x}_{\mathrm{j}} \le 0 \quad j \in J \tag{3.67}
$$

In each step, the Gradient Projection algorithm performs line search using the Golden Section Line Search Algorithm starting in initial feasible region (x_0) in a feasible direction $\boldsymbol{d}$ with a non-negative scalar α.

The FONC or Kuhn-Karush-Tucker condition for the non-linear programming problem of equations (3.63)-(3.67) is follows:

$$
\begin{aligned}
\nabla f(\mathbf{x}) + \boldsymbol{\lambda}_1^T \nabla g_1(\mathbf{x}) + \boldsymbol{\lambda}_2^T g_2(x) + \boldsymbol{\lambda}_3^T h(\mathbf{x}) &= \mathbf{0} \\
g_1(\mathbf{x}) &\le \mathbf{0} \\
g_2(\mathbf{x}) &\le \mathbf{0} \\
h(\mathbf{x}) &= \mathbf{0} \\
\boldsymbol{\lambda}^T \begin{bmatrix} g_1(\mathbf{x}) \\ g_2(\mathbf{x}) \\ h(\mathbf{x}) \end{bmatrix} &\le \mathbf{0} \\
\boldsymbol{\lambda} &\ge \mathbf{0}
\end{aligned}
\tag{3.68}
$$

The Gradient Projection method satisfies the Kuhn-Karush-Tucker condition by steepest descent starting at a feasible point. In each step, non-equality constrains (***g(x)***) are first separated as active ($g_{active}(x)$) and inactive sets ($g_{inactive}(x)$). The active set of inequality constraints equates to zero at the feasible point (***x***). On the other hand, the inactive set is strictly negative at ***x*.**

$$
\begin{aligned}
g_{active}(\mathbf{x}) &: \quad g_i(\mathbf{X}) = 0 \\
g_{inactive}(\mathbf{x}) &: \quad g_i(\mathbf{X}) < 0
\end{aligned}
\tag{3.69}
$$

These active ($g_{active}(x)$) constraints at a feasible point create the boundary of the feasible region. The equality constraints and the active set of non-equality constraints form a working set ($w(x)$).

$$
w(\mathbf{x}) = \begin{cases} g_i(\mathbf{x}) = 0 \\ h_i(\mathbf{x}) = 0 \end{cases}
\tag{3.70}
$$

This working set is the foundation of the working surface (A_w). Inactive sets are ignored because in Gradient Projection method the inactive sets lie outside of the working surface. The direction (***d***) of movement is found by projecting the negative gradient ($-\nabla f(\mathbf{x})$) of the objecting function on the working surface.

$$
d_k = -\left[\mathbf{I} - \mathbf{A}_q^T (\mathbf{A}_q \mathbf{A}_q^T)^{-1} \mathbf{A}_q\right] \nabla f(\mathbf{x}_k)^T
\tag{3.71}
$$

The length of the feasible segment is determined. Maximum distance (α_{Max}) can be found by solving the following equation:

$$
g_{inactive}(x) + \alpha_{Max} g_{inactive}(x) d = \mathbf{z}
\tag{3.72}
$$

The one dimensional matrix ***z*** is a zero matrix. In the following line search step (interspersed with the direction-finding steps), the algorithm travels from one feasible point (x_k) to a better feasible point (x_{k+1}) using a step size (α_k) such that $0 \le \alpha_k \le \alpha_{Max}$.

$$x_{k+1} = x_k + \alpha_k \mathrm{d}_k \tag{3.73}$$

By line searching through the feasible region, as in equation(3.73), the optimum point in each feasible segment can be achieved by minimizing the objective function f(***x***) as follows in each <u>step</u>:

$$\begin{aligned} &Minimize \quad f(x_k + \alpha_k \mathrm{d}_k) \\ &s.t. \quad [\mathbf{A}] \le [\mathbf{z}] \end{aligned} \tag{3.74}$$

In equation(3.74), the multi-dimensional matrix ***A*** contains the *g* and *h* matrices of equations (3.65)-(3.67).

A line search performs the movement or descent in each segment until a minimum endpoint is achieved when a new constraint becomes active. In each segment, this minimum is achieved at $d_k = 0$ such that the following FONC condition is satisfied.

$$\nabla f(\mathbf{x_k}) + \lambda_k^T \mathbf{A_q} = \mathbf{0} \tag{3.75}$$

For all active inequality constraints, if $d_k = 0$ and the Largrange Multiplier (λ) is non-negative, the Kuhn-Karush-Tucker condition is satisfied and the optimum point is achieved. We implemented the Golden Section Line search and the Gradient Projection algorithm of [50], which we reproduce in Appendix A for reference.

4.6 *Chapter Summary*

This chapter presented a mathematical non-linear programming technique to optimize—and therefore minimize—the marginal cost of providing services. Cost minimization is a necessary condition to optimize profit. Congestion of the network adds cost to providing customer-preferred services. Therefore, the minimization of network congestion is a condition to the maximization of profit. A key indicator of network congestion is the mean IP packet count in the network queue system.

An optimized routing technique minimizes the mean packet count in the network queue system. This minimization of the mean packet count reduces network congestion and equally distributes traffic around the network. The chapter described Gradient Project algorithm to optimize the mean packet count in the network that supported the well-established *M/M/1* queue system.

5 Network and Traffic Flow Design

To perform a comparative analysis of provider strategies, which optimize their profit, we need to develop identical network topology of the providers. We need to ensure that the topology fits our proposed model. We need to guarantee QoS requirements of customers as specified in Chapter 4 by developing traffic-engineering rules. To minimize marginal cost of the network, we need to avoid congestion hot spot in the network. This can be achieved by providing multiple routing options and minimizing congestion by optimized routing through these diverse options. Considering these requirements, this chapter designs a network topology, specifies traffic engineering rules, assigns network capacity, and designs Label-Switch-Path (LSP) routes, and corresponding non-linear programming matrices to conduct the analytical and session level Monte-Carlo simulation studies of Chapters 7 and 8.

5.1 Network Topology

Our duopoly market network topology consists of A.com and B.com providing services in four regions: Chicago, New York, Dallas, and Atlanta. A region surrounds large network hubs of providers and consists of multiple enterprise networks. Enterprises peer with both providers' hubs.

We assume that the sessions arrive in a network from a regional market (origin), propagate through the winning provider, and depart through a different regional market (destination). This implies that local or intra domain sessions of a regional market do not traverse through any provider. Each origin-destination (O-D) pair is unidirectional. Traffic flows from the originating E-LSR to the destination E-LSR through uni-directional Label Switch Paths (LSPs).

All links are bi-directional; for example, traffic can propagate from Chicago to Atlanta as well as from Atlanta to Chicago. Each session has two legs: origin-destination and destination-origin. These legs are symmetric, i.e. the bandwidths of the call in both directions are the same.

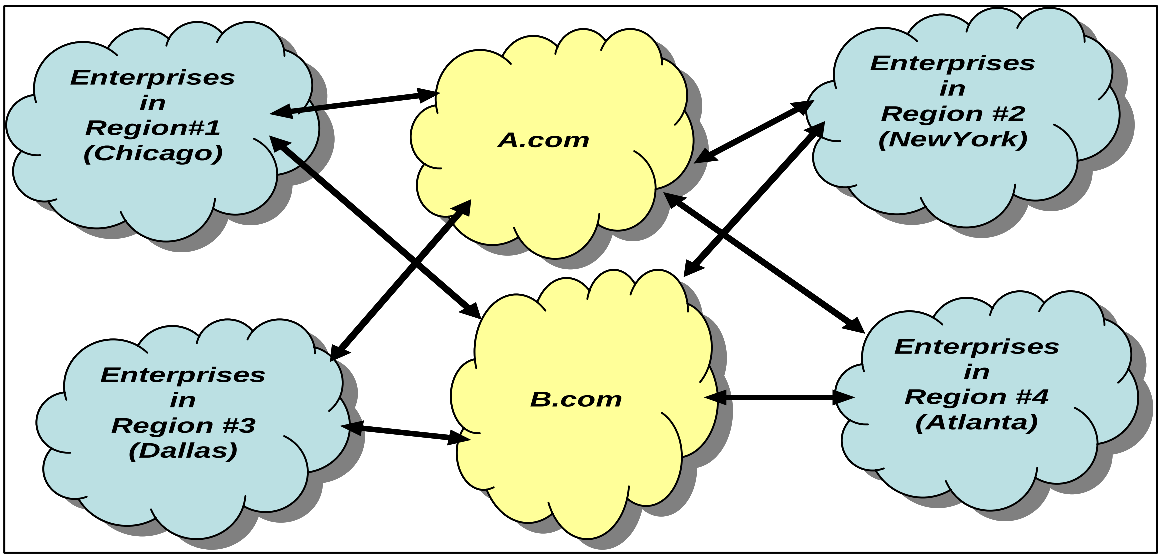


Figure 5.1: Simulation topology

Figure 5.1 depicts the topology of a provider and its connectivity with enterprise regions. There is at least one E-LSR of each provider in each region. This implies that in this duopoly market there are at least two E-LSRs in each region—one from each provider. All providers have identical networks. Either a centralized or distributed analyst along with the Call-Control-Functional-Entity (CCFE) perform the pricing negotiation, connection admission control (CAC), policy enforcement, and other control functions. We do not show control plane entities in the figures of this chapter. Assuming that the control-signaling specific traffic volume is negligible, we concentrate only on the media plane capacity.

Prior to determining market capacity, we need to develop traffic-engineering rules. Traffic engineering rules depend on packet length distribution.

5.2 *Packet Length Distribution*

Cooperative Association for Internet Data Analysis (CAIDA) observed the mean lengths of IP packets directly from the Internet in early 2000. These mean lengths were widely used to develop realistic models of Internet simulation scenarios. According to a CAIDA finding [53], the mean IP packet length observed in the internet in early 2000 was: 56% of the packets were 40 Bytes; 23% of the packets were 1500 bytes; and the rest was around 576 Bytes. In a separate study, the National Laboratory of Applied Network Research observed the following mean lengths of IP packets in the Internet: 59% of packets were 40 Bytes; 23% of the packets were 1500 Bytes, and 18% of the packets were 576 Bytes.

The CAIDA and the National Laboratory of Applied Network Research Internet packet length observations were conducted prior to the rapid growth in VoIP and IMS traffic in the Internet. We are not aware of any recent study that observed the mean packet lengths of IP packets in the Internet after the rapid growth of VoIP and IMS.

To transport voice over IP packets, Real Time Transport Protocol (RTP), and User Datagram Protocol (UDP) are used. The mean packet length of the commonly used G.711[10] coded VoIP is 200 bytes [80]: the VoIP mean payload is 160 bytes and protocol headers are 12 bytes, 8 bytes, and 20 bytes, respectively, for RTP, UDP, and IP.

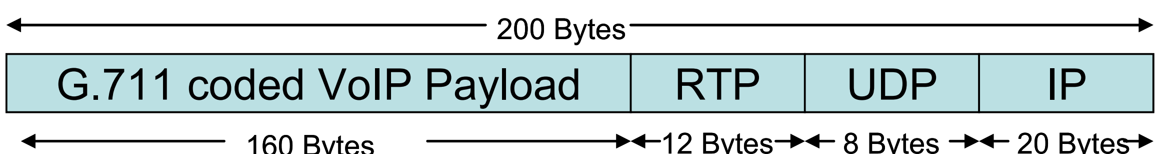


Figure 5.2: VoIP Packet Length

We assume that the mean packet length of non-VoIP packets are 576 bytes and 1500 bytes based on the CAIDA and the National Laboratory of Applied

[10] G.711 is an ITU-T standard that specifies 64-kbps Pulse Code Modulation (PCM) voice encoding technique.

Network Research packet length observations. We assume that providers deploy G.711 coded VoIP payload.

Assumption 5.1: Mean packet lengths of Blue, Green, and Red are 200 bytes, 576 bytes, and 1500 bytes, respectively.

We expect that the mean message lengths of state-of-the art IP networks will be smaller than our assumed values of Blue, Green, and Red services. Thus, our assumption reflects a worst-case scenario. Note that the packet length distribution is assumed exponential. Section 4.2 presents the rationale for assuming this distribution.

5.3 *Traffic Engineering Rule*

We need to ensure that participating providers guarantee two major performance requirements: no packet loss and mean packet delay in the queue system within tolerable limits. We accomplish this by enforcing traffic-engineering rules in the network. The objective of this section is to develop traffic-engineering rules that guarantee the following requirements:

Requirement 5.1: No packet loss in the network.

Note that for Voice and Interactive Video the packet loss should be less than 1%, and for streaming video it should be less than 5% [80]; therefore, our no packet loss requirement is a stringent requirement.

Requirement 5.2: The mean delay in the queue system of each link shall not exceed 1.0 millisecond.

The ITU standard G.114 specifies that the one-way (mouth-to-ear) delay should not exceed 150 milliseconds for voice [80]. The leading telecommunication vendor Cisco recommends that the one-way latency for Interactive-video and Streaming-video should be less than 150 milliseconds and 4 seconds, respectively. Significant portion of this delay should be attributed to the long-distance propagation. Therefore, the delay budget for a network node is much less. In the emerging Internet core networks, the node interfaces are OC48 (2.5 Giga-bits-per-second) or above. For such a high speed, the queuing delay is in microseconds. In our study, we assume that the interface speed is 100 Mega-bits-per-second. Thus, we assume a higher valued delay budget of 1.0 millisecond.

By bounding the link load of each link to an upper limit, both the packet loss and the delay budget (requirements 5.1 and 5.2) can be guaranteed.

Definition: Traffic Engineering Load is the maximum allowed load of a link.

As mentioned earlier, each network link of this study supports an integrated single queue served by a FIFO non-preemptive priority-scheduling scheme. We will develop the traffic-engineering rule based on M/M/1 queuing analysis for both the homogenous-service based and the heterogeneous-service based networks. The rationale for selecting the M/M/1 model is described in Section 4.2. When traffic with Poisson distributed arrival rate aggregates into an integrated FIFO queue, the aggregate arrival distribution continues to be Poisson. When traffic with Exponential distributed packet lengths merges into an integrated queue, the aggregate packet distribution is hyper-exponential. Thus, we need to depart from M/M/1 model and adopt M/G/1 model for the delay analysis of the queue system. This M/G/1 model will assume Poisson arrival and Generalized service time distributions.

Figure 5.3: Single Integrated FIFO Queue system

Figure 5.3 illustrates that Blue, Green, and Red classes of service arrive in a single integrated queue system with mean arrival rates of λ_b, λ_g, and λ_r, respectively. A FIFO scheduling system serves the queue at a link rate of C Mbps. Since Blue, Green, and Red classes of service share the same FIFO queue of a link, the presence the Red and Green services influence the delay variance of the Blue service. Delay variance depends on the traffic load of the link. Maintaining the link traffic load below a certain threshold can guarantee the required latency of each service.

Assumption 5.2: The link loads of Blue and the Green class of service do not exceed 20% and 30%, respectively.

Based on this assumption, we determine the maximum load of the Red service in a link to guarantee the mean packet delay of all services while maintaining a fixed Blue and Green load of $\rho_b = 20\%$ and $\rho_g = 30\%$.

Aggregate arrival rate in the queue system is as follows:

$$\lambda = \lambda_b + \lambda_g + \lambda_r \tag{5.1}$$

We denote $E[L_b], E[L_g], \text{and}, E[L_r]$ as the mean packet lengths of the Blue, Green, and Red traffic. First and second moments of service time for each service class are denoted by assuming exponentially distributed packet lengths for each service class:

$$E[\tau_b] = \frac{E[L_b]}{C}, \quad E[\tau_g] = \frac{E[L_g]}{C}, \quad E[\tau_r] = \frac{E[L_r]}{C} \tag{5.2}$$

$$E[\tau_b^2] = 2.\left(\frac{E[L_b]}{C}\right)^2, \quad E[\tau_g^2] = 2.\left(\frac{E[L_g]}{C}\right)^2, \quad E[\tau_r^2] = 2.\left(\frac{E[L_r]}{C}\right)^2 \tag{5.3}$$

We denote the first moment of integrated mean service time as $E[\hat{\tau}]$, the second moment of integrated mean service time as $E[\hat{\tau}^2]$, and variance of integrated mean service time as $Var[\hat{\tau}]$, where $\hat{\tau}$ is the delay random variable of the integrated queue.

The first moment of the integrated mean service time for the M/G/1 system is represented by the following equation where mean service time of *each flow* is exponential but the *aggregated* mean service time is hyper-exponential:

$$\begin{aligned} E[\hat{\tau}] &= \frac{\lambda_b}{\lambda} E[\tau_b] + \frac{\lambda_g}{\lambda} E[\tau_g] + \frac{\lambda_r}{\lambda} E[\tau_r] \\ &= \frac{\lambda_b}{\lambda} \frac{E[L_b]}{C} + \frac{\lambda_g}{\lambda} \frac{E[L_g]}{C} + \frac{\lambda_r}{\lambda} \frac{E[L_r]}{C} \end{aligned} \tag{5.4}$$

The second moment of the integrated mean service time for the M/G/1 system is represented by the following equation where mean service time of *each flow* is exponential but the *aggregated* mean service time is hyper-exponential:

$$\begin{aligned} E[\hat{\tau}^2] &= \frac{\lambda_b}{\lambda} E[\tau_b{}^2] + \frac{\lambda_g}{\lambda} E[\tau_g{}^2] + \frac{\lambda_r}{\lambda} E[\tau_r{}^2] \\ &= \frac{\lambda_b}{\lambda} 2.\left(\frac{E[L_b]}{C}\right)^2 + \frac{\lambda_g}{\lambda} 2.\left(\frac{E[L_g]}{C}\right)^2 + \frac{\lambda_r}{\lambda} 2.\left(\frac{E[L_r]}{C}\right)^2 \end{aligned} \tag{5.5}$$

In an M/G/1 system, mean delays experienced by Blue, Green, and Red packets are as follows:

$$E[T_b] = \frac{E[L_b]}{C} + \frac{\lambda E[\hat{\tau}^2]}{2(1 - \lambda E[\hat{\tau}])} \tag{5.6}$$

$$E[T_g] = \frac{E[L_g]}{C} + \frac{\lambda E[\hat{\tau}^2]}{2(1-\lambda E[\hat{\tau}])} \quad (5.7)$$

$$E[T_r] = \frac{E[L_r]}{C} + \frac{\lambda E[\hat{\tau}^2]}{2(1-\lambda E[\hat{\tau}])} \quad (5.8)$$

As per Assumption 5.1, the mean packet lengths of Blue, Green, and Red services are $E[L_b] = 200$ Bytes, $E[L_g] = 576$ Bytes, and $E[L_r] = 1500$ Bytes. The following figure depicts the mean packet latency in the M/G/1 queue system of link rate of 100 Mbps as per equation (5.1)-(5.5).

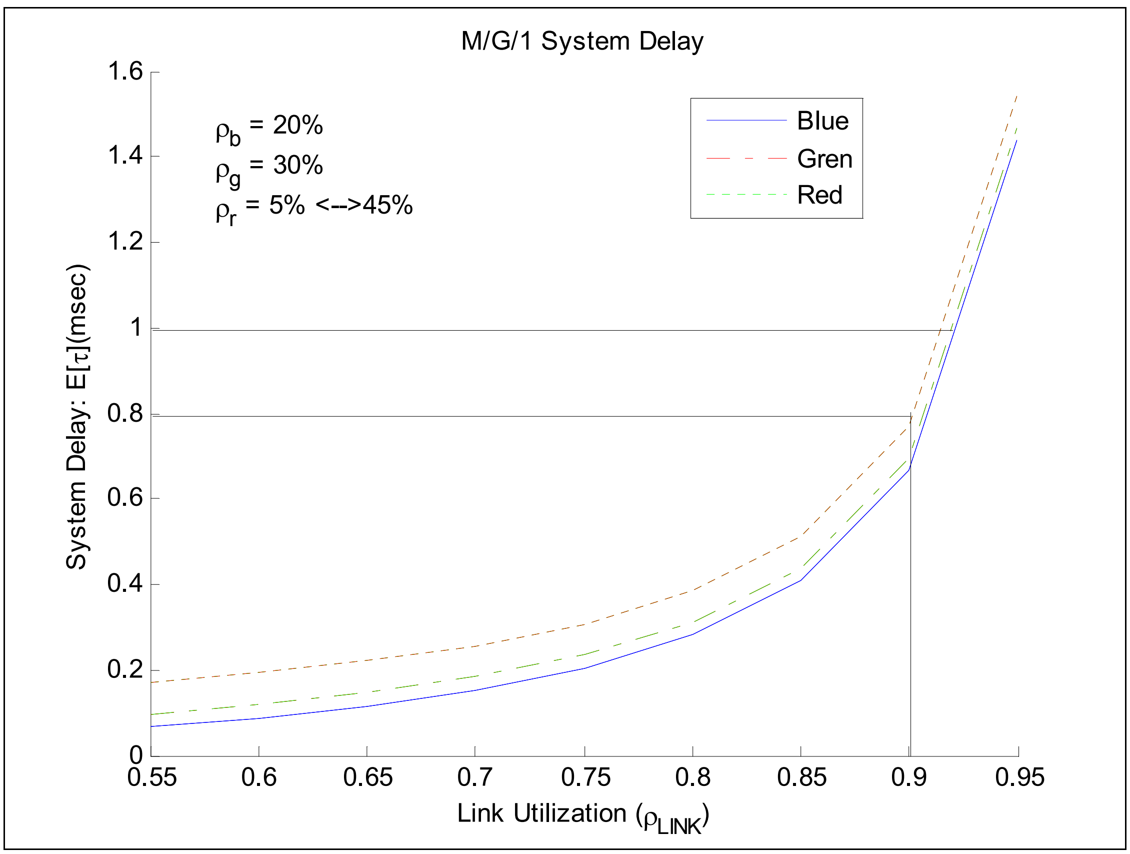


Figure 5.4: M/G/1 System Delay for Heterogeneous services

System delays in milliseconds for Blue, Green, and Red classes of service are shown with respect to the total link load from 0.55 to 0.95. Blue and Red loads are kept constant at 0.20 and 0.30. The Red load is increased from 0.05 to 0.45. The figure shows that at link utilization of 90% ($\rho_b = 20\%$, $\rho_g = 30\%$, and $\rho_r = 40\%$), the mean delays of Blue, Green, and Red classes of service are less than 1.0 millisecond, which guarantees the delay bound of requirement 5.2. At 90% link utilization, the arrival rate is less than the departure rate; thus, no packet loss occurs, which guarantees requirement 5.1 assuming infinite queue length. Network routers do not have infinite length queues in real implementation; however, queue sizes are significantly large compared to the maximum packet count in the queue. Therefore, we assume that queues have infinite length.

Based on the above discussion, we employ the following traffic engineering rules on each link.

Heterogeneous Service Network:

Maximum Traffic Engineering link load (ρ_{TE}) must adhere to the following boundaries.

$$\begin{aligned} &\rho_{TEb} \le 20\% \\ &\rho_{TEg} \le 30\% \\ &\rho_{TEr} \le 40\% \end{aligned} \tag{5.9}$$

Homogeneous (Single) Service Network:

Maximum Traffic Engineering link load (ρ_{TE}) must adhere to the following boundary.

$$\rho_{TE} \le 0.90 \tag{5.10}$$

5.4 *Capacity Assignment*

Assumption 5.3: All providers deploy identical physical capacity.

$$K_1 = K_2 = \ldots = K_N \tag{5.11}$$

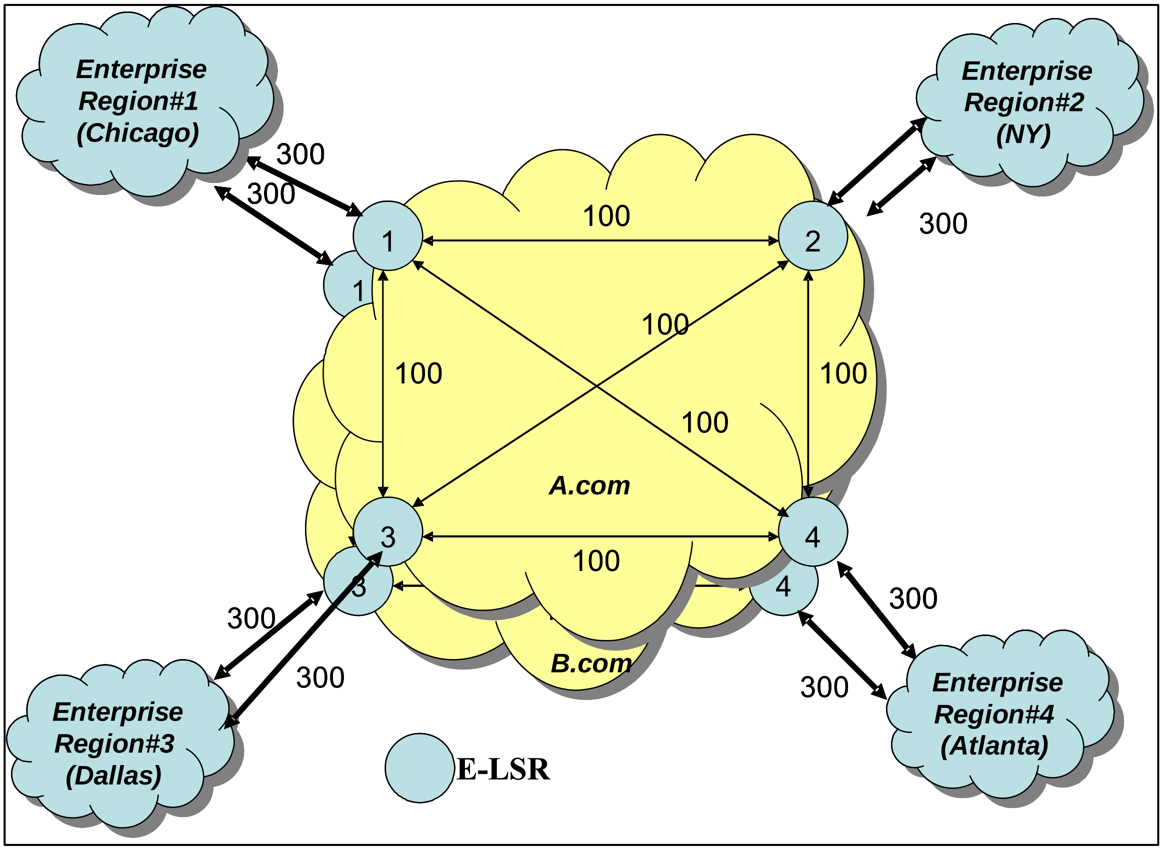


Figure 5.5: Internal Network Topology of Two providers

Figure 5.5 illustrates the internal connections of both the providers and their connections with the regions. Each provider has identical four node (Edge-LSR) network topology, where each E-LSR is connected with a region of the market. For example, all the customers in Atlanta are connected to the E-LSR #4 of both the providers. Although we have shown only one link is connecting an E-LSR of a provider to a customer region, this one link is a pictorial representation of many access links. Inside each provider, E-LSRs are interconnected in a mesh topology of 100 Mbps bi-directional links. Each E-LSR is connected with three other E-LSRs with three links; therefore, the maximum input traffic of each E-LSR towards the network core is 300 Mbps. In other words, in each region, the ingress physical capacity (K) of each provider is 300 Mbps. Similarly, in each region the egress physical capacity of each provider is 300 Mbps. Note that the maximum aggregate ingress physical traffic of all four regions to a provider core network is 300*4 = 1200 Mbps. Since we assume that the network is lossless, maximum aggregate egress traffic from a provider to all four regions is also 1200 Mbps.

As per the definition of Section 3.3, the Market Capacity (*Γ*) is represented by the following equation, where *K* is the ingress physical capacity of each provider and ρ_{TE} is the Maximum Traffic Engineered link load.

$$\Gamma = \sum_{n=1}^{N} K_n \rho_{TE} = \rho_{TE} \sum_{n=1}^{N} K_n \qquad (5.12)$$

The following table summarizes the physical capacity assignment and the market capacity of this study.

Table 5.1: Capacity Assignment

Number of providers	2	
Link Capacity of Each provider (C)	100 Mbps	
Physical Capacity of Each provider/Region	300 Mbps	
Number of Regions in the Market	4	
Physical Capacity (K) of Each provider in the Market	300*4 = 1200 Mbps	
Max Traffic Engineered Link Load (ρ_{TE})	= 0.90	$\rho_{TEb} \le 0.20$ $\rho_{TEg} \le 0.30$ $\rho_{TEr} \le 0.40$
Market Capacity of Each provider ($\rho_{TE}K$)	0.90*1200 = 1080 Mbps	
Total Market Capacity (Γ)	1080*2 = 2160 Mbps	

The session level Monte-Carlo simulation algorithm in Section 6 states the procedure of enforcing maximum market demand.

5.5 *Session Arrival Pattern*

Often in QoS simulation studies (e.g., [80]-[84]), call arrival rate distribution is assumed Poisson and call duration distribution is assumed exponential with a mean of 180 seconds. Similarly, we assume that the session arrival rate distribution is Poisson and session length distribution is exponential with a mean of 180 seconds. By changing the session arrival rate, market demand load is adjusted.

5.6 *Traffic Flow Design*

Traffic flow of each O-D pair can traverse through five different routes inside the network of each provider. For example, in Figure 5.6 traffic of the Chicago-NewYork O-D pair can flow from Edge-LSR#1 to Edge-LSR#2 through the following routes: 1→2, 1→3→2, 1→4→2, 1→3→4→2, and 1→4→3→2.

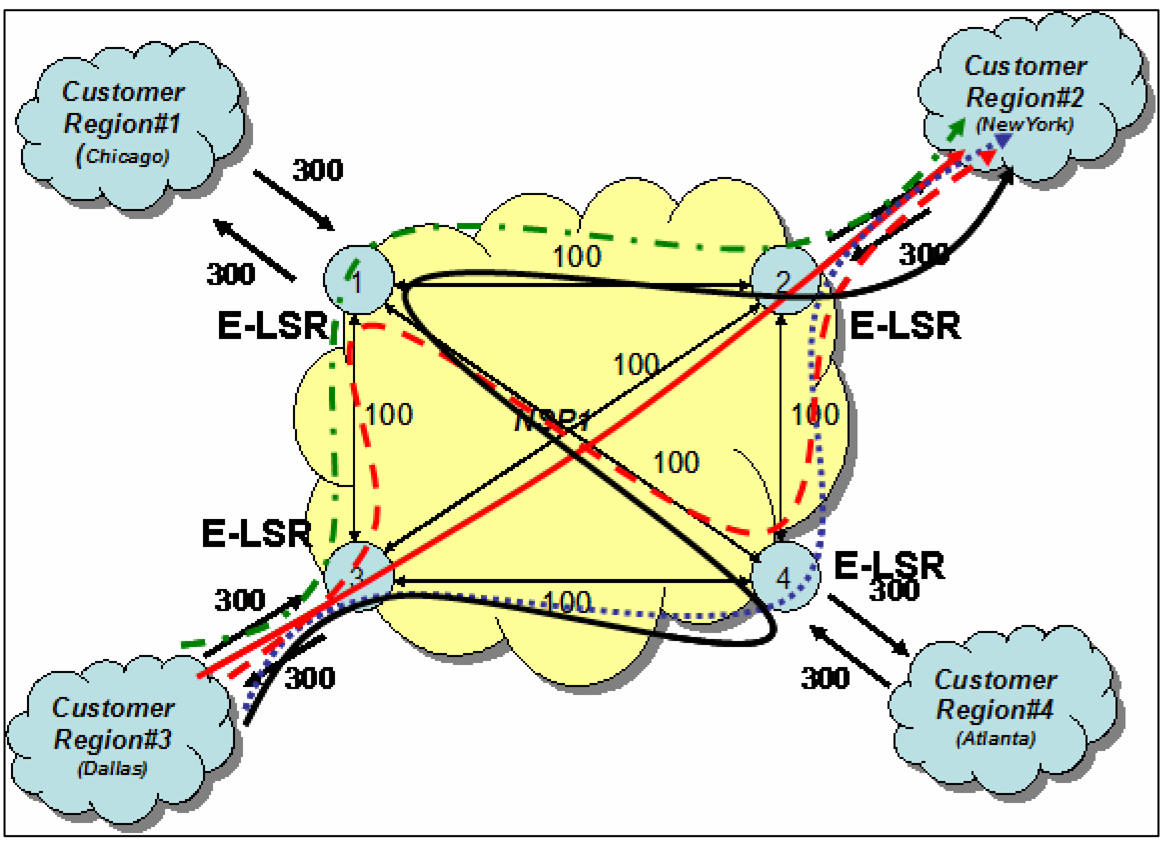


Figure 5.6: Each O-D pair has five different routes

Table 5.2 identifies the O-D pairs and their corresponding paths.

Table 5.2: O-D pairs and paths

OD Pair	**Path**	**OD Pair**	**Path**	**OD Pair**	**Path**
1 --> 2	1-->2	**1-->3**	1-->3	**1-->4**	1-->4
	1-->3-->2		1-->2-->3		1-->2-->4
	1-->4-->2		1-->4-->3		1-->3-->4
	1-->3-->4-->2		1--2-->4-->3		1-->2-->3-->4
	1-->4-->3-->2		1-->4-->2-->3		1-->3-->2-->4
2-->1	2-->1	**2-->3**	2-->3	**2-->4**	2-->4
	2-->4-->1		2-->1-->3		2-->3-->4
	2-->3-->1		2-->4-->3		2-->1-->4
	2-->4-->3-->1		2-->1-->4-->3		2-->1-->3-->4
	2-->3-->4-->1		2-->4-->1-->3		2-->3-->1-->4
3-->1	3-->1	**3-->2**	3-->2	**3-->4**	3-->4
	3-->2-->1		3-->1-->2		3-->1-->4
	3-->4-->1		3-->4-->2		3-->2-->4
	3-->2-->4-->1		3-->1-->4-->2		3-->1-->2-->4
	3-->4-->2-->1		3-->4-->1-->2		3-->2-->1-->4
4-->1	4-->1	**4-->2**	4-->2	**4-->3**	4-->3
	4-->2-->1		4-->1-->2		4-->1-->3
	4-->3-->1		4-->3-->2		4-->2-->3
	4-->2-->3-->1		4-->1-->3-->2		4-->1-->2-->3
	4-->3-->2-->1		4-->3-->1-->2		4-->2-->1-->3

Table 5.2 depicts the traffic matrix between origin-destination (O-D) pairs where r_{ij} is the traffic from an origin (i) to a destination (j).

Table 5.3: O-D Traffic Matrix

Destination	→	1	2	3	4
	1	0	R_{12}	R_{13}	R_{14}
Origin	2	R_{21}	0	R_{23}	R_{24}
	3	R_{31}	R_{32}	0	R_{34}
	4	R_{41}	R_{42}	R_{43}	0

We denote capacity between node i and j as C_{ij}. Following table shows the Capacity matrix of the network.

Table 5.4: Capacity Matrix of Each Network

$$\left[{}_{12\times 1}\mathbf{C}\right]=\begin{bmatrix}C_{12}\\C_{21}\\C_{13}\\C_{31}\\C_{14}\\C_{41}\\C_{42}\\C_{24}\\C_{23}\\C_{32}\\C_{34}\\C_{43}\end{bmatrix}=\begin{bmatrix}100\\100\\100\\100\\100\\100\\100\\100\\100\\100\\100\\100\end{bmatrix}$$

The sum of traffic flows in a link should not be greater than the capacity of the link. Based on Table 5.2, we develop the equations for the inequality constraints as follows:

Table 5.5: Inequality Constraint

$$\begin{aligned}
&x_{12}+x_{123}+x_{124}+x_{1243}+x_{1234}+x_{312}+x_{3124}+x_{3412}+x_{4123}+x_{412}+x_{4312}-\rho_{TE}C_{12}\le 0\\
&x_{21}+x_{321}+x_{421}+x_{3421}+x_{4321}+x_{213}+x_{4213}+x_{2143}+x_{3214}+x_{214}+x_{2134}-\rho_{TE}C_{21}\le 0\\
&x_{13}+x_{134}+x_{213}+x_{413}+x_{132}+x_{1342}+x_{1324}+x_{4132}+x_{2134}+x_{2413}+x_{4213}-\rho_{TE}C_{13}\le 0\\
&x_{31}+x_{431}+x_{312}+x_{314}+x_{231}+x_{2431}+x_{4231}+x_{2314}+x_{4312}+x_{3142}+x_{3124}-\rho_{TE}C_{31}\le 0\\
&x_{14}+x_{142}+x_{214}+x_{314}+x_{143}+x_{1432}+x_{1423}+x_{2314}+x_{3214}+x_{3142}+x_{2143}-\rho_{TE}C_{14}\le 0\\
&x_{41}+x_{241}+x_{412}+x_{413}+x_{341}+x_{2341}+x_{3241}+x_{4132}+x_{4123}+x_{2413}+x_{3412}-\rho_{TE}C_{41}\le 0\\
&x_{42}+x_{142}+x_{342}+x_{421}+x_{423}+x_{1342}+x_{1423}+x_{3142}+x_{3421}+x_{4213}+x_{4231}-\rho_{TE}C_{42}\le 0\\
&x_{24}+x_{241}+x_{243}+x_{124}+x_{324}+x_{2431}+x_{3241}+x_{2413}+x_{1243}+x_{3124}+x_{1324}-\rho_{TE}C_{24}\le 0\\
&x_{23}+x_{231}+x_{234}+x_{123}+x_{423}+x_{1234}+x_{1423}+x_{2314}+x_{2341}+x_{4123}+x_{4231}-\rho_{TE}C_{23}\le 0\\
&x_{32}+x_{132}+x_{432}+x_{321}+x_{324}+x_{4321}+x_{3241}+x_{4132}+x_{1432}+x_{3214}+x_{1324}-\rho_{TE}C_{32}\le 0\\
&x_{34}+x_{134}+x_{234}+x_{341}+x_{342}+x_{1342}+x_{1234}+x_{2341}+x_{2134}+x_{3421}+x_{3412}-\rho_{TE}C_{34}\le 0\\
&x_{43}+x_{431}+x_{432}+x_{143}+x_{243}+x_{2431}+x_{4321}+x_{1432}+x_{4312}+x_{1243}+x_{2143}-\rho_{TE}C_{43}\le 0
\end{aligned}$$

The individual flows are assigned y index in the next table:

X12	Y1	x1234	y13	x3421	Y25	X1324	y37	x1432	Y49
X21	Y2	x3124	y14	x4321	Y26	X4132	y38	x142	Y50
X13	Y3	x3412	y15	x213	Y27	X2413	y39	x241	Y51
X31	Y4	x4123	y16	x4213	Y28	X431	y40	x341	Y52
X14	Y5	x4312	y17	x2143	Y29	X314	y41	x2341	Y53
X41	Y6	x1243	y18	x3214	Y30	X231	y42	x3241	Y54
X42	Y7	x412	y19	x214	Y31	X2431	y43	x342	Y55
X24	Y8	x312	y20	x2134	Y32	X4231	y44	x423	Y56
X23	Y9	x124	y21	x134	Y33	X2314	y45	x243	Y57
X32	Y10	x123	y22	x1342	Y34	X3142	y46	x324	Y58
X34	Y11	x321	y23	x413	Y35	X143	y47	x234	Y59
X43	Y12	x421	y24	x132	Y36	X1423	y48	x432	Y60

Then, the capacity inequality conditions appear below:

$$\begin{aligned}
&y_1+y_{13}+y_{14}+y_{15}+y_{16}+y_{17}+y_{18}+y_{19}+y_{20}+y_{21}+y_{22}-\rho_{TE}C_{12}\le 0\\
&y_2+y_{23}+y_{24}+y_{25}+y_{26}+y_{27}+y_{28}+y_{29}+y_{30}+y_{31}+y_{32}-\rho_{TE}C_{21}\le 0\\
&y_3+y_{27}+y_{28}+y_{32}+y_{33}+y_{34}+y_{35}+y_{36}+y_{37}+y_{38}+y_{39}-\rho_{TE}C_{13}\le 0\\
&y_4+y_{14}+y_{17}+y_{20}+y_{40}+y_{41}+y_{42}+y_{43}+y_{44}+y_{45}+y_{46}-\rho_{TE}C_{31}\le 0\\
&y_5+y_{29}+y_{30}+y_{31}+y_{41}+y_{45}+y_{46}+y_{47}+y_{48}+y_{49}+y_{50}-\rho_{TE}C_{14}\le 0\\
&y_6+y_{15}+y_{16}+y_{19}+y_{35}+y_{38}+y_{39}+y_{51}+y_{52}+y_{53}+y_{54}-\rho_{TE}C_{41}\le 0\\
&y_7+y_{24}+y_{25}+y_{28}+y_{34}+y_{44}+y_{46}+y_{48}+y_{50}+y_{55}+y_{56}-\rho_{TE}C_{42}\le 0\\
&y_8+y_{14}+y_{18}+y_{21}+y_{37}+y_{39}+y_{43}+y_{51}+y_{54}+y_{57}+y_{58}-\rho_{TE}C_{24}\le 0\\
&y_9+y_{13}+y_{16}+y_{22}+y_{42}+y_{44}+y_{45}+y_{48}+y_{53}+y_{56}+y_{59}-\rho_{TE}C_{23}\le 0\\
&y_{10}+y_{23}+y_{26}+y_{30}+y_{36}+y_{37}+y_{38}+y_{49}+y_{54}+y_{58}+y_{60}-\rho_{TE}C_{32}\le 0\\
&y_{11}+y_{13}+y_{15}+y_{25}+y_{32}+y_{33}+y_{34}+y_{52}+y_{53}+y_{55}+y_{59}-\rho_{TE}C_{34}\le 0\\
&y_{12}+y_{17}+y_{18}+y_{26}+y_{29}+y_{40}+y_{43}+y_{47}+y_{49}+y_{57}+y_{60}-\rho_{TE}C_{43}\le 0
\end{aligned}\tag{5.13}$$

In addition, we add 60 rows of non-negative constraints for each flow. Therefore, the inequality constraint matrix has 72 rows and 60 columns. This constraint is denoted as **g(x)** in the Gradient Projection non-linear program of Section 4.4. We denote this inequality matrix as **G**.

The inequality and non-negative conditions are described as a matrix notation by equation(5.14). **G** is a 72x60 matrix containing the indices of the inequality and non-negative constraints.

$$\begin{bmatrix} {}_{(12\times 60)}\mathbf{G}_{Inequaltiy} \\ {}_{(60\times 60)}\mathbf{G}_{Non-negative} \end{bmatrix} \begin{bmatrix} {}_{(60\times 1)}\mathbf{Y} \end{bmatrix} - \begin{bmatrix} {}_{(12\times 1)}\rho_{TE}\mathbf{C} \\ {}_{(60\times 1)}\mathbf{0} \end{bmatrix} \leq \begin{bmatrix} {}_{72\times 1}\mathbf{0} \end{bmatrix} \quad (5.14)$$

The first twelve rows of **G** matrix are the indices of the inequality constraints. The last sixty rows of **G** matrix are the indices of the non-negativity constraints. Each column of **G** matrix represents a flow. For example, $g_{11} = 1$ represents the flow x_{12} = LSP y_1 that passes through link 1. The following is the first 12 rows of column 44 to 60 of G matrix.

Table 5.6: A portion of G Matrix

$$G(1:12, 44:60) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

In Table 5.6, column 44 represents the LSP indicator y_{44}, which maintains a route $x_{4\text{->}2\text{->}3\text{->}1}$. In this column, $g_{44,4}$=$g_{44,7}$ = $g_{44,9}$ = 1. In Table 5.4, the rows 4, 7, and 9 respectively, represent links 31, 42, and 23. This implies that flow $y_{44} = x_{4\text{->}2\text{->}3\text{->}1}$ passes through links C_{31}, C_{42}, and C_{23}.

The sum of the individual flows between the origin-destination pair is equal to the O-D flow, which is represented by the equality constraint equations of Table 5.7.

Table 5.7: Equality Constraints

$$
\begin{gathered}
x_{12}+x_{132}+x_{142}+x_{1342}+x_{1432}-R_{12}=0\\
x_{21}+x_{231}+x_{241}+x_{2431}+x_{2341}-R_{21}=0\\
x_{13}+x_{123}+x_{143}+x_{1243}+x_{1423}-R_{13}=0\\
x_{31}+x_{321}+x_{341}+x_{3421}+x_{3241}-R_{31}=0\\
x_{14}+x_{124}+x_{134}+x_{1324}+x_{1234}-R_{14}=0\\
x_{41}+x_{421}+x_{431}+x_{4231}+x_{4321}-R_{41}=0\\
x_{42}+x_{412}+x_{432}+x_{4132}+x_{4312}-R_{42}=0\\
x_{24}+x_{214}+x_{234}+x_{2314}+x_{2134}-R_{24}=0\\
x_{23}+x_{213}+x_{243}+x_{2143}+x_{2413}-R_{23}=0\\
x_{32}+x_{312}+x_{342}+x_{3412}+x_{3142}-R_{32}=0\\
x_{34}+x_{314}+x_{324}+x_{3124}+x_{3214}-R_{34}=0\\
x_{43}+x_{413}+x_{423}+x_{4213}+x_{4123}-R_{43}=0
\end{gathered}
$$

After mapping with y, the equality constraints appear as follows.

$$
\begin{gathered}
y_1+y_{36}+y_{50}+y_{34}+y_{49}-R_{12}=0\\
y_2+y_{42}+y_{51}+y_{43}+y_{53}-R_{21}=0\\
y_3+y_{22}+y_{47}+y_{18}+y_{48}-R_{13}=0\\
y_4+y_{23}+y_{52}+y_{25}+y_{54}-R_{31}=0\\
y_5+y_{21}+y_{33}+y_{37}+y_{13}-R_{14}=0\\
y_6+y_{24}+y_{40}+y_{44}+y_{26}-R_{41}=0\\
y_7+y_{19}+y_{60}+y_{38}+x_{17}-R_{42}=0\\
y_8+y_{31}+y_{59}+y_{45}+y_{32}-R_{24}=0\\
y_9+y_{27}+y_{57}+y_{29}+y_{39}-R_{23}=0\\
y_{10}+y_{20}+y_{55}+y_{15}+y_{46}-R_{32}=0\\
y_{11}+y_{41}+y_{58}+y_{14}+y_{30}-R_{34}=0\\
y_{12}+y_{35}+y_{56}+y_{28}+y_{16}-R_{43}=0
\end{gathered}
\tag{5.15}
$$

The equality constraint matrix is the h(x) of the gradient projection non-linear program of Section 4.4. By denoting the equality matrix as **H**, it is derived as equation (5.16).

$$
\begin{gathered}
\left[{}_{12\times 60}\mathbf{H}_{LSP}\right]\left[{}_{60\times 1}\mathbf{Y}\right]-\left[{}_{12\times 1}\mathbf{R}\right]=\left[{}_{12\times 1}\mathbf{0}\right]\\
\left[{}_{12\times 1}\mathbf{H}\right]=\left[{}_{12\times 1}\mathbf{0}\right]
\end{gathered}
\tag{5.16}
$$

Y is a 60x1 matrix representing the sixty LSPs. **H** is a 12x60 matrix with twelve rows representing twelve equality constraints of O-D pairs. Since there are sixty LSPs, this equality matrix defines the sum of rates of LSPs between origin and destination, which is equal to the O-D traffic.This **H** matrix and the active constraints of **G** matrix form the Working matrix (**W**).

$$\left[\mathbf{W}\right] = \begin{bmatrix} \mathbf{G}_{\text{Active}} \\ \mathbf{H} \end{bmatrix} \tag{5.17}$$

In our session level Monte-Carlo simulation, the Gradient Project algorithm uses this **W** matrix to optimize the traffic flow by minimizing the mean packet count in the network.

6 A Snapshot of the Algorithm

This chapter outlines a layered view of the *Providers Optimized Game in Internet Traffic* algorithm. It combines the proposed oligopoly model to determine price of services, the non-linear programming technique to minimize cost (which optimizes the profit of the providers) and the traffic engineering rules. The chapter also presents performance measurement matrices, and session level Monte-Carlo simulation algorithm.

6.1 The Layered View of the Algorithm

The algorithm consists of two major mechanisms: i) price negotiation between an enterprise and providers, and ii) the provider's method of computing a price. The signaling and control layer performs the price negotiation. The media layer routes traffic. A provider enforces Call Admission Control (CAC), performs optimized routing, deploys traffic-engineering rules, and computes cost of producing a service based on the media layer traffic load.

Each provider computes traffic engineered load based on M/G/1 queuing analysis (See rationale in Chapter 5). This load is a CAC parameter. By this load, a provider also computes market capacity as per equation (3.6). Each session initiation request is an instance of the game. The session arrival distribution is assumed Poisson and the session duration distribution is assumed exponential. When a session initiation request arrives, a provider first performs CAC to see whether the session can be supported based on the traffic engineered load of the network. If the session cannot be supported, a rejection is sent by appropriate SIP messages. In session level Monte-Carlo simulation, we model it as sending an infinite bid for the service.

Figure 6.1 depicts the layered view of this algorithm:

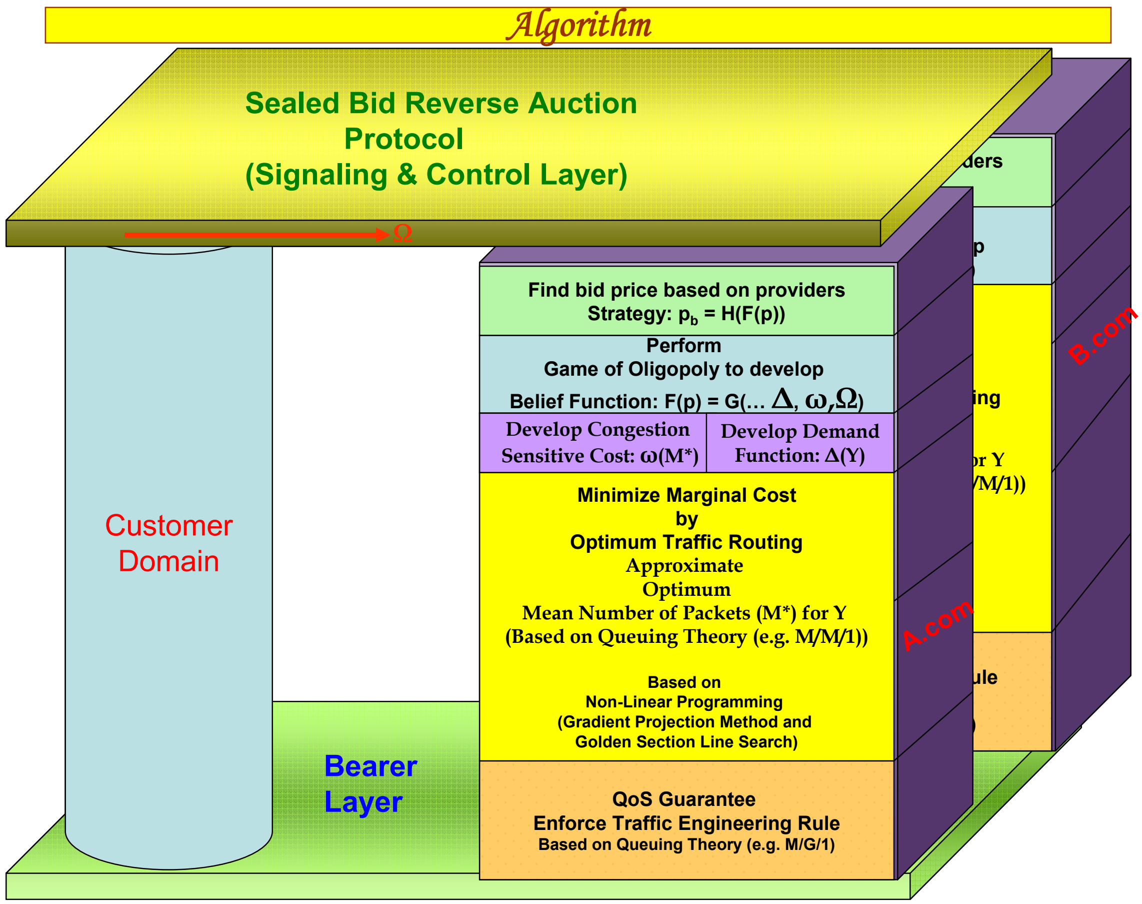


Figure 6.1: Layered View of the Proposed Algorithm

If the session can be supported, a provider first finds initial Origin-Destination (O-D) and Destination-Origin (D-O) routes of the bi-directional flow based on the minimum-hop routing scheme. These initial routes are used as the initial value of non-linear optimization program. By using non-linear program methods (Gradient Projection and Golden Section line search), providers approximate the optimum mean packet count in the queue system using M/M/1 model (See rationale in Chapter 4 and 5). From this optimum mean packet count information, each provider develops its optimum marginal cost function. The provider also computes perceived market demand for adding this session and the market demand of the network as per equation (3.13).

Based on the reservation price of the enterprise, market capacity, market demand, and service marginal cost, the provider develops a belief function as per equation (3.36). Then, the provider determines the price of service from the belief function based on the strategy from Table 3.3. (See Chapter 3).

6.2 *Performance Measurement Metrics*

The performance metrics are market prices of services, unit profit, expected unit profit, market shares of profit, market shares of throughput, and optimum strategies of providers.

The following are the measurement metrics of this research in both homogeneous and heterogeneous service-based markets.

The *unit profit* of a provider is the profit per unit duration (e.g. one second) measured at an instant of the steady state throughput ($\hat{Y}$) when the bid price and the marginal cost of the provider converge to $\hat{p}$ and $\hat{\omega}$.

$$u(p) = (\hat{p} - \hat{\omega})\hat{Y} \tag{6.1}$$

The *steady state mean profit* or *steady state mean profit* is the average of the profit curve during the steady state.

A provider (n) computes *profit* or *total profit* from a session as a function of the price (p), the marginal cost (ω), the duration (d), and the bandwidth (y) of the session. Although the price and the marginal cost vary with time, profit is computed based on their values at each session start time. The total profit of the provider is the sum of profits from all ($\forall k$) sessions until the end of simulation (i.e. end of short-term game).

$$profit_n = \sum_{\forall k} (\tilde{p}_{s,t,k} - \tilde{\omega}_{n,s,t,k}) d_{n,k} y_{n,s,k} \tag{6.2}$$

Our equations to compute the profit share acquired by A.com and B.com are as follows:

$$\% \, Market\ Share\ of\ \ profit_A = \frac{profit_A}{profit_A + profit_B}$$
$$\% \, Market\ Share\ of\ \ profit_B = \frac{profit_B}{profit_A + profit_B} \qquad (6.3)$$

The Network load of a provider at a time (t) is computed as the ratio between the provider throughput ($Y_{n,t}$) and the provider physical capacity (K_n):

$$Network\ Load \quad = \rho_{n,t} = \frac{Y_{n,t}}{K_n} = \frac{Y_{n,t}}{12C} = \frac{Y_{n,t}}{1200} \qquad (6.4)$$

The Market load at a time (t) is computed as the ratio between the providers' aggregate throuput and the market physcial capacity:

$$Market\ Load = \frac{Y_{A,t} + Y_{B,t}}{K_A + K_B} = \frac{Y_{A,t} + Y_{B,t}}{24C} = \frac{Y_{A,t} + Y_{B,t}}{2400} \qquad (6.5)$$

6.3 *Session Level Monte-Carlo Simulation Algorithm*

The following steps describe the simulation algorithm. In Figure 6.2, circled numbers identify the steps.

1. The simulation starts with Market Capacity (Γ), individual Network physical capacity (K), Time of next session (T_{next_call}), Maximum Regional Demand (*MRD*), and Current Regional Demand (*CRD*) values.
2. The simulator performs the desired duration in second as specified in step 2. Each iteration corresponds to one time slot, which is one-tenth of a second. The algorithm of Figure 5-7 shows that the duration of simulation is one million seconds. Note that this simulation is a continuous time process quantized to a one-tenth of a second. A provider is identified as a Network Service Provider (NSP) in the figure.
3. In each time slot, the algorithm performs four loops for four regions. If current time (T_{now}) is the time of next session (T_{next_call}), proceed to the step 4. If current time is the time of ending a session (T_{tear_down}), proceed to the step 5.

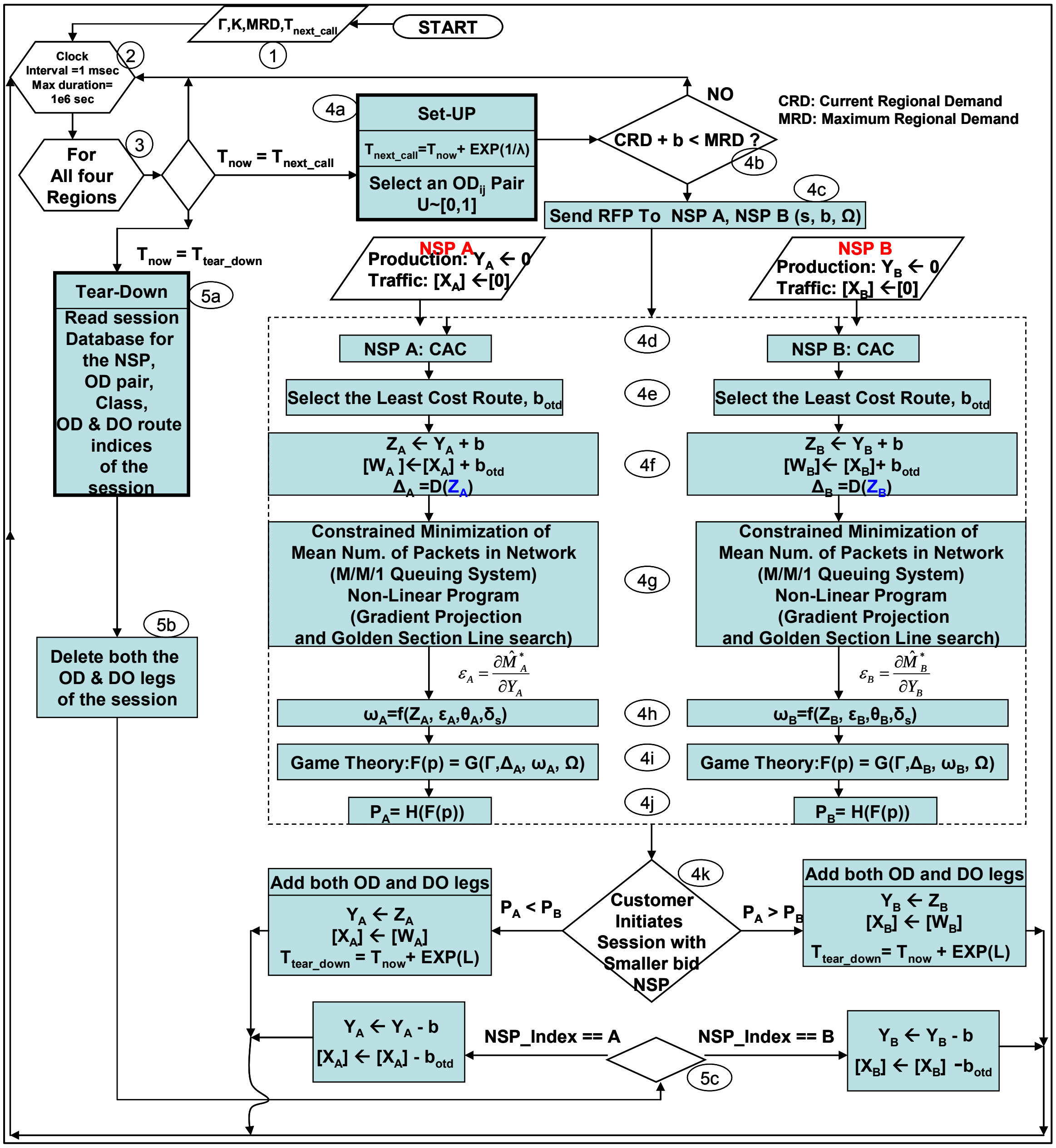


Figure 6.2: Session Level Monte-Carlo Simulation Flow Algorithm for Duopoly Market

4. Setup: In each region, the session arrives in exponential distribution with a mean inter-arrival rate of $1/\lambda$ second. (Note, each iteration is one-tenth of a second). A session remains active in exponential distribution with a mean session duration of L second. The traffic demand load level in the network is the function of the mean arrival rate and the mean session duration. The simulator performs steps 4a to 4k.

a. Select an O-D pair with uniform distribution.
b. Check to see whether the market demand of a class of service is within the maximum market demand for this class. If the market demand for this class is fulfilled, then the loop ends for this iteration in this region. Go to the step 2.
c. If the market demand for this class of service is less than the maximum market demand for this class, then send Request for Purchase (RFP) to both providers, specifying service class, reservation price, and requested bandwidth.
d. For each provider, perform call admission control (CAC) to see whether this session can be supported in both the O-D leg and the D-O leg of the route. If the session cannot be supported, send an infinite bid and proceed to the step 4k. If the session can be supported, proceed to step 5.
e. Select an initial origin-destination route and an initial destination-origin route using the lowest cost routing scheme; e.g. minimum number of hops.
f. Compute the anticipated market demand and initial flow matrix.
g. Perform constrained minimization of the mean packet count for the M/M/1 queuing system by non-linear programming. We use Gradient Projection algorithm with Golden Section Line search. This computes the anticipated optimized routes and optimum mean packet count in the network. Compute anticipated change (ε) in the mean packet count for adding this session.
h. Compute the marginal cost of supporting the session.
i. Perform game theory to develop belief function or the mixed strategy profile.
j. Determine a bid price from the belief function based on the preferred strategy and send bid to the enterprise.

k. An enterprise selects the lowest bidding provider. Setup a session leg in the O-D route and a session leg in the D-O route. Adjust provider's current traffic flow matrix equal the anticipated traffic flow matrix. Determine the time of the end of this session from an exponential distribution with the mean duration of call (L).

5. Tear Down

    a. Retrieve the session from the session database. Read the provider, O-D Pair, Class, O-D route, and D-O route of the session.

    b. Delete both the O-D and D-O legs of the session.

Adjust the traffic flow matrix to reflect the removal of the session and go to step 2.

7 *Mathematical Analyses and Validation*

This chapter presents mathematical analyses of the providers' game strategies. It also validates the mathematical model by means of simulation. The objective of the mathematical analyses is to determine the best strategies that optimize providers' profit.

In Chapter 3, we developed a provider's mixed strategy profile (belief function) and associated parameters: service class, market capacity and demand functions, marginal cost functions, reservation price of an enterprise, profit functions, and a set of game strategies. We also explained the properties of the belief function.

In Chapter 4, we developed a mathematical optimization method to maximize a provider's profit by minimizing marginal cost. This is performed by minimizing the mean packet count in the M/M/1 queue system of the network.

In Chapter 5, we designed a network topology, specified traffic-engineering rules, assigned network capacity, designed traffic routes, and developed associated non-linear programming matrices.

In this chapter, we will synthesize the belief functions and game strategies of Chapter 3 with the M/M/1 optimum mean packet count of Chapter 4 using the network topology and traffic flows of Chapter 5. We will assign reservation price and service cost coefficient values in Sections 7.1 and 7.2. In Sections 7.3 and 7.4, respectively, we will analyze homogeneous and heterogeneous service-based markets.

7.1 *The Reservation Price*

We assume that the homogenous service-based market only supports Green service. Since it is easier to analyze results from a perspective of 100 percent, we assign a reservation price of $100 for Green service in the homogenous service-based market.

According to the network design of Chapter 5, the demand for Blue, Green, and Red services is 20%, 30%, and 40% of total physical capacity, respectively. These demands are 22.22%, 33.33%, and 44.44% of total market capacity. For heterogeneous service-based market, we assign reservation prices for Blue, Green, and Red services based on the percentage of market demand by the following equation:

$$(22.22\%)*160 + (33.33\%)*100 + (44.44\%)*70 = 100 \tag{7.1}$$

This equation ensures that the reservation prices of Blue and Red services are appropriately scaled with the market demand share of each service. The following table presents the reservation price for all these services as per equation (7.1).

Table 7.1: The Reservation price of different types of services

Blue = $160	Green=$100	Red = $70

7.2 *Service Cost Coefficient Values in Marginal Cost*

In Chapter 3, we developed marginal cost function in relation to service cost coefficients, providers' fixed costs, and the optimized mean packet count in networks. We also presented rationales for these cost parameters. In this section, we develop marginal cost as a function of network throughput and assign appropriate service cost coefficient values to Blue, Green, and Red services.

7.2.1 *Analytical Marginal Cost Function*

In Figure 7.1, the provider network connects to four regions. Network throughput is the total traffic entering or leaving a provider's network because of the assumed lossless nature. The following equation represents the throughput of this network:

$$Y_{n,t} = y_{n,t}^{Chicago} + y_{n,t}^{Dallas} + y_{n,t}^{Atlanta} + y_{n,t}^{Newyork} \tag{7.2}$$

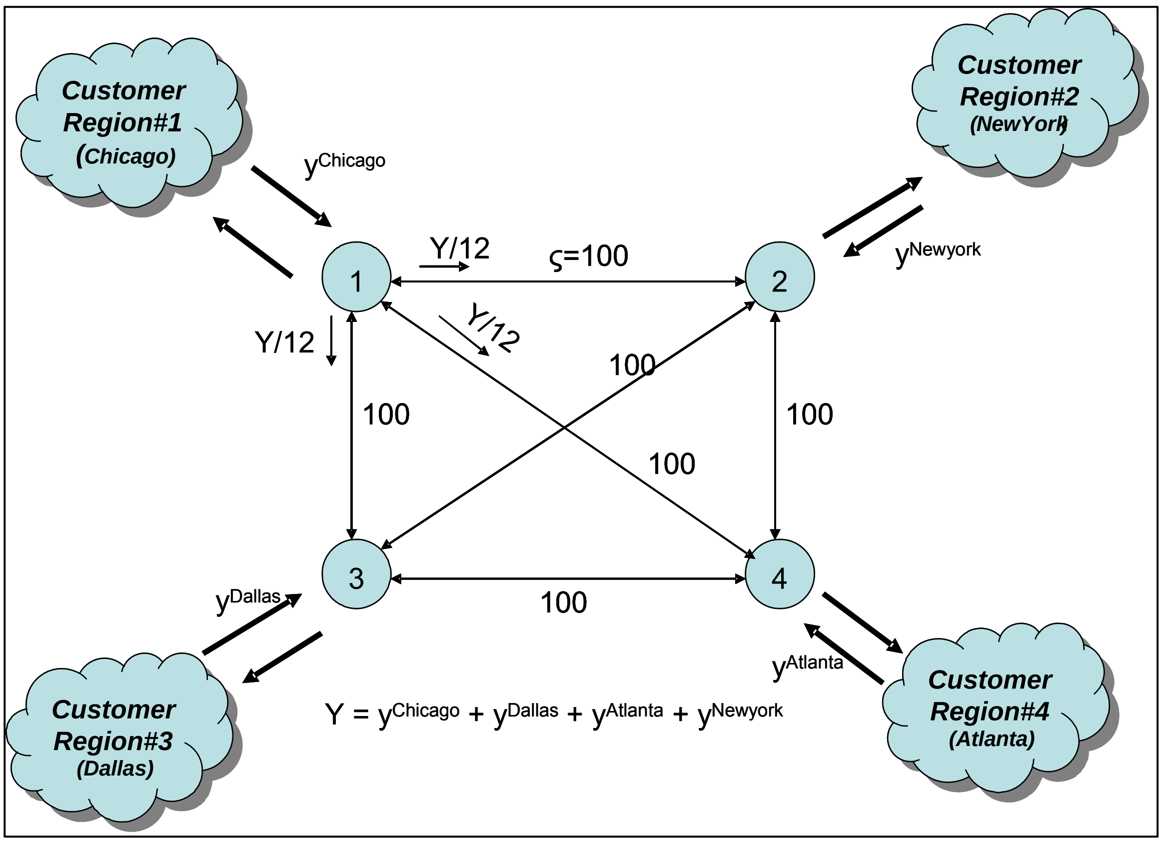


Figure 7.1: Uniform traffic flow across the network in optimized load

A large set of session level Monte-Carlo simulations verifies our assumption in this analysis that the traffic is equally load-balanced among network links by optimum routing during the steady state operating point. In Figure 7.1, the provider has 12 bidirectional links. When the traffic is equally load-balanced, each unidirectional link (l) will transport 1/12 of the total throughput of the network:

$$y_{l,t} = \frac{Y_{n,t}}{12} \qquad (7.3)$$

In Section 4.3, we developed an equation for the mean packet count in the network queue system. From equations (3.58) and (7.3), we derive the mean packet count in this network for the optimized link throughput.

$$\hat{M}^*_{n,t} = \sum_l \frac{\sum_{p:l\in p} x_p}{C_l - \sum_{p:l\in p} x_p} = \sum_{l=1}^{12} \frac{y_{l,t}}{C_l - y_{l,t}} = \frac{\frac{Y_{n,t}}{12}}{C - \frac{Y_{n,t}}{12}} + \ldots + \frac{\frac{Y_{n,t}}{12}}{C - \frac{Y_{n,t}}{12}} = \frac{Y_{n,t}}{C - \frac{Y_{n,t}}{12}} \qquad (7.4)$$

The change in the optimum mean packet count in the network due to the change in network throughput can be derived from equation (7.4) by considering the optimum mean packet count in the queue system as a continuous function of throughput (Y):

$$\frac{\partial \hat{M}^*_{n,t}}{\partial Y_{n,t}} = \frac{\partial}{\partial Y_{n,t}} \left(\frac{Y_{n,t}}{C - \frac{Y_{n,t}}{12}} \right)$$

$$= \frac{\left(C - \frac{Y_{n,t}}{12}\right) \frac{\partial}{\partial Y_{n,t}} Y_{n,t} - Y_{n,t} \frac{\partial}{\partial Y_{n,t}} \left(C - \frac{Y_{n,t}}{12}\right)}{\left(C - \frac{Y_{n,t}}{12}\right)^2}$$

$$= \frac{\left(C - \frac{Y_{n,t}}{12}\right).1 - Y_{n,t}\left(0 - \frac{1}{12}\right)}{\left(C - \frac{Y_{n,t}}{12}\right)^2}$$

$$= \frac{C}{(C - \frac{1}{12} Y_{n,t})^2} \qquad (7.5)$$

Below, we rewrite the marginal cost equation (3.17) for reference:

$$\omega_{n,s,t}(\hat{M}_{n,t}) = \delta_s (Y_{n,t} \frac{\partial \hat{M}_{n,t}}{\partial Y_{n,t}} + \hat{M}_{n,t}) + \theta_n$$

Equations (7.5) and (3.17) yield the following analytical marginal cost equation:

$$\omega_{n,s,t}(\hat{M}_{n,t}) = \omega_{n,s,t}(Y_{n,t}) = \delta_s (Y_{n,t} \frac{C}{(C - \frac{1}{12} Y_{n,t})^2} + \frac{Y_{n,t}}{C - \frac{Y_{n,t}}{12}}) + \theta_n$$
$$= \delta_s (Y_{n,t} \frac{2CY_{n,t} - \frac{Y_{n,t}^2}{12}}{(C - \frac{1}{12} Y_{n,t})^2}) + \theta_n \qquad (7.6)$$

7.2.2 *Simulated Marginal Cost Function*

In our research, the throughput (Y) is the total amount of traffic served by the network per unit of time. It is the sum of the egress traffic ($Y_{n,t}$) towards the enterprises. Change in optimum packet count is measured for future—if the requested bid is successful, traffic will be added when the session is activated. Thus, change in optimum packet is approximated as follows:

$$\frac{\partial \hat{M}^*_{n,t+1}}{\partial Y_{n,t+1}} \approx \frac{\hat{M}^*_{n,t+1} - \hat{M}^*_{n,t}}{Y_{n,t+1} - Y_{n,t}} \qquad (7.7)$$

Each session consists of bi-directional connections (O-D and D-O). When a session is activated or deactivated, the change in production is the sum of the sessions' bandwidths in both directions. In the simulation, for each session request, we compute $(\hat{M}^*_{n,t+1} - \hat{M}^*_{n,t})$. We then compute $\frac{\partial \hat{M}^*_{n,t}}{\partial Y_{n,t}}$ as $(\hat{M}^*_{n,t+1} - \hat{M}^*_{n,t})$ divided by the sum of the session's bandwidth (b) in both directions.

$$\frac{\partial \hat{M}^*_{n,t+1}}{\partial Y_{n,t+1}} \approx \frac{\hat{M}^*_{n,t+1} - \hat{M}^*_{n,t}}{(b_{OD} + b_{DO})} \qquad (7.8)$$

This use of this near sighted one-step history makes our game a myopic Markovian-Bayesian game.

The following is the simulation marginal cost equation:

$$\omega_{n,s,t}(\hat{M}^*_{n,t}) = \delta_s (Y_{n,t} \frac{\hat{M}^*_{n,t+1} - \hat{M}^*_{n,t}}{(b_{OD} + b_{DO})} + \hat{M}^*_{n,t}) + \theta_n \qquad (7.9)$$

7.2.3 *Service Cost Coefficient Values*

In Section 3.3.2, we described the rationale of having a unique service cost coefficient for each service class. We also noted that the service cost coefficient reflects the cost of security technology used to provide a service. The higher the security level required, the higher the processing cost for enforcing deep packet inspection. The higher the traffic load, the greater the time required for the deep packet inspection.

We have no service cost coefficient values at this time. In this section, we assign service cost coefficient values to Blue, Green, and Red services based on the following discussion:

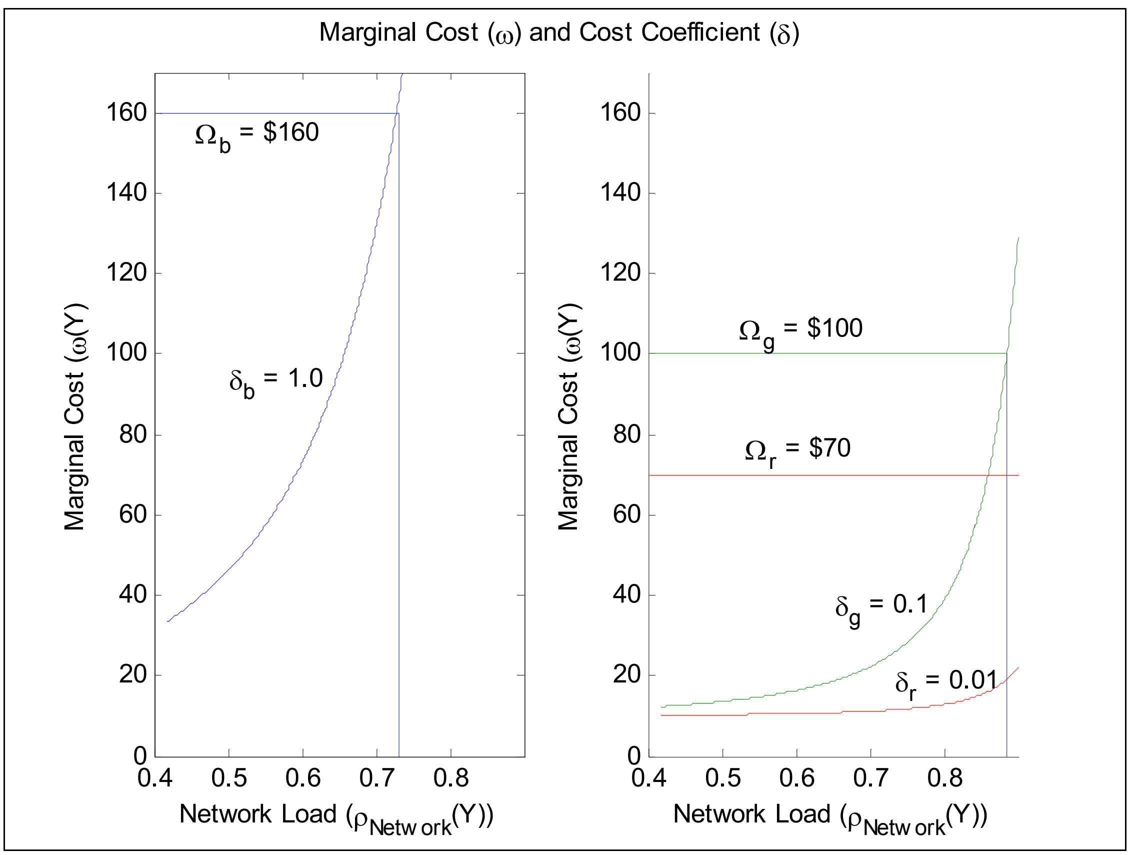


Figure 7.2: Marginal Cost as a Function of Service Cost Coefficient and Network Load

Figure 7.2 depicts the marginal cost as a function of network throughput for service cost coefficient values. The marginal cost is plotted against network load.

The Blue service provides the highest level of security. Thus, the cost of technology used for Blue is likely to be very high. We assume that the Blue service will exact a high penalty for operating in a high network load. This is because the higher the network load, the more the delay is added to application (e.g. VoIP) packets during deep packet inspection to enforce a high level of security. In other words, this is a result of the greater delay accompanying the greater share of Blue network load. We will emulate this penalty by having a high Blue cost coefficient value such that the marginal cost exceeds the reservation price at a certain network load. In Figure 7.2, the left plot shows that for a service cost coefficient (δ_m) of 1.0, the marginal cost exceeds Blue reservation Price ($\Omega_b = 160$) at around 73% of network load. We select Blue service cost coefficient (δ_b) of 1.0 because it is a unit number and it emulates providers' penalty at a reasonable load around 70%[11].

Green service provides the medium level of security. We want to select a Green cost coefficient suitably scaled down from the Blue cost coefficient. We assume that the Green service will cause minimal penalty for operating in a high network load. At one-tenth of a Blue cost coefficient, a marginal cost causes minimal impact to the Green service because as shown in Figure 7.2 right-hand plot, the Green marginal cost exceeds the Green reservation price ($\Omega_g = 100$) only above 88.5% of network load for $\delta_g = 0.10$. Therefore, we select a Green cost coefficient (δ_g) of 0.10.

We assume that the lowest security requiring Red service will not cause any penalty for operating in a high network load. As Green cost coefficient is 1/10th of the Blue cost coefficient, we scale Red cost coefficient to 1/10th of the Green cost coefficient. Figure 7.2, right-hand plot, illustrates that at $\delta_r = 0.01$, the marginal cost always remains well below the Red reservation price ($\Omega_r = 70$); thus, a provider does not pay any penalty for operating in high load.

[11] The magic number 70% is often used as a safe operating load for the Internet because of its wide acceptance in PSTN network based on M/M/1 queuing theory.

Table 7.2: The Service Cost Coefficient values

Class of Service	Service Cost Coefficient
Blue (b)	$\delta_b = 1.00$
Green (g)	$\delta_g = 0.10$
Red (r)	$\delta_r = 0.01$

Note that the major part of this research is a comparative study of two different providers' strategies. Since both providers use same service cost coefficient set, a service cost coefficient value does not influence the comparative results of providers' strategies.

7.3 *Homogeneous Service-based Market*

In this section, we develop analytical models of market price, providers' bid price, and providers' profit for homogeneous service-based market and validate analytical results with by means of session level Monte-Carlo simulation.

Section 7.3.1 concentrates on a market where both providers adopt the *Rejection Neutral* strategy. In Section 7.3.2, we develop an analytical model of general market price function for all strategies. We present analyses of providers' profit and throughputs when both adopt identical strategies in Section 7.3.3 and non-identical strategies in Section 7.3.4.

A strategy set that optimizes all providers' profit is the best strategy. As stated in Chapter 1, the Bayesian-Nash equilibrium strategy set represents such a strategy set. According to [2], a strategy set is Pareto efficient if it is impossible to improve a providers' profit without harming another provider. In section 7.3.5, we will explore an analytical method to find the Bayesian Nash equilibrium and the Pareto efficient outcome strategy set.

In Chapter 5, we discussed the parameters of this research in detail. We summarize the main parameters of both the simulation and the analytical study of Section 7.3 in Table 7.3 for reference.

Table 7.3: Parameters for homogeneous service-based network

The Class of Service	Homogeneous: Green
Market	Duopoly
Strategy	Strategy set of Figure 7.3
Network Topology and TE Rules	The topology and Rules of Chapter 5
Reservation Price (Ω)	\$100.00
The service cost coefficient (δ_s)	0.10
Provider fixed cost coefficient (θ)	10.0

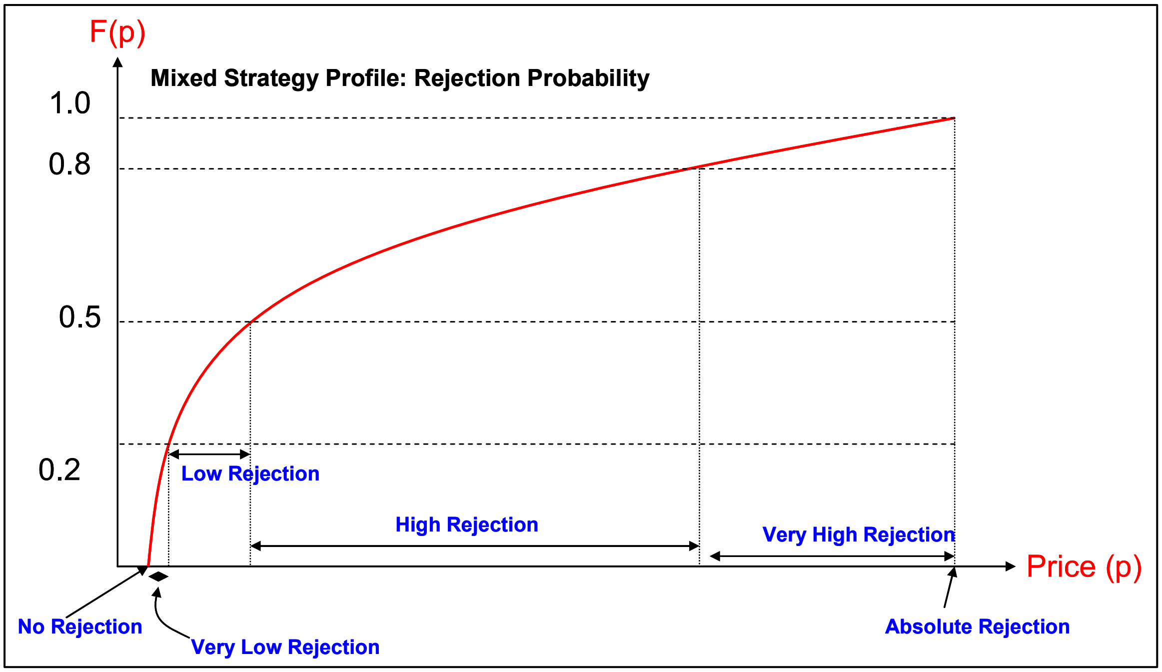


Figure 7.3: Strategy set of experiments

7.3.1 *Study of the Rejection Neutral Strategy Set*

The objective of this section is to develop a mathematical model of the market price—when both providers adopt the *Rejection Neutral* (RN) strategy—and, to measure providers' profit and throughput. Another objective is to determine a desired load that optimizes a provider's profit using the *Rejection Neutral* strategy.

Let us assume that the strategy set adopted by A.com and B.com is as follows such that $\gamma_A^j = \gamma_A^k = 0.5$:

Provider	*Strategy*	*Rejection Probability* (γ)
A.com	$strategy\ h_{Aj} : p_{A,g,t}^{bid} \longleftarrow (F_{A,g,t}(p \le p_{A,g,t}^{bid}) = \gamma_{A,g}^{j})$	$\gamma_A^j = 0.5$
B.com	$strategy\ h_{Bk} : p_{B,g,t}^{bid} \longleftarrow (F_{B,g,t}(p \le p_{B,g,t}^{bid}) = \gamma_{B,g}^{k})$	$\gamma_B^k = 0.5$

Assume at a steady state market demand (Δ^*), throughput of A and B are Y_A and Y_B. Since they adopt the same strategy, we expect that both will enjoy a fair share of profit and throughput:

$$\begin{aligned} Y_A^* &= Y_B^* = Y^* \\ \Delta^* &= 2Y^* \\ u_A^*(.) &= u_B^*(.) \end{aligned} \qquad (7.10)$$

The belief function of equation (3.36) for a duopoly (n = 2) market is as follows:

$$\begin{aligned} F_{n,s,t}(p_{n,s,t}) &= \left[\frac{\left\{p_{n,s,t} - \omega_{n,s,t}(M_{n,t}^*)\right\} \rho_{TE} K - \left\{\Delta(Y_{n,t}^*) - \rho_{TE} K\right\} \left\{\Omega_s - \omega_{n,s,t}(M_{n,t}^*)\right\}}{\left\{p_{n,s,t} - \omega_{n,s,t}(M_{n,t}^*)\right\} \left\{2\rho_{TE} K - \Delta(Y_{n,t}^*)\right\}} \right] \\ &= \frac{1}{\left\{2\rho_{TE} K - \Delta(Y_{n,t}^*)\right\}} \left[\frac{\left\{p_{n,s,t} - \omega_{n,s,t}(M_{n,t}^*)\right\} \rho_{TE} K}{\left\{p_{n,s,t} - \omega_{n,s,t}(M_{n,t}^*)\right\}} - \frac{\left\{\Delta(Y_{n,t}^*) - \rho_{TE} K\right\} \left\{\Omega_s - \omega_{n,s,t}(M_{n,t}^*)\right\}}{\left\{p_{n,s,t} - \omega_{n,s,t}(M_{n,t}^*)\right\}} \right] \\ &= \frac{1}{\left\{2\rho_{TE} K - \Delta(Y_{n,t}^*)\right\}} \left[\rho_{TE} K - \frac{\left\{\Delta(Y_{n,t}^*) - \rho_{TE} K\right\} \left\{\Omega_s - \omega_{n,s,t}(M_{n,t}^*)\right\}}{\left\{p_{n,s,t} - \omega_{n,s,t}(M_{n,t}^*)\right\}} \right] \end{aligned} \qquad (7.11)$$

Here, p is a price of a service, $\omega(.)$ is the marginal cost function of a provider, $\Delta(.)$ is the provider's market demand function, M_n* is the optimum mean packet count in the network, Y is the provider throughput or production, and ρ_{TE} is the traffic-engineered load.

As described in Chapter 3, $F(p)$ is a continuous function of price. Thus, the probability density function of the mixed strategy profile is obtained by differentiating (7.11) with respect to p and performing algebra as follows:

$$\begin{aligned} f_{n,s,t}(p_{n,s,t}) &= \frac{\partial F_{n,s,t}(p_{n,s,t})}{\partial p_{n,s,t}} \\ & \frac{1}{\left\{2\rho_{TE} K - \Delta(Y_{n,t}^*)\right\}} \left[\frac{\partial \rho_{TE} K}{\partial p_{n,s,t}} - \frac{\partial}{\partial p_{n,s,t}} \left(\frac{\left\{\Delta(Y_{n,t}^*) - \rho_{TE} K\right\} \left\{\Omega_s - \omega_{n,s,t}(M_{n,t}^*)\right\}}{\left\{p_{n,s,t} - \omega_{n,s,t}(M_{n,t}^*)\right\}} \right) \right] \\ &= \frac{1}{\left\{2\rho_{TE} K - \Delta(Y_{n,t}^*)\right\}} \left[\frac{\left\{\Delta(Y_{n,t}^*) - \rho_{TE} K\right\} \left\{\Omega_s - \omega_{n,s,t}(M_{n,t}^*)\right\}}{\left\{p_{n,s,t} - \omega_{n,s,t}(M_{n,t}^*)\right\}^2} \right] \\ &= \frac{(\Delta(Y_{n,t}^*) - \rho_{TE} K)(\Omega_s - \omega_{n,s,t}(M_{n,t}^*))}{(p_{n,s,t} - \omega_{n,s,t}(M_{n,t}^*))^2 (2\rho_{TE} K - \Delta(Y_{n,t}^*))} \end{aligned} \qquad (7.12)$$

The *No Rejection* strategy price (p_{Min}) of a provider is the lower bound price. The *Absolute Rejection* strategy price, or the reservation price (*Ω)*, is the upper bound price. This implies that the price of a service (s) at an instant of time (t) is bounded by p_{Min} and Ω:

$$p_{n,s,t} \in [p_{Min,n,s,t}, \Omega_s] \tag{7.13}$$

From equations (7.12)-(7.13), the mean price of a service ($\overline{p}_{n,s,t}$) is the *Rejection Neutral* strategy price and it is derived by the following equations:

$$\overline{p}_{n,s,t} = \int_{p_{Min,n,s,t}}^{\Omega_s} p_{n,s,t} f_{n,s,t}(p_{n,s,t}) dp_{n,s,t} \tag{7.14}$$

$$\overline{p}_{n,s,t} = \int_{p_{Min,n,s,t}}^{\Omega_s} p_{n,s,t} \frac{(\Delta(Y_{n,t}^*) - \rho_{TE}K)(\Omega_s - \omega_{n,s,t}(M_{n,t}^*))}{(p_{n,s,t} - \omega_{n,s,t}(M_{n,t}^*))^2 (2\rho_{TE}K - \Delta(Y_{n,t}^*))} dp_{n,s,t} \tag{7.15}$$

$$\overline{p}_{n,s,t} = \frac{(\Delta(Y_{n,t}^*) - \rho_{TE}K)(\Omega_s - \omega_{n,s,t}(M_{n,t}^*))}{(2\rho_{TE}K - \Delta(Y_{n,t}^*))} \int_{p_{Min,n,s,t}}^{\Omega_s} \frac{p_{n,s,t}}{(p_{n,s,t} - \omega_{n,s,t}(M_{n,t}^*))^2} dp_{n,s,t} \tag{7.16}$$

$$\begin{aligned} \overline{p}_{n,s,t} &= \frac{(\Delta(Y_{n,t}^*) - \rho_{TE}K)(\Omega_s - \omega_{n,s,t}(M_{n,t}^*))}{(2\rho_{TE}K - \Delta(Y_{n,t}^*))} \int_{Z=p_{Min,n,s,t}-\omega(.)}^{Z=\Omega_s-\omega(.)} \frac{Z+\omega(.)}{Z^2} dZ \\ &= \frac{(\Delta(Y_{n,t}^*) - \rho_{TE}K)(\Omega_s - \omega_{n,s,t}(M_{n,t}^*))}{(2\rho_{TE}K - \Delta(Y_{n,t}^*))} \int_{Z=p_{Min,n,s,t}-\omega(.)}^{Z=\Omega_s-\omega(.)} \left[\frac{1}{Z} + \frac{\omega(.)}{Z^2}\right] dZ \end{aligned} \tag{7.17}$$

$$\overline{p}_{n,s,t} = \frac{(\Delta(Y_{n,t}^*) - \rho_{TE}K)(\Omega_s - \omega_{n,s,t}(M_{n,t}^*))}{(2\rho_{TE}K - \Delta(Y_{n,t}^*))} \left[\begin{aligned} &\ln\left(\frac{\Omega_s - \omega_{n,s,t}(M_{n,t}^*)}{p_{Min,n,s,t} - \omega_{n,s,t}(M_{n,t}^*)}\right) \\ &+\omega_{n,s,t}(M_{n,t}^*)\left(\frac{1}{p_{Min,n,s,t} - \omega_{n,s,t}(M_{n,t}^*)} - \frac{1}{\Omega_s - \omega_{n,s,t}(M_{n,t}^*)}\right) \end{aligned} \right] \tag{7.18}$$

For the network topology described in Chapter 5 and presented in Figure 7.1, the market physical capacity (K) is represented as a function of the link capacity in equation (7.19). Here, all the network links are bi-directional and have equal physical capacity (C).

$$K = 12C \tag{7.19}$$

In Chapter 4, we described the optimum mean packet count in the network as a function of a provider's network throughput (Y) at each instant of time.

$$M_{n,t}^* \leftarrow f(Y_{n,t}) \tag{7.20}$$

Equations (7.18) - (7.20) yield the following:

$$\overline{p}_{n,s,t} = \frac{(\Delta(Y_{n,t}^{*}) - 12\rho_{TE}C)(\Omega_s - \omega_{n,s,t}(Y_{n,t}^{*}))}{(24\rho_{TE}C - \Delta(Y_{n,t}^{*}))}\left[\begin{array}{l} \ln\left(\dfrac{\Omega_s - \omega_{n,s,t}(Y_{n,t}^{*})}{p_{Min,n,s,t} - \omega_{n,s,t}(Y_{n,t}^{*})}\right) \\ +\omega_{n,s,t}(Y_{n,t}^{*})\left(\dfrac{1}{p_{Min,n,s,t} - \omega_{n,s,t}(Y_{n,t}^{*})} - \dfrac{1}{\Omega_s - \omega_{n,s,t}(Y_{n,t}^{*})}\right) \end{array}\right]$$

(7.21)

For Green service, by denoting the service cost coefficient (δ_s) of Green as $\frac{1}{10}$ and a provider's fixed cost coefficient as 10, the marginal cost function of the network as per equation (7.6) can be represented as follows:

$$\omega_{n,g,t}(Y_{n,t}) = 0.10\frac{200Y_{n,t} - \frac{Y_{n,t}^2}{12}}{(100 - \frac{Y_{n,t}}{12})^2} + 10 \qquad (7.22)$$

This equation (7.22) represents the marginal cost function of the mean price equation (7.18).

A provider needs to estimate the market demand function $\Delta(Y)$ to compute the Nash equilibrium price of service. The rationale for the following market demand function was presented in Chapter 3. Applying the Traffic Engineering Rule of Chapter 5 to equation (3.13) for a duopoly market yields the following network demand function:

$$\Delta(Y_t) = \begin{cases} \rho_{TE}K + \varepsilon & 2Y_t \le \rho_{TE}K, \varepsilon > 0 \\ 2Y_t & \rho_{TE}K < 2Y_t \le \Delta_{Max} \end{cases} = \begin{cases} (0.90)(1200) + 1 & 2Y_t \le (0.90)(1200) \\ 2Y_t & (0.90)(1200) < 2Y_t \le \Delta_{Max} \end{cases} \qquad (7.23)$$

In this analysis, we consider Δ_{Max} as follows:

$$\Delta_{Max} = 1.90\rho_{TE}K \qquad (7.24)$$

Here 1.90 represents the market demand when the lowest price provider sells 100% of its market capacity and the other provider sells 90% of its market capacity. This is equivalent to 90% of the physical capacity of the lowest priced provider and 81% of the physical capacity of the higher priced provider.

We increase demand from $\rho_{TE}K = (0.90)(1200)$ Mbps to $\Delta_{Max} = (1.90)(0.90)(1200)$ Mbps to compute the market price of Green service and the providers' marginal cost, unit profit, and network loads. The *network load* of a provider at an instant of time is the ratio of the throughput and the physical capacity (K_n) of the provider.

$$\rho_{Network,n,t} = \frac{Y_{n,t}}{K_n} = \frac{Y_{n,t}}{12C} = \frac{Y_{n,t}}{1200} \tag{7.25}$$

Note that a provider cannot change the market demand; however, it can change its network load by changing its strategy. In this section, we do not change the strategy; thus, network load is a linear function of market demand. When both providers adopt the same strategy, both of them will enjoy fair profit shares and fair throughput shares. Thus, analysis of one provider is sufficient. By using the above equations, we sketch the analytical results for a provider that adopts the *Rejection Neutral* strategy for a homogeneous market (Green: $\Omega_g = 100$) in Figure 7.4.

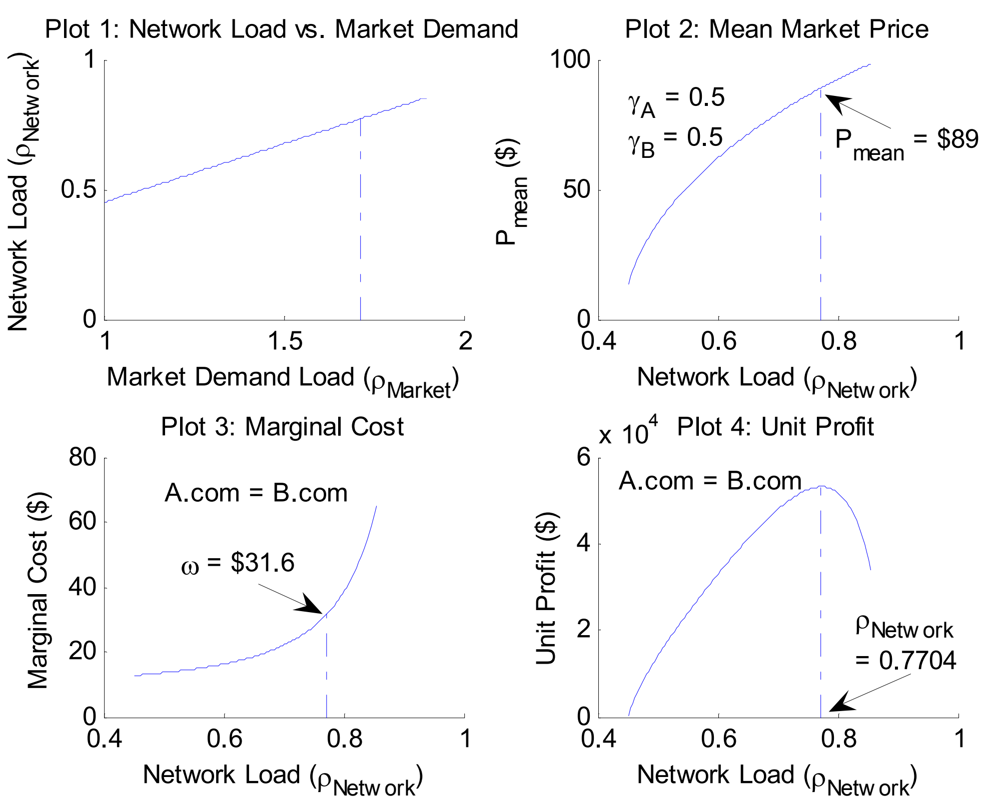


Figure 7.4: Analytical Result for *Rejection Neutral* Strategy (Homogeneous Service)

In Figure 7.4, Plot 1 illustrates the increase in the network load due to the increase in market demand as per equations (7.23)-(7.25). Market demand increases as the multiplicative (1.00 to 1.90) of a provider's market capacity as shown on the x-axis. (Note, here we use a very high market demand load to observe clearly the concavity of the profit function). Plot 2 depicts the analytical *Rejection Neutral* price. The mean price logarithmically increases to the customer's reservation price as load increases. The analytical marginal cost function depicted in Plot 3 increases exponentially. At a high load, the marginal cost increases rapidly, and so does the price of a service.

Proposition: As network utilization converges to 100%, the price approaches infinity.

$$\rho \longrightarrow 1.0 \Rightarrow p^*_{Mean} \longrightarrow \infty$$

Proof:

As network utilization converges to 100%,

$$(C - \frac{Y^*}{12}) \xrightarrow{\rho=1.0} 0 \Rightarrow M^* \longrightarrow \infty \Rightarrow \omega(Y^*) \longrightarrow \infty \Rightarrow p^*_{mean} \longrightarrow \infty \qquad \square$$

Because price approaches infinity when network load converges to 1.0, we prevent network load from converging to 100% of network capacity by implementing Call Admission Control (CAC) and capacity constraint in optimized routing that enforced Traffic Engineering Rules of Chapter 5.

Chapter 3 defines the unit profit as follows: The *unit profit* of a provider is the profit per unit duration (e.g. one second) measured at an instant of the steady state throughput ($\hat{Y}$) when the bid price and the marginal cost of the provider converge to $\hat{p}$ and $\hat{\omega}$.

$$u(p) = (\hat{p} - \hat{\omega})\hat{Y} \qquad (7.26)$$

From the above, we compute the *unit profit* as follows, where $\overline{p}_t$ represents equation (7.18).

$$u_t(p) = (\overline{p}_t - \omega_t)Y_t = (\overline{p}_t - 0.10\frac{200Y_{n,t} - \frac{Y_t^2}{12}}{(100 - \frac{Y_t}{12})^2} + 10)Y_t \qquad (7.27)$$

In Figure 7.4, the Plot 4 illustrates the unit profit of a provider with respect to the increase in traffic demand. This plot exhibits all three main properties of a profit function:

i) it monotonically increases with the throughput to a maximum point.

ii) it is bound because the profit cannot be increased beyond the providers' load of 0.7704.

iii) it is concave because the cost of producing a service increases in high throughput causing diminishing return. The following is true for network load (ρ_n):

$$u(\psi\rho_{n,1} + (1-\psi)\rho_{n,2}) \geq \psi u(\rho_{n,1}) + (1-\psi)u(\rho_{n,2}), \quad \psi \in [0,1] \qquad (7.28)$$

The increase in the load increases the market demand and the marginal cost to provide the service; thus, the provider's price of service increases. The price increases faster than the marginal cost up to a load of 0.7704. Beyond this load, the rate of marginal cost increase is faster than that of price. Therefore, profit diminishes beyond 0.7704. In this load, the provider perceives that the market demand is equal to $1.712\rho K$.

From these results, we observe that for the network scenario of Chapter 5 and parameters of this chapter, a provider should maintain a load of 0.7704 to optimize profit. Table 7.4 summarizes the analytical optimum values.

Table 7.4: Analytical Result (Homogeneous Service Market)

Market Demand	1.712ρK
Network Load	0.7704
Price	89.0357
Marginal Cost	31.6
Unit Profit	5.31e4

Figure 7.5 compares the analytical and session level Monte-Carlo simulation results of A.com. The asterisks represent the simulation results. The curved lines represent the analytical results. Note that these analytical results also represent B.com. The differences in the simulation results are not significant enough to plot for both A.com and B.com.

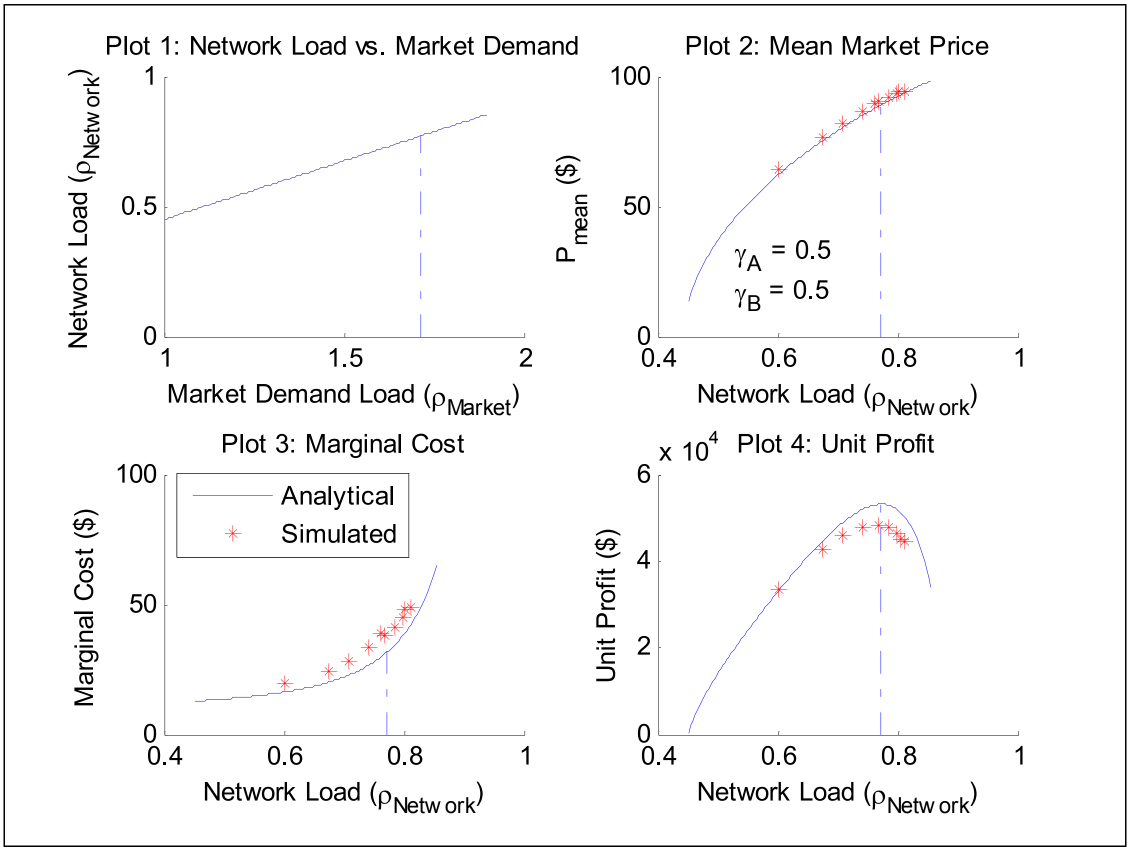


Figure 7.5: A.com: Analytical vs. Simulated Results ({A.com RN, B.com RN})

The figure shows that the simulated mean price and the analytical mean price were similar. The simulated marginal cost was slightly higher than the analytical marginal cost and the simulated profit was slightly lower than the analytical unity profit. Although we applied the same parameters and assumptions for both analytical and simulation models, the simulation model was subjected to the oscillatory traffic load due to the SIP call arrivals and departures. As described in Chapter 3, the marginal cost is a function of the optimum mean packet count and the change in mean packet count in the network queue system. Due to the high oscillation in the traffic load in simulation, the marginal cost was higher in simulation than the analytical model. Thus, the profit curve of the simulation model was slightly lower. Note that in the above simulation plots, we illustrate the mean of the oscillatory profit and marginal cost; we do not indicate their variances. Nevertheless, the simulation and analytical results are close to each other. Since these simulation results were approximations of the analytical results, the proposed model and the implementation in MATLAB were verified.

7.3.2 General Equation of Bid Price for All Strategies

In this section, we develop a bid price function that can be used to determine market price for any strategy for a homogenous service-based network. Assume in a game instant (*t*), if a provider (*n*) selects a bid ($p_{n,s,t}^{bid}$) for a class of service (*s*), the rejection probability is γ. In the context of a belief function, this rejection probability can be stated by the following equation:

$$\gamma = \int_{p_{Min,n,s,t}}^{p_{n,s,t}^{bid}} f_{n,s,t}(p_{n,s,t})dp_{n,s,t} \tag{7.29}$$

The rejection probability can be found as a function of the parameters of the proposed model though the following equations:

$$\gamma_{n,s} = \int_{p_{Min,n,s,t}}^{p_{n,s,t}^{bid}} \frac{(\Delta(Y_{n,t}^{*}) - \rho_{TE}K)(\Omega_s - \omega_{n,s,t}(M_{n,t}^{*}))}{(p_{n,s,t} - \omega_{n,s,t}(M_{n,t}^{*}))^2(2\rho_{TE}K - \Delta(Y_{n,t}^{*}))} dp_{n,s,t} \tag{7.30}$$

$$\gamma_{n,s} = \frac{(\Delta(Y_{n,t}^{*}) - \rho_{TE}K)(\Omega_s - \omega_{n,s,t}(M_{n,t}^{*}))}{(2\rho_{TE}K - \Delta(Y_{n,t}^{*}))} \int_{p_{Min,n,s,t}}^{p_{n,s,t}^{bid}} \frac{1}{(p_{n,s,t} - \omega_{n,s,t}(M_{n,t}^{*}))^2} dp_{n,s,t} \tag{7.31}$$

$$\gamma_{n,s} = \frac{(\Delta(Y_{n,t}^{*}) - \rho_{TE}K)(\Omega_s - \omega_{n,s,t}(M_{n,t}^{*}))}{(2\rho_{TE}K - \Delta(Y_{n,t}^{*}))} \int_{Z=p_{Min,n,s,t}-\omega(.)}^{Z=p_{n,s,t}^{bid}-\omega(.)} \frac{1}{Z^2} dZ \tag{7.32}$$

$$\gamma_{n,s} = \frac{(\Delta(Y_{n,t}^{*}) - \rho_{TE}K)(\Omega_s - \omega_{n,s,t}(M_{n,t}^{*}))}{(2\rho_{TE}K - \Delta(Y_{n,t}^{*}))} \left[\left(\frac{1}{p_{Min,n,s,t} - \omega_{n,s,t}(M_{n,t}^{*})} - \frac{1}{p_{n,s,t}^{bid} - \omega_{n,s,t}(M_{n,t}^{*})} \right) \right] \tag{7.33}$$

By algebraic manipulation, we find the bid price equation as follows:

$$p^{bid}_{n,s,t} = \omega_{n,s,t}(M^*_{n,t}) + \left[\frac{1}{\left(p_{Min,n,s,t} - \omega_{n,s,t}(M^*_{n,t})\right)} - \frac{\gamma_{n,s}}{\left(\frac{(\Delta(Y^*_{n,t}) - \rho_{TE}K)(\Omega_s - \omega_{n,s,t}(M^*_{n,t}))}{(2\rho_{TE}K - \Delta(Y^*_{n,t}))} \right)} \right]^{-1} \quad (7.34)$$

The intersection of two bid price functions of two providers for a market demand is the steady state market price. Thus, this bid price function allows a provider to determine the market price function and expected profit for a set of strategies.

7.3.3 Study of Identical Strategies

In this section, we analytically determine the market price, the marginal cost of a provider, profit curve of providers in different network load, and the optimum throughput of network when providers adopt identical strategies. Let us assume that the strategy set adopted by A.com and B.com is as follows such that $\gamma^j_A = \gamma^k_A$:

Provider	*Strategy*	*Rejection Probability (γ)*
A.com	$\text{strategy } h_{Aj} : p^{bid}_{A,g,t} \longleftarrow (F_{A,g,t}(p \le p^{bid}_{A,g,t}) = \gamma^j_{A,g})$	γ^j_A
B.com	$\text{strategy } h_{Bk} : p^{bid}_{B,g,t} \longleftarrow (F_{B,g,t}(p \le p^{bid}_{B,g,t}) = \gamma^k_{B,g})$	γ^k_B

Assume at a steady state market demand (Δ^*), throughput of A and B are Y_A and Y_B. At the steady state, the bid prices of A.com and B.com converge at the steady state market price ($p^*_{Market,s,t}$). This price can be found by solving bid price functions of A.com and B.com.

$$p^*_{Market,s,t} = p^{bid}_{A,s,t}(Y_{A,t}) = p^{bid}_{B,s,t}(Y_{B,t}) \quad (7.35)$$

Since they adopt the same strategy, we expect that both will enjoy a fair share of profit and throughput:

$$
\begin{aligned}
&Y_A^* = Y_B^* = Y^* \\
&\Delta^* = 2Y^* \\
&u_A^*(.) = u_B^*(.)
\end{aligned}
\tag{7.36}
$$

Figure 7.6 and Figure 7.7 compare analytical and simulated market price, marginal cost and profit of A.com when both A.com and B.com adopt the *Very High Rejection* (VHR) strategy (Figure 7.6) or the *Very Low Rejection* (VLR) strategy (Figure 7.7). The results for B.com are nearly identical to those of A.com; thus, these analytical results also represent B.com. The differences in the simulation results are not significant enough to plot for both A.com and B.com

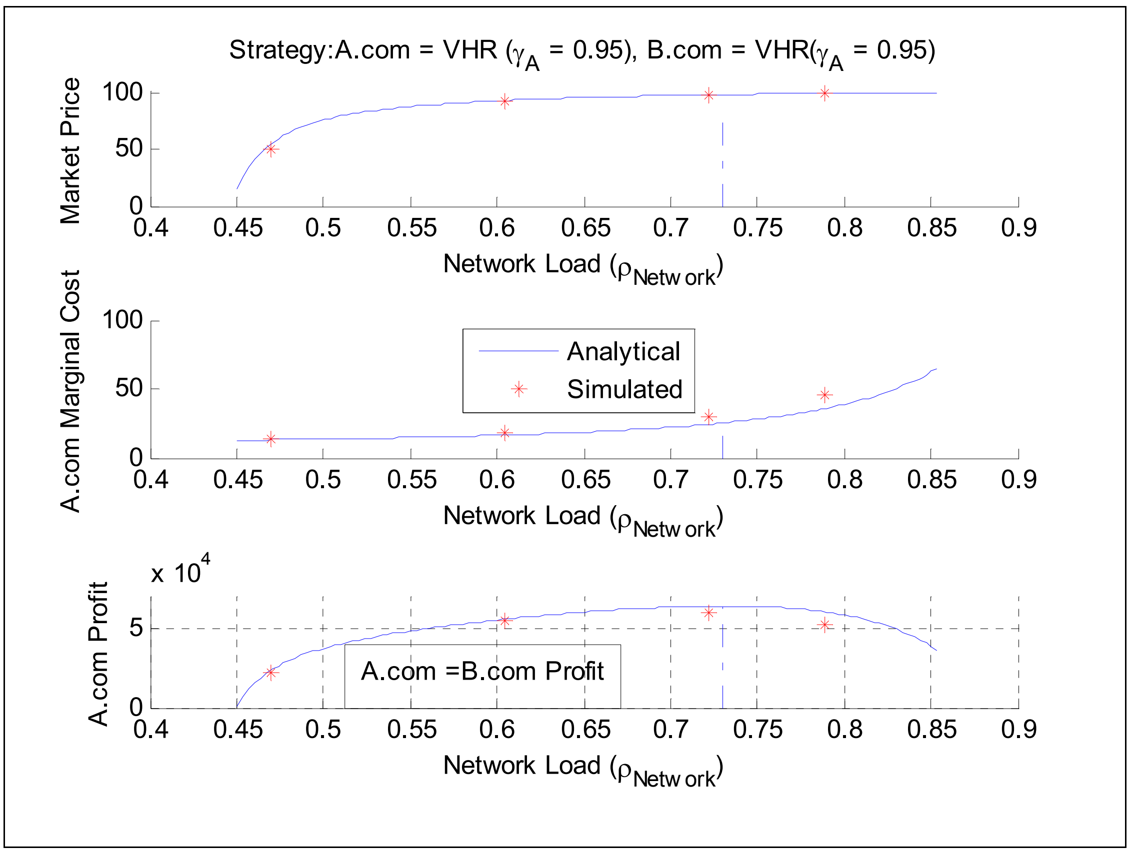


Figure 7.6: Analytical vs. Simulated Results ({A.com VHR, B.com VHR})

In both figures, the upper plot compares the analytical market price (equations (7.34) and (7.35)) with the simulated market price. The center plot compares a provider's analytical marginal cost (equation (7.22)) with A.com's simulated mean steady state marginal cost. The lower plot compares analytical unit profit (equation (7.27)) with the simulated mean steady state unit profit of A.com. The analytical and simulation price, marginal cost, and profit are close to each other. Thus, simulation results verify analytical results.

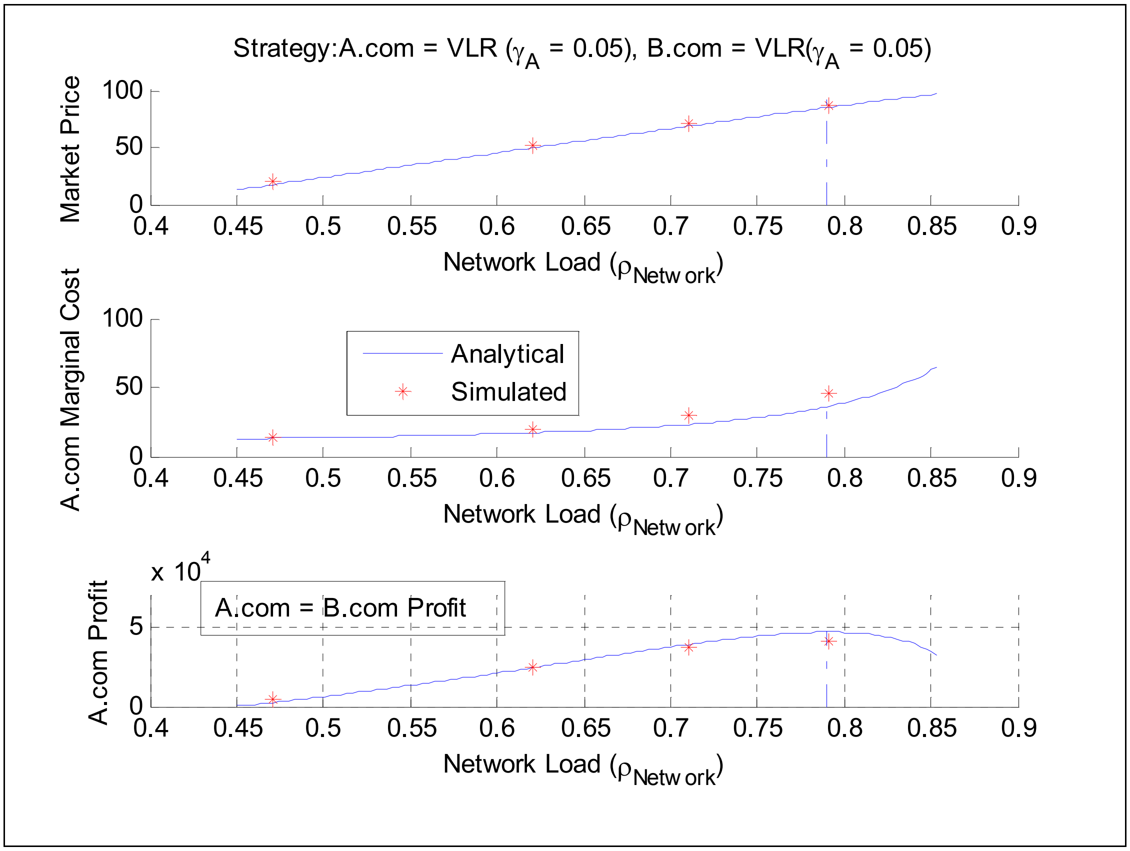


Figure 7.7: Analytical vs. Simulated Results (Strategy: {A.com VLR, B.com VLR})

Comparison of Figure 7.6 and Figure 7.7 shows that both providers achieved higher profit for adopting the *Very High Rejection* (VHR) strategy set than that of the *Very Low Rejection* (VLR) strategy set. This is because the *Very High Rejection* strategy set drove the market price higher than that of the *Very Low Rejection* strategy set; however, marginal costs in both cases remain close to each other in identical loads. Like the *Rejection Neutral* strategies, the optimum network loads were around 0.74 ~0.77 in the *Very High Rejection* and the *Very Low Rejection* strategy sets.

7.3.4 *Study of Non-Identical Strategy Set*

In this section, we analytically determine the market price, the marginal cost of a provider, profit curve of providers in different network load, and the optimum throughput of network when providers adopt non-identical strategies. Let us assume that the strategy set adopted by A.com and B.com is as follows such that $\gamma_A^j \neq \gamma_A^k$:

Provider	*Strategy*	*Rejection Probability (γ)*
A.com	$strategy\ h_{Aj} : p^{bid}_{A,g,t} \longleftarrow (F_{A,g,t}(p \leq p^{bid}_{A,g,t}) = \gamma^j_{A,g})$	γ_A^j
B.com	$strategy\ h_{Bk} : p^{bid}_{B,g,t} \longleftarrow (F_{B,g,t}(p \leq p^{bid}_{B,g,t}) = \gamma^k_{B,g})$	γ_B^k

When providers adopt non-identical strategies, their bid prices will converge to the market price in steady state; however, their profit and throughputs will be different. In this section, we develop profit functions for both A.com and B.com.

Assume at a steady state market demand (Δ^*), throughput of A.com and B.com are Y_A^* and Y_B^*. Since they adopt different strategies, we expect that their steady state throughput and profit will not be the same.

$$\begin{aligned} &Y_A^* \neq Y_B^* \\ &\Delta^* = Y_A^* + Y_B^* \\ &u_A^*(.) \neq u_B^*(.) \end{aligned} \tag{7.37}$$

The bid price of A.com and B.com can be represented by the following equations:

$$p^{bid}_{A,s,t} = \omega_{A,s,t}(Y^*_{A,t}) + \left[\frac{1}{\left(p_{Min,A,s,t} - \omega_{A,g,t}(Y^*_{A,t})\right)} - \frac{\gamma^j_{A,s}}{\left(\dfrac{(\Delta(Y^*_{A,t}) - \rho_{TE}K)(\Omega_s - \omega_{A,s,t}(Y^*_{A,t}))}{(2\rho_{TE}K - \Delta(Y^*_{A,t}))} \right)} \right]^{-1} \tag{7.38}$$

$$p_{B,s,t}^{bid} = \omega_{B,s,t}(\Delta^* - Y_{A,t}^*) + \left[\frac{1}{\left(p_{Min,B,s,t} - \omega_{B,s,t}(\Delta^* - Y_{A,t}^*)\right)} - \frac{\gamma_{B,s}^{k}}{\left(\frac{(\Delta(\Delta^* - Y_{A,t}^*) - \rho_{TE}K)(\Omega_g - \omega_{B,s,t}(\Delta^* - Y_{A,t}^*))}{(2\rho_{TE}K - \Delta(\Delta^* - Y_{A,t}^*))} \right)} \right]^{-1}$$

(7.39)

At the steady state, the bid prices of A.com and B.com converge at the steady state market price ($p_{Market,s,t}^*$).

$$p_{Market,s,t}^* = p_{A,s,t}^{bid}(Y_{A,t}^*) = p_{B,s,t}^{bid}(Y_{B,t}^*) \qquad (7.40)$$

Here, by solving two equations representing bid functions of A.com and B.com, we can find market price at $Y_{A,t}^*$ and $Y_{B,t}^* = \Delta^* - Y_{A,t}^*$.

For the strategy set (h_{Aj}, h_{Bk}) of A.com and B.com (i.e. rejection probability set $\{\gamma_{A,s}^{j}, \gamma_{B,s}^{k}\}$, equations (7.38)-(7.40) can be solved to find unique $Y_{A,t}^*$. By using A.com's steady state throughput ($Y_{A,t}^*$) and B.com's steady state throughput ($Y_{B,t}^* = \Delta^* - Y_{A,t}^*$) we can determine the steady state market price (equations (7.39) and (7.40)) , the marginal cost (equation (7.6)), and unity profit (equation (6.1)) of both the providers.

Providers' bid price equations (7.38) and (7.39) are hyperbolic functions. In addition, the marginal cost equations ($\omega(Y_{n,t}^*)$) are also hyperbolic function. Solving equations (7.38)-(7.40) to find $Y_{A,t}^*$ by algebraic manipulation is seemingly intractable. In addition, we need to find a point where $p_{A,s,t}^{bid}(Y_{A,t}^*)$ *and* $p_{B,s,t}^{bid}(Y_{B,t}^*)$ intersects each other. Therefore, we solve them by numerical analysis method using MATLAB.

We develop an array of A.com's bid prices (equation (7.38)) for a range of throughput ($Y_{A,t}^*$). Then, we develop an array of B.com bid prices (equation (7.39)) for a range of throughput ($Y_{B,t}^* = \Delta^* - Y_{A,t}^*$). By using MATLAB search algorithm we find $Y_{A,t}^*$ when $p_{A,s,t}^{bid}(Y_{A,t}) = p_{B,s,t}^{bid}(Y_{B,t})$ within the window of continuous hyperbolic function in the market demand range.

Figure 7.8 shows that bid prices of A.com and B.com converge at the Green market price of $90.7 at an A.com throughput of 984 Mbps when A.com adopts the *Very High Rejection* strategy and B.com adopts the *Very Low Rejection* strategy in a 70% market load. In this case, the throughput of B.com is 696 Mbps. Note that the B.com's throughput is less than the throughput of A.com because of B.com's higher rejection probability.

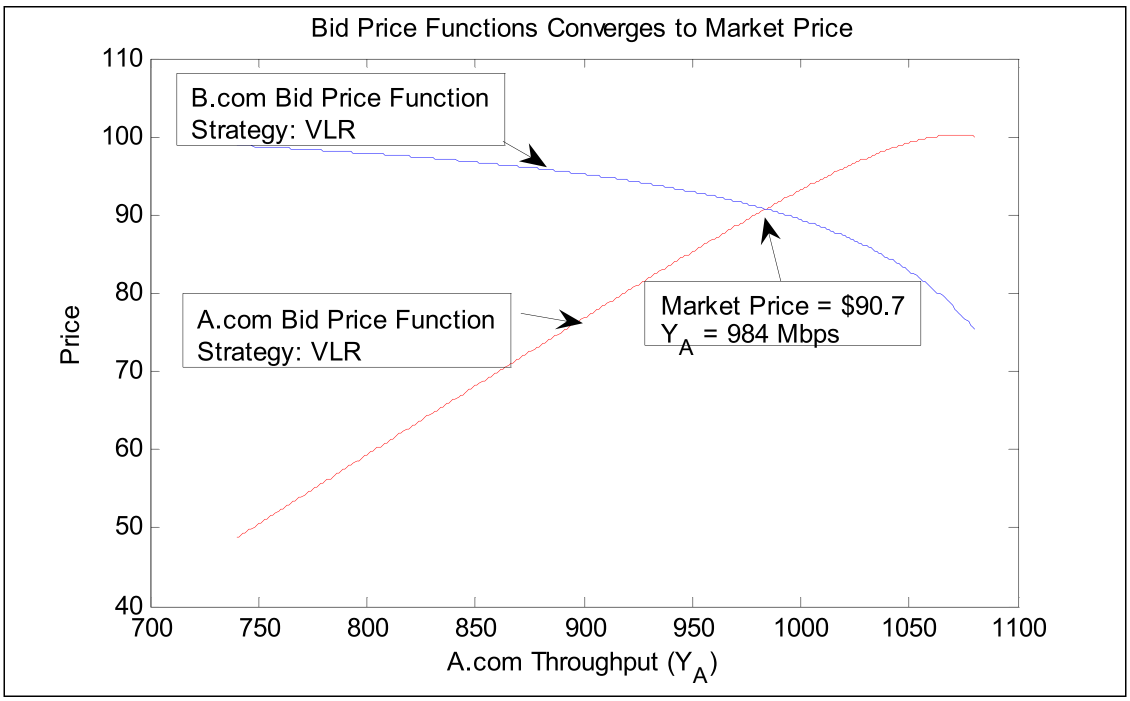


Figure 7.8: Solving Non-Identical Strategies Bid Price Equations by Numerical Analysis

Figure 7.9 presents analytical values for the strategy set $\{h_{A_j} = VLR, h_{B_j} = VHR\}$ and validates with simulation results.

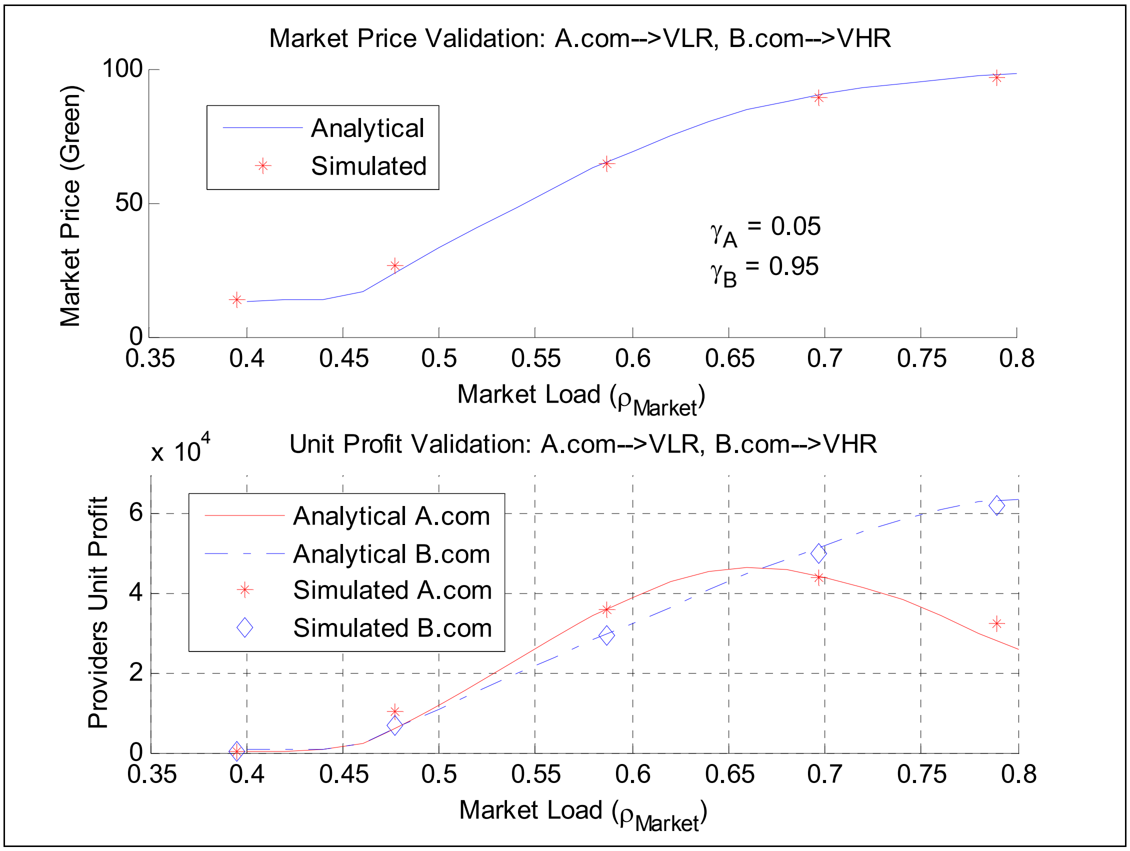


Figure 7.9: Comparison of Dissimilar strategies

A.com's lower rejection strategy caused it to operate in a smaller optimum profit than that of B.com. In the lower plot of Figure 7.9, A.com's optimum throughput (around 67%) is lower than B.com throughput (around 80%). Here, higher rejection strategy yields higher optimum profit; thus, it is the dominant strategy. Note that when both providers played the VHR strategies, their unit profit were higher (more than 6e4 in Figure 7.6) in comparison to their unit profit (less than 5e4 in Figure 7.7) for both playing the VLR strategies. These results further strengthen our argument of the VHR being a dominant strategy.

7.3.5 *Bayesian-Nash and Pareto-Efficient Strategy*

In the Bayesian-Nash equilibrium, a provider maximizes their expected profit [50]. A strategy space $Strategy = (h_1, h_2, ..., h_j)$ constitutes a Bayesian-Nash equilibrium of a game $G = [\{A.com, B.com\}, \{Strategy_A, Strategy_B\}, \{u_A, u_B\}]$ for every j = 1, …, J such that:

$$E[u_A(h^*_{Aj}, h^*_{Bj})] \geq E[u_A(h^{\forall j}_{Aj}, h^*_{Bj})]\,. \tag{7.41}$$

To find a Bayesian-Nash equilibrium, we need to find the best strategy of A.com $\{h^*_{Aj}\}$ maximizing its expected profit when B.com adopts its best strategy $\{h^*_{Bj}\}$. Note that, in this strategy set both providers optimize their expected profit.

Since market demand varies and the market demand patterns are unknown, we show a framework to locate a Bayesian-Nash equilibrium based on a hypothetical market load distribution. We assume that the market demand varies from 50% to 80% of market capacity and the demand pattern represents the discrete pseudo Gaussian Normal distribution with $prob(\rho_{Market}) \sim N[0.65, 0.01]$:

$$prob(\rho_{Market}) = \frac{1}{\sqrt{2\pi(0.01)}} \exp^{-\frac{(\rho_{Market}-0.65)^2}{2(0.01)}} \tag{7.42}$$

Figure 7.10 illustrates the market load probability density function (pdf) that indicates market demand probability. This distributions sums to 1.0 within $\rho_{Market} \sim [.5, .8]$.

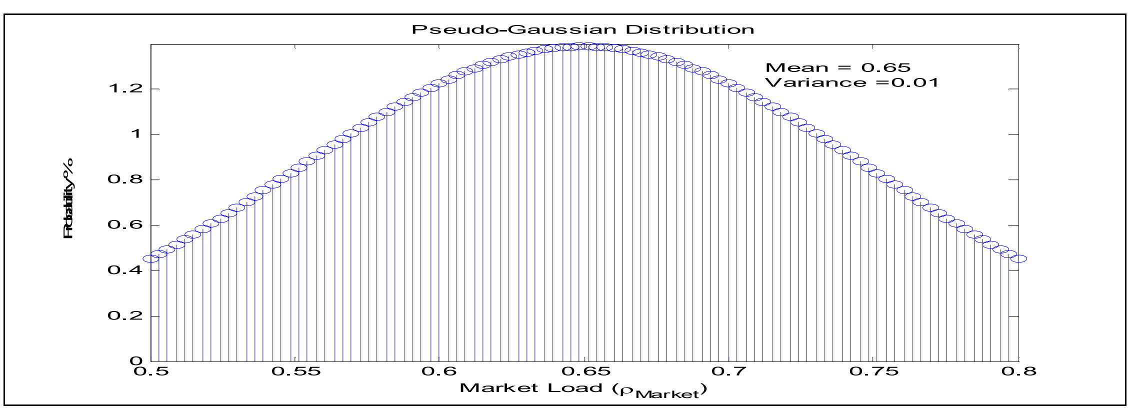


Figure 7.10: Probability Density Funciton (pdf) of Market Load

We compute the expected *unit profit* as follows:

$$E[u_A(.)] = \sum_{\forall \rho_{Market}} prob(\rho_{Market}) u_A(.)$$
$$E[u_B(.)] = \sum_{\forall \rho_{Market}} prob(\rho_{Market}) u_B(.) \tag{7.43}$$

The expected *unit profit* pair ($E[u_A(.)] | \gamma_A, E[u_B(.)] | \gamma_B$) of A.com and B.com for each strategy set (γ_A, γ_B) is presented in Table 7.5.

Table 7.5: Expected Unit Profit of Providers for different combination of strategies.

		B.com				
h_{nj}		*VLR*	*LR*	*RN*	*HR*	*VHR*
A.com	*VLR*	(.50,.50)	(.54,.55)	(.57,.58)	(.60,.61)	(.66,.73)
	LR	(.55,.54)	(.59,.59)	(.62,.62)	(.65,.66)	(.74,.77)
	RN	(.58,.57)	(.62,.62)	(.65,.65)	(.69,.69)	(.79,.80)
	HR	*(.61,.60)*	*(.66,.65)*	*(.69,.69)*	*(.73,.73)*	*(.84,.85)*
	VHR	*(.73,.66)*	*(.77,.74)*	*(.80,.79)*	*(.85,.84)*	*(1.00,1.00)*√√

The table shows that higher rejection strategies (i.e. higher rejection probability) yield higher expected profit compared to lower rejection strategies. The *Very High Rejection* strategy yields highest profit of all other strategies. Thus, the *Very High Rejection* is the dominant strategy of this game.

In addition, Table 7.5 shows that for strategies $h_{n,j}^{\forall j} = \{VLR, LR, RN, HR, VHR\}$ the following is true for A.com:

$$E[u_A(h^*_{A\_Very\_high\_Rejection}, h^*_{B\_Very\_High\_Rejection})] \geq E[u_A(h^{\forall j}_{Aj}, h^*_{B\_Very\_High\_Rejection})] \tag{7.44}$$

This implies that the Bayesian-Nash equilibrium strategy set for both providers is {*Very High Rejection*, *Very High Rejection}*, which is marked by √√ in Table 7.5. From the {VHR, VHR} strategy set, if a provider (e.g. A.com) switches to another strategy in the last column by moving upward, it hurts its expected profit.

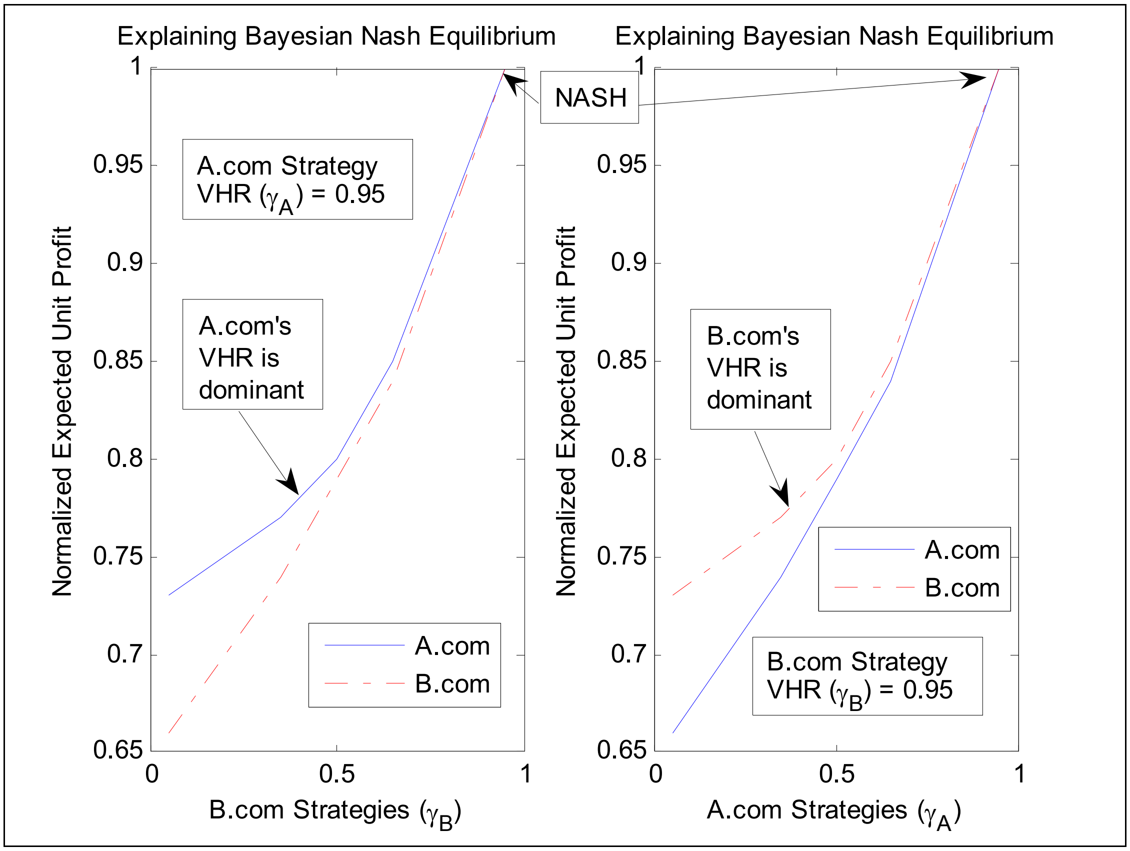


Figure 7.11: 2D-Plot: Analytical Bayesian Nash Equilibrium in Homogeneous Market

Figure 7.11 provides a pictorial representation of the dominant strategy plotting row five (left hand figure) and column five (right hand figure) of Table 7.5. The left-hand figure depicts the normalized expected unity profit of both providers when A.com adopts VHR strategy and B.com changes strategy from VLR to VHR. The plot shows that VHR strategy is the dominant strategy. The right hand figure plots the normalized expected profit when B.com adopts VHR strategy and A.com changes strategy from VLR to VHR. Again, the VHR is the dominant strategy. The figure also identifies the Nash Equilibrium strategy set {VHR, VHR} because if any provider changes its strategy from this strategy, it will hurt both of them.

The strategy set {VHR, VHR} is also the Unique Bayesian Nash Equilibrium among these strategies because there is no other Bayesian Nash Equilibrium in this game.

Similarly, if B.com switches to another strategy in the bottom row moving left from √√ combination, it hurts its expected profit.

$$E[u_B(h^*_{A\_Very\_high\_Rejection}, h^*_{B\_Very\_High\_Rejection})] \geq E[u_B(h^*_{A\_Very\_High\_Rejection}, h^{\forall j}_{Bj})] \quad (7.45)$$

In this scenario, the A.com and B.com profit are equivalent when they adopt the same strategy.

$$E[u_A(h^*_{A\_Very\_high\_Rejection}, h^*_{B\_Very\_High\_Rejection})] \Leftrightarrow E[u_B(h^*_{A\_Very\_high\_Rejection}, h^*_{B\_Very\_High\_Rejection})]$$

This equilibrium does not imply that two providers will always enjoy fair market share at a Nash Equilibrium strategy set. In our study, providers enjoy fair market share at the Nash Equilibrium strategy set because the network topology, traffic flow paths, network capacity, and traffic engineering rules are identical for both providers.

The strategy set {*Very High Rejection, Very High* Rejection} is a Pareto efficient outcome strategy set because there is no other strategy set (α) to meet the following criterion with strict inequality for at least one strategy (*j*):

$$u_j(\alpha) > u_j(a = \{Very\_High\_Rejection, Very\_High\_Rejection\}) \quad \forall j \quad (7.46)$$

This strategy set yields the Pareto-efficient outcome when averaged across the market demand profiles of Pseudo-Gaussian Normal (N[.85,0.01]) depicted in Figure 7.10. However, this set is not safe to adopt because a provider can change its strategy to *Low Rejection* strategy in low market demand to obtain higher profit as described in Section 8.1.4.1, where the safe strategy set is identified as {*Rejection Neutral, Rejection Neutral*}.

We can graphically view the Nash Equilibrium in 3-D plot.

Both upper and lower plots in Figure 7.12 represent the same picture viewed from different angles. There are two surfaces in each plot representing the normalized expected unit profit of A.com and B.com. The A.com's rejection probabilities are input values on the x-axis. B.com's rejection probabilities are input values in y-axis. The z-axis represents the normalized unit profit.

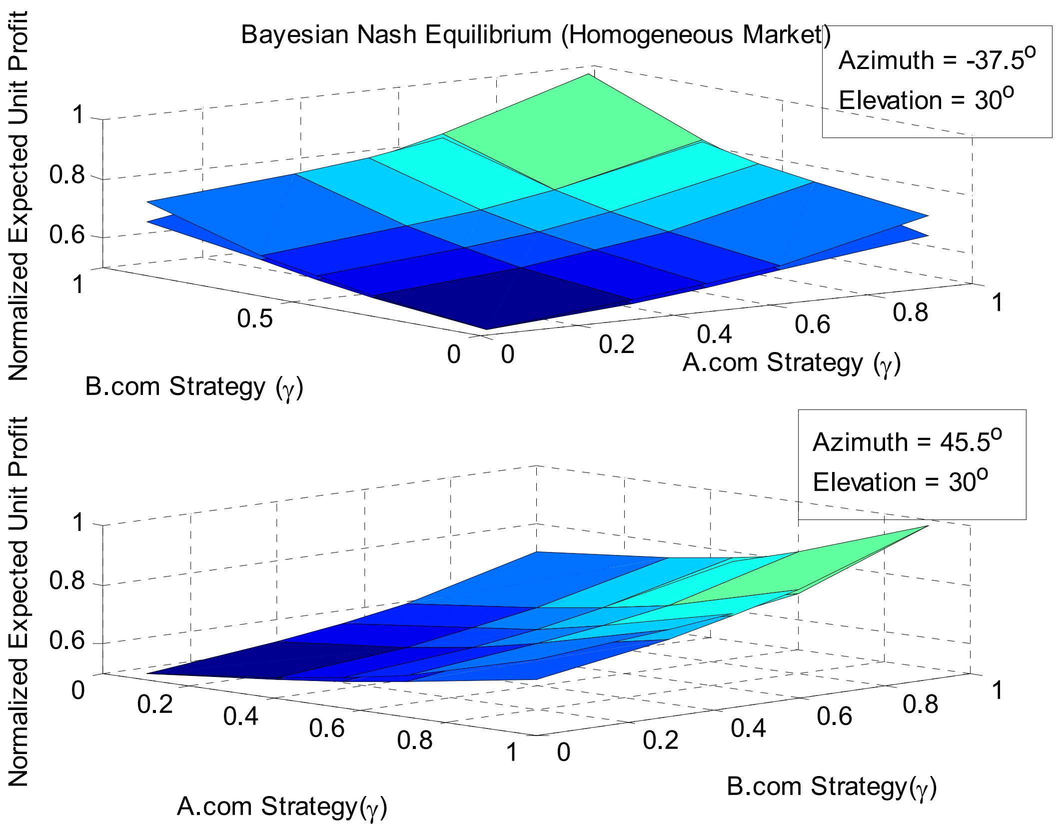


Figure 7.12: 3D Plot: Analytical Bayesian Nash Equilibrium in Homogeneous Market

We introduce this figure to illustrate Nash Equilibriums and to determine whether the unique Nash Equilibrium and Pareto-efficient outcome strategy set exists. The figures show that the unique Bayesian Nash equilibrium and Parteto-efficient outcome strategies are at $\gamma_A^* = 0.95, \gamma_B^* = 0.95$. This can be understood by viewing only one peak on this surface at ($\gamma_A^* = 0.95, \gamma_B^* = 0.95$) and observing a decrease in normalized expected unit profit while moving from ($\gamma_A^* = 0.95, \gamma_B^* = 0.95$) to lower values of rejection probability either in x-axis or y-axis.

7.4 *Heterogeneous Service-based Market*

In this section, we develop analytical models of market price, providers' bid price, and providers' profit for heterogeneous service-based market and validate analytical values by simulation results. Section 7.4.1 studies a market in which both providers adopt the *Rejection Neutral* strategy, and Section 7.4.1.2 presents results when two providers adopt other identical strategies.

Table 7.6 summarizes the main parameters of the analytical studies:

Table 7.6: Summary of Parameter for Heterogeneous services

The Class of Service	Heterogeneous: Blue, Green, Red
Market	Duopoly
Strategy	Strategy set of Figure 7-3
Network Topology and TE Rules	The topology and Rules of Chapter 5
Reservation Price (Ω)	Blue = \$160.00
	Green = \$100.00
	Red = \$70.00
Service cost coefficients (δ_s)	Blue = 1.0
	Green = 0.10
	Red = 0.01
Product rule	Service cannot be switched. For example, an application requiring Blue security cannot switch to Green security.
Provider fixed cost coefficient (θ)	10.0

7.4.1 *Study of Identical Strategy Set*

In this section, we analytically determine the market price, the marginal cost to a provider, profit curve of providers in different network load, and the optimum throughput of network when providers adopt identical strategy sets. We also compare the analytical results with those from simulations.

Let us assume that at a steady state market demand (Δ^*), throughputs of A and B are Y_A and Y_B. Since they adopt the same strategy set, we expect that both will enjoy fair share of profit and throughput. Each service class throughput will be exactly scaled to the percentage of traffic type in the market.

$$
\begin{aligned}
&Y_A^* = Y_B^* = Y^* \\
&\Delta^* = 2Y^* \\
&u_A^*(.) = u_B^*(.) \\
&Y_{n,b}^* = \frac{20}{90}Y_n^*, Y_{n,g}^* = \frac{30}{90}Y_n^*, Y_{n,r}^* = \frac{40}{90}Y_n^*
\end{aligned}
\tag{7.47}
$$

In a steady state market, bid prices of both providers for each service class will converge at the market price of the service.

$$
p_{Market,s,t}^* = p_{A,s,t}^*(Y_{A,t}) = p_{B,s,t}^*(Y_{B,t}) \tag{7.48}
$$

As stated in Chapter 3, the service cost coefficient differentiates the service class. We use the general bid price equation ((7.34)) of the homogeneous service-based market for the heterogeneous service-based market by assigning appropriate service cost coefficients of Blue, Green, and Red services. Similarly, we assign appropriate service cost coefficient values in the marginal cost function (7.6) for the Blue, Green, and Red classes.

We expand the profit function of (7.27) to take into the account the presence of Blue, Green, and Red services in the network as follows:

$$
\begin{aligned}
u_n^*(.) &= (p_{n,b,t}^* - \omega_{n,b,t}^*)Y_{n,b,t}^* + (p_{n,g,t}^* - \omega_{n,g,t}^*)Y_{n,g,t}^* + (p_{n,r,t}^* - \omega_{n,r,t}^*)Y_{n,r,t}^* \\
&= (p_{n,b,t}^* - \omega_{n,b,t}^*)(\frac{20}{90})Y_{n,t}^* + (p_{n,g,t}^* - \omega_{n,g,t}^*)(\frac{30}{90})Y_{n,t}^* + (p_{n,r,t}^* - \omega_{n,r,t}^*)(\frac{40}{90})Y_{n,t}^*
\end{aligned}
\tag{7.49}
$$

7.4.1.1 *The Rejection Neutral Strategy Set*

Let us assume that the strategy set adopted by A.com and B.com is as follows:

Provider	*Strategy*	*Rejection Probability (γ)*
A.com	$h_{Aj} = \{RN, RN, RN\}$	$\gamma_{A,b}^j = 0.5, \gamma_{A,g}^j = 0.5, \gamma_{A,r}^j = 0.5$
B.com	$h_{Bj} = \{RN, RN, RN\}$	$\gamma_{B,b}^j = 0.5, \gamma_{B,g}^j = 0.5, \gamma_{B,r}^j = 0.5$

The *Rejection Neutral* bid price for each service class can be obtained by appropriately assigning service cost coefficient values from Table 7.6 to the equation (7.50).

$$\overline{p}_{n,s,t} = \frac{(\Delta(Y^*_{n,t}) - 12\rho_{TE}C)(\Omega_s - \omega_{n,s,t}(Y^*_{n,t}))}{(24\rho_{TE}C - \Delta(Y^*_{n,t}))}\left[\begin{array}{l} \ln\left(\dfrac{\Omega_s - \omega_{n,s,t}(Y^*_{n,t})}{p_{Min,n,s,t} - \omega_{n,s,t}(Y^*_{n,t})}\right) \\ +\omega_{n,s,t}(Y^*_{n,t})\left(\dfrac{1}{p_{Min,n,s,t} - \omega_{n,s,t}(Y^*_{n,t})} - \dfrac{1}{\Omega_s - \omega_{n,s,t}(Y^*_{n,t})}\right)\end{array}\right] \tag{7.50}$$

Figure 7.13 plots the analytical values of market price, provider's marginal cost, and their profit with respect to market load and validates the analytical values by the simulation results.

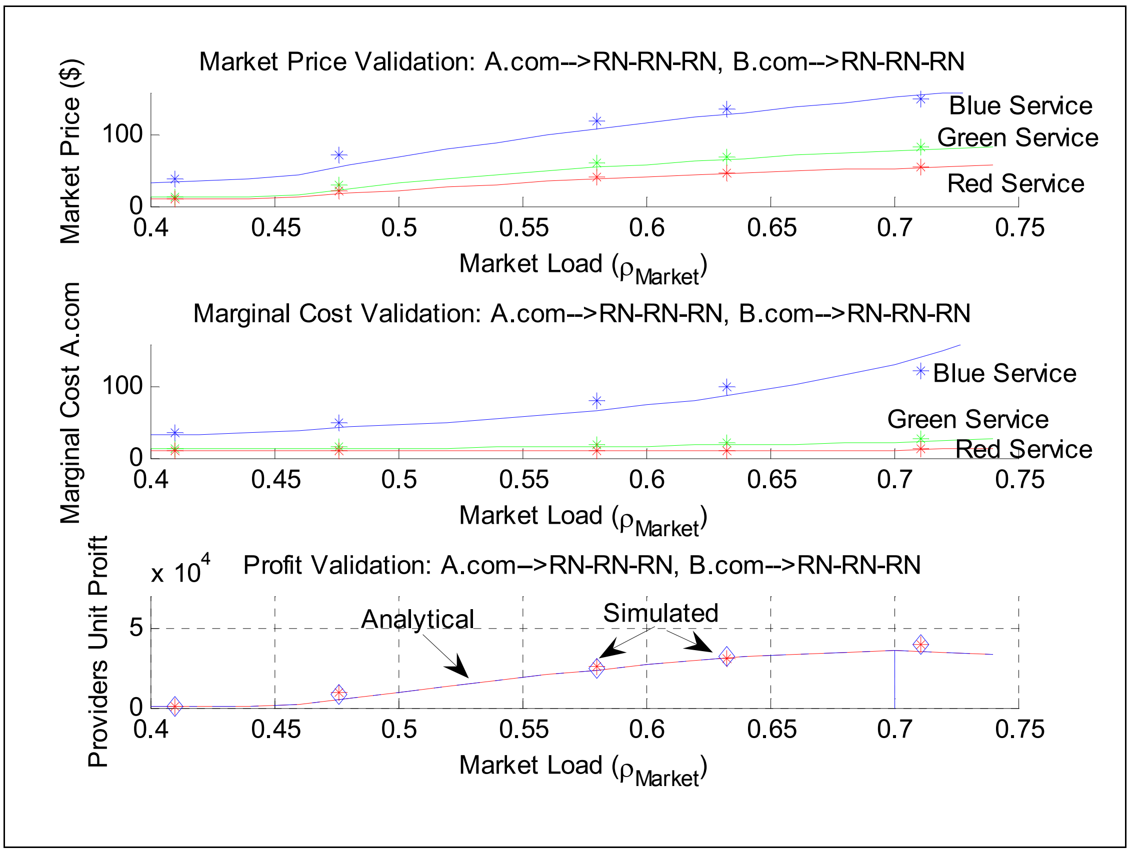


Figure 7.13: Heterogeneous based-Market: Analytical and Simulation Results (RN strategy sets)

The asterisks in the figure represent simulation results and the continuous curved lines represent analytical results. All plots show that the simulated results approximated the analytical results. The curve of Plot 3 in Figure 7.13 exhibits all three properties (monotonous, bound, and concave) of the profit function. This function shows that at an approximate network load of 0.7, a provider optimizes profit.

Note that, depending on the traffic mix, cost function parameters, and reservation prices, this optimum load may be slightly different. Nevertheless, we emphasize that if a provider knows the traffic mix, cost function parameters, and reservation prices, it can determine the optimum load when applying our analytical model.

7.4.1.2 *Study of Other Strategy Sets*

Let us assume that the strategy set adopted by A.com and B.com is as follows:

Provider	*Strategy*	*Rejection Probability (γ)*
A.com	$h_{Aj} = \{VHR, RN, VLR\}$	$\gamma^{j}_{A,b} = 0.95, \gamma^{j}_{A,g} = 0.50, \gamma^{j}_{A,r} = 0.05$
B.com	$h_{Bj} = \{VHR, RN, VLR\}$	$\gamma^{j}_{B,b} = 0.95, \gamma^{j}_{B,g} = 0.50, \gamma^{j}_{B,r} = 0.05$

By appropriately assigning service cost coefficient values and the rejection probability values ($\gamma^{j}_{n,s}$) to the equation (7.51), we find the bid price of each service class.

$$p^{bid}_{n,s,t} = \omega_{n,s,t}(M^{*}_{n,t}) + \left[\frac{1}{\left(p_{Min,n,s,t} - \omega_{n,s,t}(M^{*}_{n,t})\right)} - \frac{\gamma_{n,s}}{\left(\dfrac{(\Delta(Y^{*}_{n,t}) - \rho_{TE} K)(\Omega_{s} - \omega_{n,s,t}(M^{*}_{n,t}))}{(2\rho_{TE} K - \Delta(Y^{*}_{n,t}))} \right)} \right]^{-1}$$

(7.51)

Figure 7.14 plots the analytical values of market price, provider's marginal cost, and their profit with respect to market Load and validates the analytical values by the simulation results.

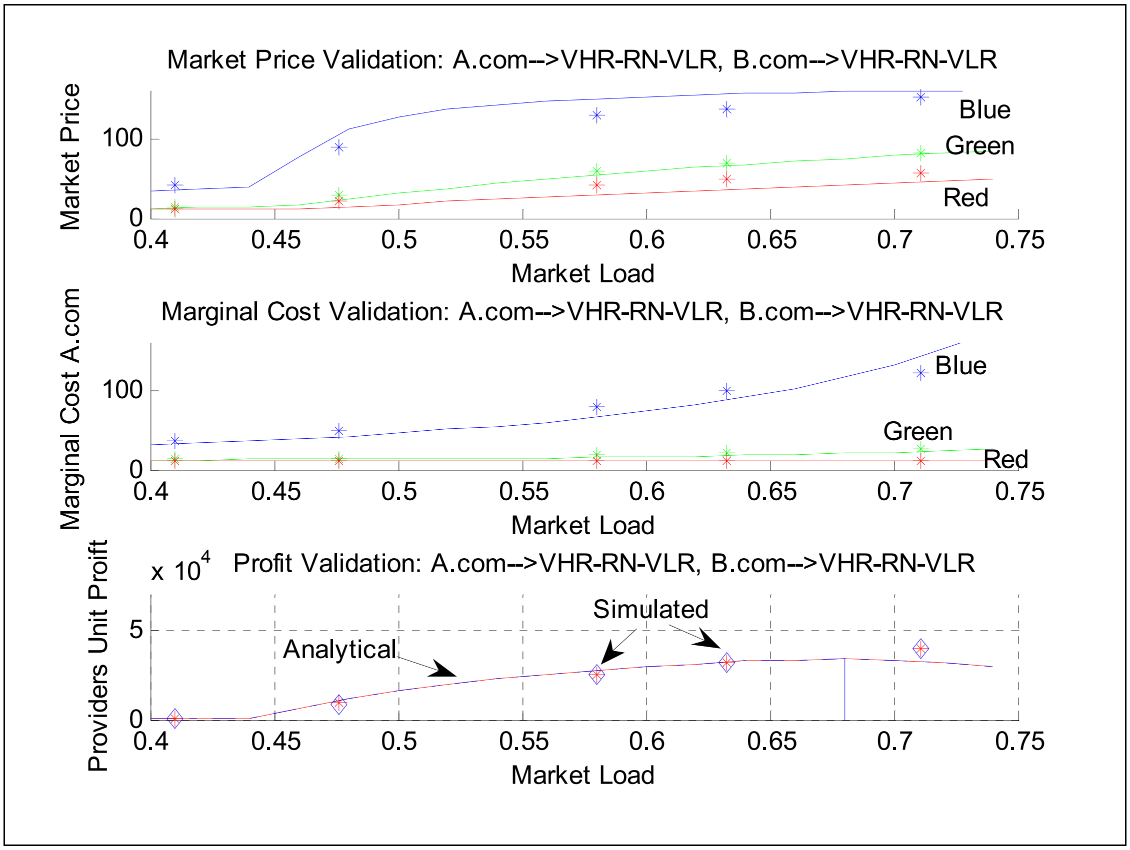


Figure 7.14: Heterogeneous based-Market: Analytical and Simulation Results (Other strategy sets)

Providers' profit functions show that an approximate market demand load of 0.68, each provider optimizes profit. The optimum profit and optimum network load of this strategy set are different from those of the last section. This difference implies that the optimum network load and profit depend upon the strategy choice of providers.

7.4.2 *Non-Identical Strategy Set*

Determining the profit of a provider requires a knowledge of each service throughput ($Y_{n,s,t}$) as shown in the following equation.

$$u_n^*(.) = (p_{n,b,t}^* - \omega_{n,b,t}^*)Y_{n,b,t}^* + (p_{n,g,t}^* - \omega_{n,g,t}^*)Y_{n,g,t}^* + (p_{n,r,t}^* - \omega_{n,r,t}^*)Y_{n,r,t}^* \tag{7.52}$$

The bid prices are a function of a network throughput and the network throughput is the sum of the individual service throughput as shown in the following equations:

$$p_{n,s,t}^{bid} = f(Y_{n,t},...) \tag{7.53}$$

$$Y_{n,t} = Y_{n,b,t} + Y_{n,g,t} + Y_{n,r,t} \tag{7.54}$$

When two providers adopt dissimilar strategies in a heterogeneous service based market, we cannot determine *unique* individual service throughput ($Y_{n,s,t}$) because the bid price is not a function of $Y_{n,s,t}$ and each service throughput is not equally distributed among providers.

By applying the analytical method of determining price as described in previous sections, we can determine the steady state market price of each service and the corresponding steady state network throughput ($Y_{n,t}^*$) for non-identical sets of strategies. However, since equation (7.54) is one equation with three unknowns, we cannot analytically determine the *unique* individual throughputs of Blue, Green, and Red services. As such, we cannot compute the profit of each provider through analytical method. Consequently, we cannot also analytically determine the Bayesian Nash Equilibrium and the Pareto-efficient strategy sets in the heterogeneous market.

In our session level Monte-Carlo simulation method, each provider keeps records of each service throughput; thus, we can determine the profit of each provider, the Bayesian Nash Equilibrium and the Pareto-efficient strategy sets in the heterogeneous market. In Chapter 8, we will illustrate a method of determining Bayesian Nash Equilibrium and Pareto-efficient outcome strategy sets.

7.5 *Chapter Summary*

This chapter analytically synthesized belief functions and game strategies with the M/M/1 optimum mean packet count functions in a predetermined network topology and traffic flow matrices. The chapter assigned service reservation price and service cost coefficient values.

The chapter developed analytical models of market price, providers' bid price, and providers' profit for a homogeneous service-based market and validated analytical values with those of session level Monte-Carlo simulations. The chapter also analytically determined the best strategy set (Unique Bayesian Nash Equilibrium and Pareto-Efficient outcome) for the homogeneous service-based market. For the heterogeneous service-based market, the chapter developed analytical models of market price, providers' bid price, and providers' profit when providers adopt identical strategy sets. When strategy sets are not identical, it is a seemingly intractable task to analytically determine the network throughput of each service. Thus, profit of providers and the best strategy set cannot be determined by analytical method for all strategy sets in a heterogeneous service-based market. We will determine these by session level Monte-Carlo simulation in the next chapter.

A key lesson learned from this chapter is that each provider can determine the operating load of a network that optimizes its profit by mathematical analysis for a set of strategies in homogenous service-based market. Providers can also predict the market price of services. Another lesson learned is that the network loads that optimize providers' profit are different for different sets of strategies.

In the next chapter, we will present our session level Monte-Carlo simulation methods to determine the best strategy set, the preferred strategy set, and the safe strategy set.

8 Session Level Monte-Carlo Simulation, Applications, and Advantages

This chapter contains session level Monte-Carlo simulation results and their analyses. In addition, it contains traffic engineering applications and advantages of the model. Sections 8.1 and 8.2 present results of homogeneous and heterogeneous service-based markets. Each section outlines the research objective, the common parameters, and the results of each experiment. Section 8.3 summarizes the lessons learned.

8.1 Homogeneous Service-based Market

The main objectives of the experiments discussed in this section are to find preferred strategies, and examine the applications and advantages of the model in homogeneous service-based market.

8.1.1 Experiment Objectives

- **Validation of the model**
  - **Functional validation:** One method of functional validation is to compare the outcome of two similar strategies. Stochastically, a mean price should yield the same expected outcome as a random price from the same probability distribution. Therefore, the *Rejection Neutral* strategy and the *Random Rejection* strategy should yield the same performance. In Section 8.1.3.1, we will investigate whether they yield equal profit.
- **Application**
  - **Safe Strategy:** A safe strategy set should be indifferent to the dynamic nature of Internet traffic. Here, by the safe strategy set we imply a strategy set that ensures fair market share of profit in all market demands. We will conduct simulations at various market demand levels to find the safe strategy set in Section 8.1.4.1. These simulation methods include

assigning the *Rejection Neutral* strategy to one provider while varying the strategies of the opponent in each simulation. We will observe the influence of the different strategies on the *Rejection Neutral* strategy by comparing profit shares obtained by both providers.

  - **Best Strategy Set (Bayesian-Nash and Pareto-Efficient Strategy):** Providers want to determine the best strategy that will optimize their profit. According to the game theory, the Bayesian-Nash Equilibrium and the Pareto-Efficient outcome strategy set represents the best strategy set. In Section 8.1.4.2, we will show an application of determining the best strategy set.
  - **Routing Scheme:** Providers generally support multiple routing schemes (e.g. min-hop or max-hop) in their networks. We will address the question as to whether the min-hop and the max-hop routing schemes influence the providers' profits in section 8.1.4.3.

- **Advantages of the Model:** We will illustrate the advantages of our model in Section 8.1.4.4. Since our model is an extension to the classical Bertrand model of price, we will inquire whether the proposed model has advantages over the Bertrand model. We will also examine whether both enterprise and providers benefit by implementing this model. The comparison parameters of these experiments are market price and profit.

8.1.2 *Parameters*

In Chapter 5, we discussed the parameters of this research in detail. Unless otherwise explicitly stated, Table 8.1 summarizes the main parameters of the simulation:

Table 8.1: Parameters for simulation and analytical studies

The Class of Service	Homogeneous: Green
Market	Duopoly
Strategy	Strategy set of Figure 8.1
Network Topology and TE Rules	The topology and Rules of Chapter 5
Reservation Price (Ω)	$100.00
The service cost coefficient (δ_s)	0.10
Provider fixed cost coefficient (θ)	10.0

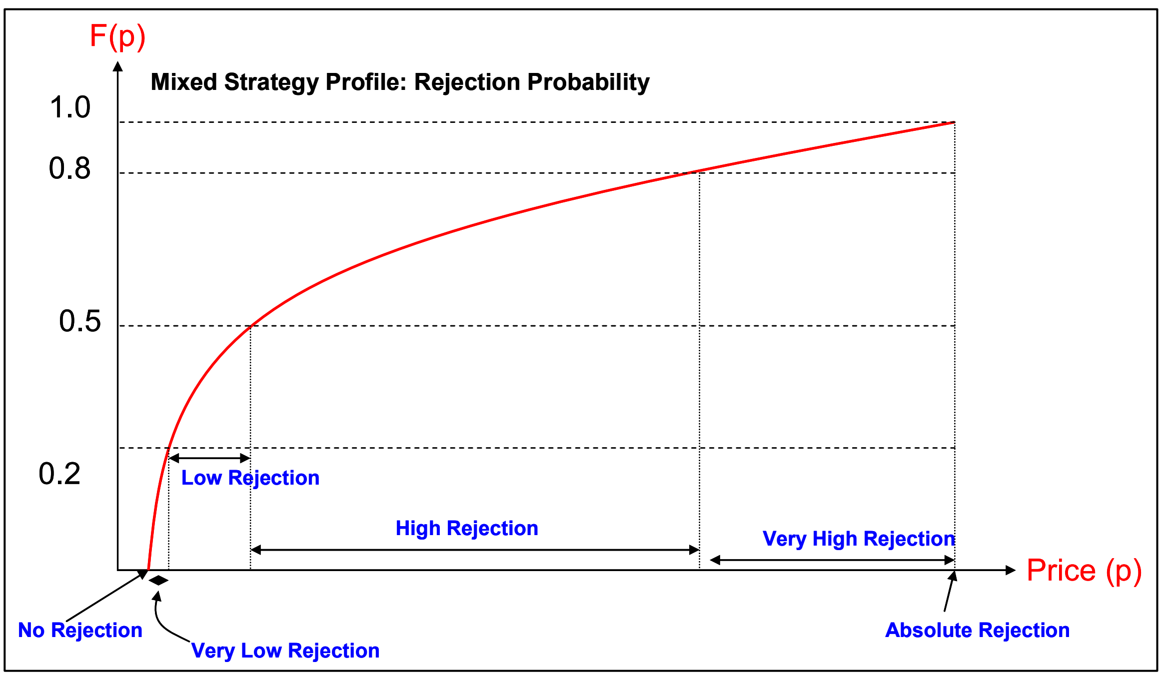


Figure 8.1: Strategy set of experiments

We will evaluate the market price, the profit share, the steady state mean profit, the marginal cost, and the network load of providers.

8.1.3 *Validation*

8.1.3.1 *Functional Validation*

In this experiment, we compared the *Random Rejection* and the *Rejection Neutral* strategies by studying the steady state marginal cost and profit of the providers in the proposed model. We also observed the profit share and market price. The strategies assigned to the providers are shown below:

Strategy	Provider
Random Rejection	A.com
Rejection Neutral	B.com

For the RFP of each session initiation request at a time (t), A.com first developed a mixed-strategy profile ($F_{n,s,t}(p)$) using equation (7.8) and then selected a bid price ($p_{n,s,t}^{random}$) within the interval [$p_{Min,n,s,t}$, Ω_s] with a probability of $F_{n,s,t}(p)$. This interval was the region between the *No Rejection* strategy price and the *Absolute Rejection* strategy price. We briefly describe the procedure of developing $F_{n,s,t}(p)$ and selecting a price. For each session request, an *analyst* of a provider drew a discrete graph of the belief function $F_{n,s,t}(p)$ using equation (3.36). The graph was drawn for the interval [$p_{Min,n,s,t}$, Ω_s] of 1000 bins. A price was then uniform randomly selected from this distribution.

The following algorithm describes the process.

Algorithm *Random:*

BEGIN

$price^1 \leftarrow p_{Min,n,s,t}$

FOR j = 1 TO 1000 DO

$$F^{j}{}_{n,s,t}(price^{j}) \leftarrow BeliefFunction(price^{j})$$
$$price^{j} \leftarrow price^{j} + \frac{(\Omega_s - p_{Min,n,s,t})}{1000}$$

END

$$rand \leftarrow Uniform[0,1]$$
$$p^{random}_{n,s,t} \leftarrow \{p^{bid}_{n,s,t} : F_{n,s,t}(price \le p^{bid}_{n,s,t}) \approx rand\}.$$

END #

For the RFP of each session initiation request, B.com first developed $f_{n,s,t}(p)$ and then selected a mean price within the interval [$p_{Min,n,s,t}$, Ω_s] with a probability of $f_{n,s,t}(p)$. For each session request, an *analyst* of a provider drew a discrete graph of $f_{n,s,t}(p)$ for 1000 bins and determined the *rejection neutral* price ($p^{neutral}_{n,s,t}$) as the mean price of the distribution $f_{n,s,t}(p$ according to the following algorithm:

Algorithm *Neutral:*

BEGIN

$$price^{1} \leftarrow p_{Min,n,s,t}$$
$$F^{0}{}_{n,s,t}(price^{1}) = 0;$$

FOR j = 1 TO 1000 DO

$$F^{j}{}_{n,s,t}(price^{j}) \leftarrow BeliefFunction(price^{j})$$
$$f^{j}{}_{n,s,t} \leftarrow F^{j}{}_{n,s,t}(price^{j}) - F^{j-1}{}_{n,s,t}(price^{j})$$
$$price^{j} \leftarrow price^{j} + \frac{(\Omega_s - p_{Min,n,s,t})}{1000}$$

END

$$p^{Neutral}_{n,s,t} = \left[price^{1}, price^{2}, ..., price^{1000} \right] \begin{bmatrix} f^{1} \\ f^{2} \\ \vdots \\ f^{1000} \end{bmatrix}$$

END #

Note that the discrete version of price computation was conducted to mimic the method of the hardware or software computation of price in a network device.

Figure 6-2 compares simulation results of the steady state mean marginal costs and the mean profit of A.com and B.com for a range of market demand.

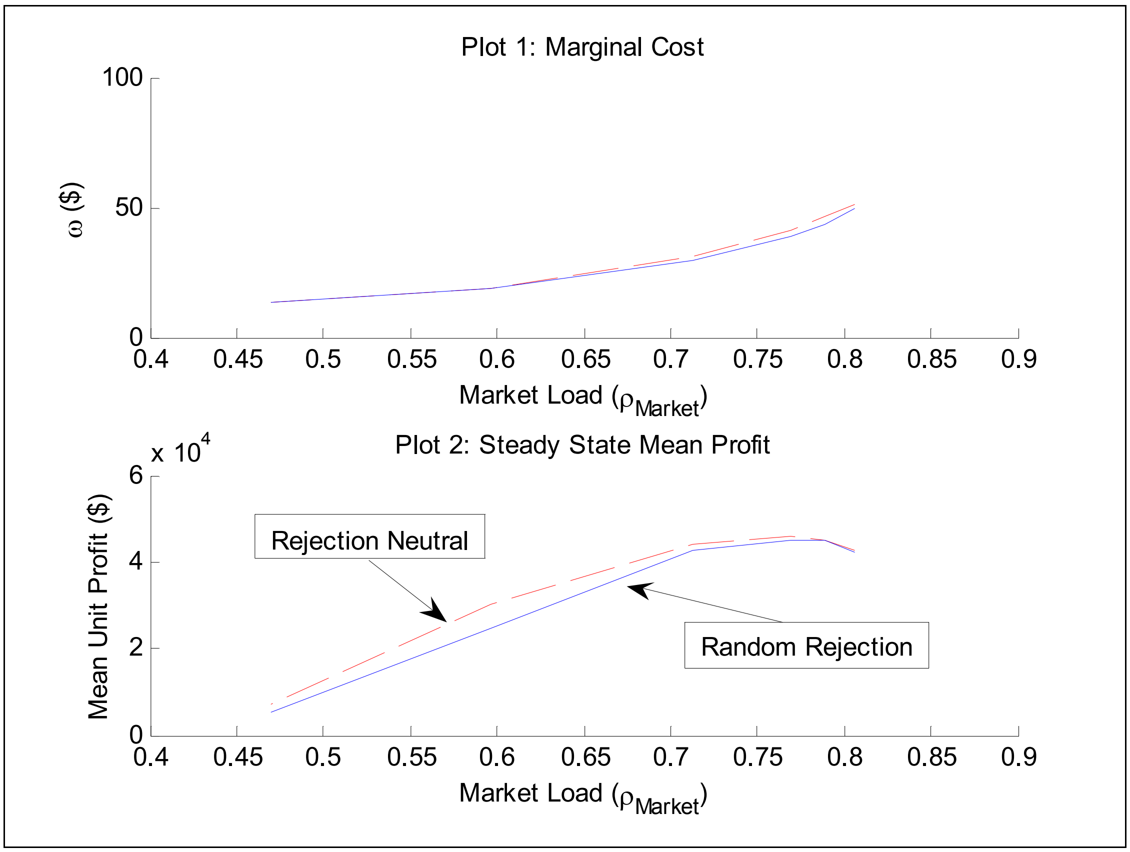


Figure 8.2: Comparison of *Random Rejection* and *Rejection Neutral* Strategies.

The figure shows that the marginal cost and the steady state mean profit of the providers for the *Rejection Neutral* and the *Random Rejection* strategies were approximately equal. We also observed that both providers' profit share or total profit share was almost the same ($\approx 50\%$) in all ranges of market demand.

$$U_B(H(.) : Rejection\ Neutral) \approx U_A(H(.) : Random\ Rejection) \approx 0.50 \qquad (8.1)$$

For the *Random Rejection* strategy, for the RFP of each session, an *Analyst* of a provider will need to perform about 1003 iterations to determine a bid price as shown in *Algorithm Random*. However, in the following implementation of the

Rejection Neutral, the *Analysist* will have to perform only about 2 iteratons to deterime a bid price:

Algorithm *Algebraic Neutral:*

```
BEGIN Algebraic Neutral
```

$price^1 \leftarrow p_{Min,n,s,t}$

$p_{n,s,t}^{Neutral} \leftarrow Equation(7.18)(price^1,...)$

```
END
```

In Section 7.3.1, we have shown that the analytical method using equation (7.18) yields the closely approximated results of the simulation method (using the *Algorithm Neutral*) . In this section, we have shown that the *Random Rejection* using the *Algorithm Random* provide closely approximated results of the *Rejection Neutral* using the *Algorithm Neutral*. Therefore, we claim the followings: while our algebraic method of the *Rejection Neutral* strategy yields approximately same uitlity of the *Random Rejection*. A simplified version of the *Random Rejection* strategy can also be implmented which requires only three iterations to determine a price.

Algorithm *Algebraic Random:*

```
BEGIN Algebraic Random
```

$price^1 \leftarrow p_{Min,n,s,t}$

$rand \leftarrow Uniform[0,1]$

$p_{n,s,t}^{Random} = Equation(3.1)(price^1, rand,...)$

```
END                              #
```

8.1.4 *Application*

8.1.4.1 Finding a Safe Strategy

In this section, we will find a safe strategy for all market demand by simulation. Assume that B.com adopts the *Rejection Neutral* strategy. What is the safe strategy of A.com? We will answer this question by observing the profit share of A.com in a range of market demand for all the strategy pairs of the following table:

Experiment	A.com	B.com
6..1.3.1	*Very High Rejection* strategy	*Rejection Neutral* strategy
6.1.3.2	*High Rejection* strategy	*Rejection Neutral* strategy
6.1.3.3	*Rejection Neutral* strategy	*Rejection Neutral* strategy
6.1.3.4	*Low Rejection* strategy	*Rejection Neutral* strategy
6.1.3.5	*Very Low Rejection* strategy	*Rejection Neutral* strategy
6.1.3.6	*No Rejection* strategy	*Rejection Neutral* strategy

Figure 8.3 depicts the simulated profit share of A.com in a range of market demand.

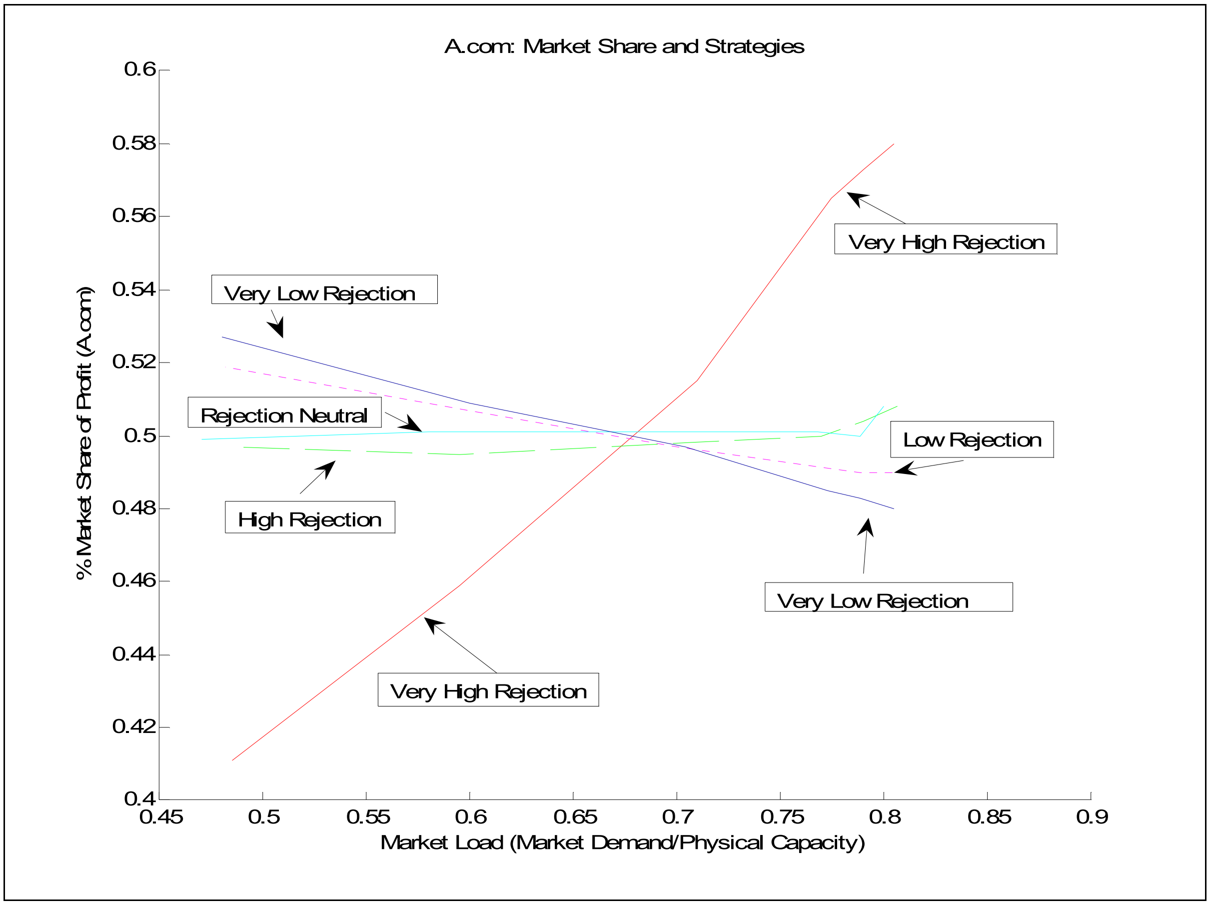


Figure 8.3: Comparison of all strategies with the *Rejection Neutral* strategy

By adopting the *Rejection Neutral* strategy, A.com gained almost equal profit as B.com at all market demand levels ($\Delta(y)$):

$$U_A(h(.): Rejection\ Neutral) \approx U_B(h(.): Rejection\ Neutral) \approx 0.50 \quad \forall \Delta(y) \qquad (8.2)$$

We also observed that both providers' experienced almost the same level of load in their networks when they adopted the *Rejection Neutral* strategy.

The *High Rejection* strategy yielded similar results as the *Rejection Neutral* strategy. It is because their rejection probabilities are close to each other.

$$U_A(h(.): Rejection\ Neutral) \approx U_A(h(.): High\ Rejection) \quad \forall \Delta(y) \qquad (8.3)$$

Comparison of A.com's profit share for the *Very High Rejection* or the *Rejection Neutral* strategy can be summarized as follows:

$$\begin{aligned} U_A(h(.): Very\ High\ Rejection) &< U_A(h(.): Rejection\ Neutral) \quad \Delta(y) < 0.65 \\ U_A(h(.): Very\ High\ Rejection) &> U_A(h(.): Rejection\ Neutral) \quad \Delta(y) > 0.70 \end{aligned} \qquad (8.4)$$

In the following discussion, we will explain the cause of the results of equation (8.4). Let us denote $\Delta(y) < 0.65$ as low market demand and $\Delta(y) > 0.70$ as high market demand.

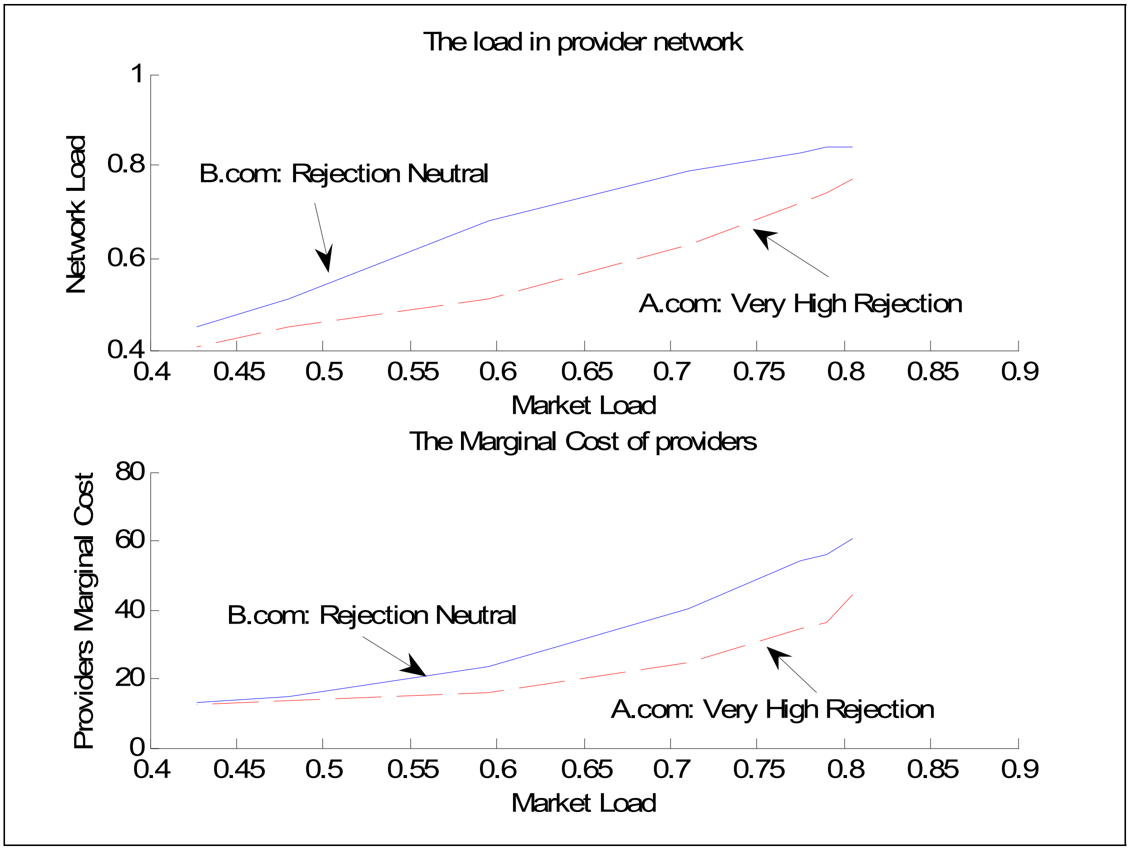


Figure 8.4: Very High and Neutral strategy providers' load and marginal cost

Figure 8.4 depicts the network load and the marginal cost of the providers when one provider adopted the *Rejection Neutral* and the other provider adopted the *Very High Rejection* strategy. In high market demand, the *Very High Rejection* strategy of A.com yielded a higher price of service compared to the *Rejection Neutral* strategy of B.com. Therefore, B.com won the majority of the bids and its operating load was higher than A.com; consequently, B.com's marginal cost of production was also comparatively higher. As a result, B.com's profit share was lower. On the other hand, A.com sold the residual bandwidth with a higher price and lower marginal cost. Therefore, A.com's profit share was higher than that of B.com. This result indicates that the in high market demand, the *Very High Rejection* strategy yields higher profit than the *Rejection Neutral* strategy. In low market demand, B.com captured almost all the market with a lower price; thus, the A.com's profit share was lower. This result indicates that in low market demand, the *Very High Rejection* strategy needs to be avoided.

The following equation compares profit achieved (as shown in Figure 6-9) for the *No Rejection*, the *Very Low Rejection*, the *Low Rejection* strategies compared to the *Rejection Neutral* strategy:

$$U_A(h(.): \textit{No Rejection}, \textit{Very Low Rejection}, \textit{Low Rejection}) > U_A(h(.): \textit{Rejection Neutral}) \quad \Delta(y) < 0.65$$
$$U_A(h(.): \textit{No Rejection}, \textit{Very Low Rejection}, \textit{Low Rejection}) < U_A(h(.): \textit{Rejection Neutral}) \quad \Delta(y) > 0.70 \tag{8.5}$$

For clarity, we do not illustrate the plot of the *No Rejection* strategy in Figure 8.3. The result of the *No Rejection* strategy was almost the same as the *Very Low Rejection* strategy. It is because their rejection probabilities are close to each other.

In low to moderate market demand levels, for the *Very Low Rejection*, and the *Low Rejection* strategies, A.com acquired a slightly better profit share than the *Rejection Neutral* strategy. At high market demand levels, A.com acquired the better profit share with the *Rejection Neutral* strategy than the *No Rejection*, the *Very Low Rejection*, and the *Low Rejection* strategies. The cause of these results can be explained as the opposite to the discussion of Figure 8.4 and equation (8.4).

Due to the dynamic nature of the Internet, traffic demand changes with the time of the day, the day of the week, and the holidays of the year. When a provider adopts the *Rejection Neutral* strategy, the other providers might be relatively disadvantaged if their strategies are not appropriately suited to network demand as shown in Figure 8.3. However, the *Rejection Neutral* strategy set, if implemented by both providers, ensures both providers to receive a fair share of profit at all levels of market demand.

Also, note that if both providers adopt *Very High Rejection* strategies they will also maximize their profits. However, at a market load less than approximately 0.7, a provider can reduce the rejection probability to obtain a higher profit share. Then, the other provider may retaliate by further lowering the rejection probability. This will result in a price war. Therefore, it is safe for both providers to adopt the *Rejection Neutral* strategy all the time to obtain equal profit shares at all market demand levels for a homogeneous service-based market. This is one of the **major findings** of this research. This major finding is important for providers because by implementing this strategy they can optimize their profit even though the dynamic nature of Internet traffic is unpredictable.

8.1.4.2 Finding Pareto-Efficient Outcome Strategy Set

In Section 7.3.5, we have shown an analytical method to locate the best strategy set for a homogeneous service-based market. In this section, we will determine the best strategy set by session level Monte-Carlo simulation. Note that the simulation emulates a real time network. In the simulation, the performance measurement metric is normalized *expected profit* as opposed to the normalized *expected steady state profit* of the analytical method.

In Figure 8.3, the plots of the profit share show an area surrounded by a diagonal. The plots of the *Very Low Rejection* and the *Very High Rejection* strategies show opposite and maximum influence on the profit share. As expected, the *Rejection Neutral* strategy always maintained equal profit shares. Therefore, we use the *Very High Rejection*, the *Rejection Neutral,* and the *Very High Rejection* strategies to postulate the Bayesian-Nash equilibrium of a game.

Table 8.2 illustrates these strategies:

Table 8.2: Reduced set of providers' feasible strategies

J	Strategy	Feasible strategies
1	*Very Low Rejection*	$p_b^{s,n,t} : F_{n,t}^s(p \le p_b^s) = 0.05$
2	*Rejection Neutral*	$p_b^{s,n,t} = Mean(F_{n,t}^s(p)) = 0.50$
3	*Very High Rejection*	$p_b^{s,n,t} : F_{n,t}^s(p \le p_b^s) = 0.950$

Since market demand varies and the market demand patterns are unknown, we show a framework to locate a Bayesian-Nash equilibrium based on a hypothetical market load distribution that relates market demand. The market demand varies from 40% to 80% of market capacity and the demand pattern represents the two scenarios in Figure 8.5.

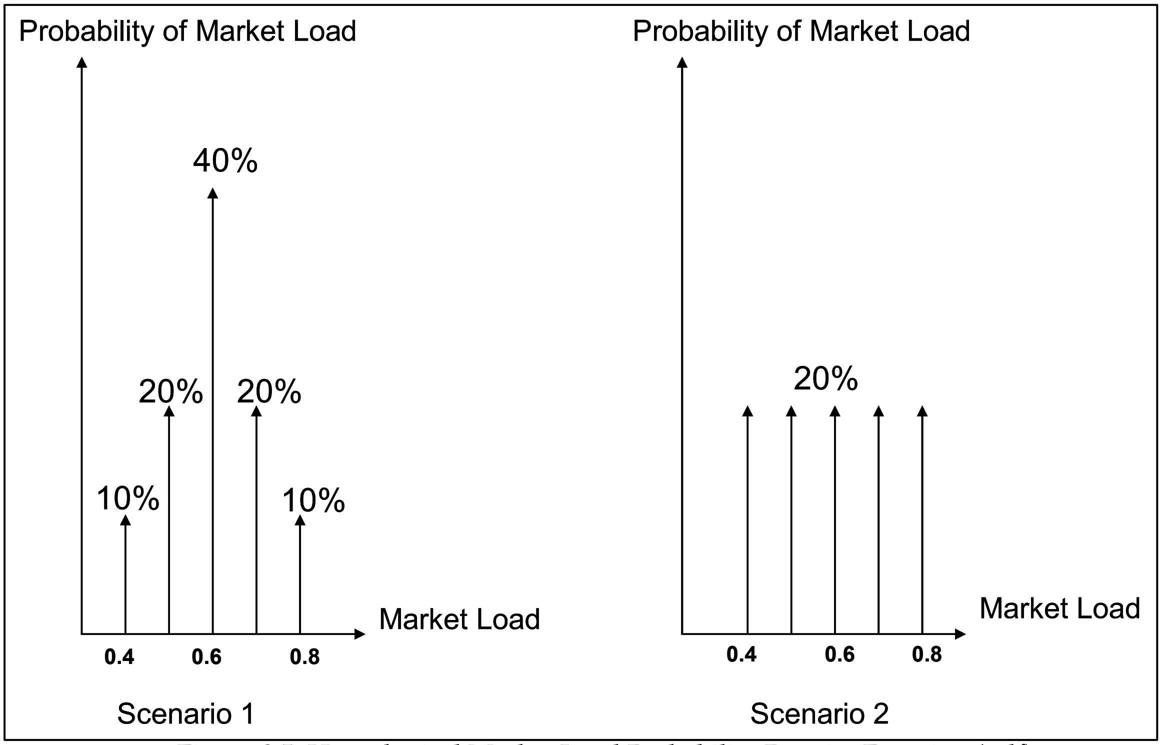


Figure 8.5: Hypothetical Market Load Probability Density Function (pdf)

In Scenario 1, the demand distribution is pseudo Gaussian Normal and in Scenario 2, the demand distribution is Uniform.

The following equations compute the expected profit share:

$$\begin{aligned} &E[\text{Proft}_A] = \sum_{\forall \rho_{Market}} \text{Load\_Probability*Profit}_A \quad \rho_{Market} = \{0.4, 0.5, 0.6, 0.7, 0.8\} \\ &E[\text{Proft}_B] = \sum_{\forall \rho_{Market}} \text{Load\_Probability*Profit}_B \end{aligned} \tag{8.6}$$

Table 8.3 and Table 8.4 present the normalized expected profit achieved by A.com and B.com for the traffic load of scenarios 1 and 2, respectively. Figure 8.7 and Figure 8-8 depict the surface 3D plots of the normalized expected profits.

In the followings, we first present tables and figures of both scenarios, and then discuss them together.

Scenario 1:

Table 8.3: Scenario 1 – The Normalized Expected Profit in Homogeneous market

		B.com		
	h_{nj}	*Very Low Rejection*	*Rejection Neutral*	*Very High Rejection*
A.com	*Very Low Rejection*	(0.51,0.51)	(0.51,0.50)	(0.65,0.59)
	Rejection Neutral	(0.50,0.51)	(0.65,0.65)	(0.82,0.76)
	Very High Rejection	(0.59,0.65)	(0.76,0.82)	(1.00,1.00) √√

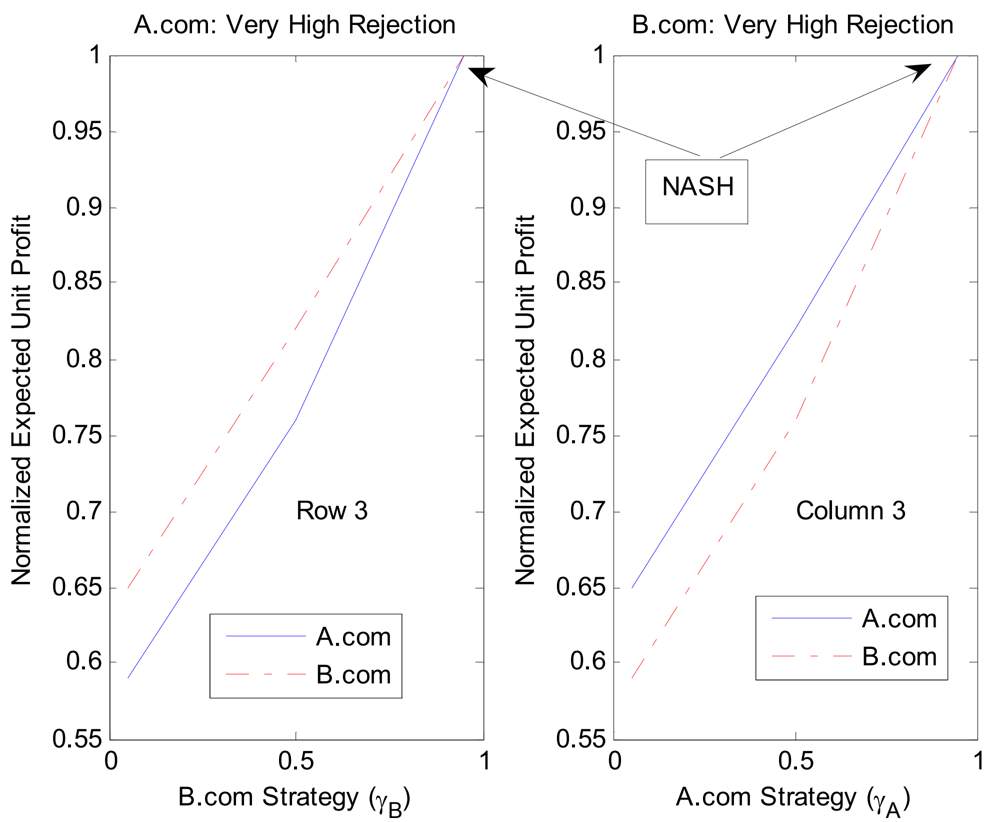


Figure 8.6: 2D Plot: Simulated Bayesian Nash Equilibrium in Homogeneous Market (Scenario 1)

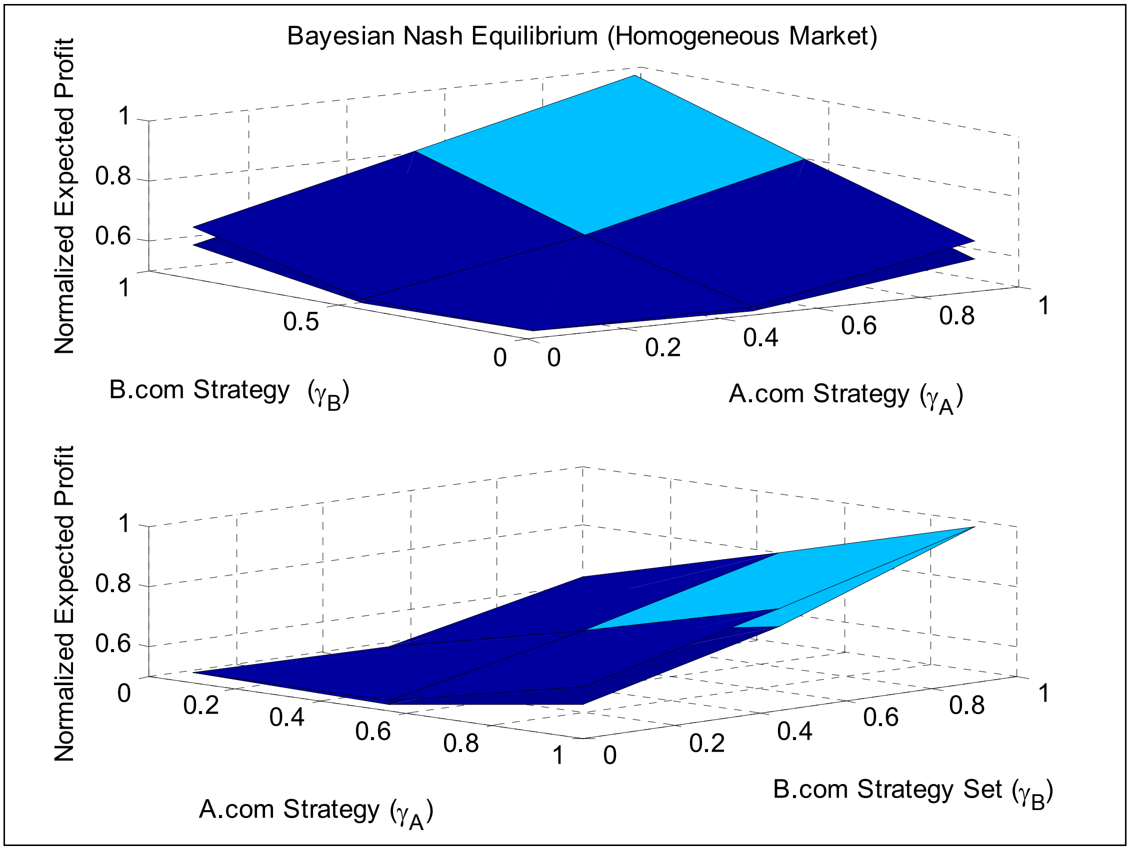


Figure 8.7: 3D Plot: Simulated Bayesian Nash Equilibrium in Homogeneous Market (Scenario 1)

Scenario 2:

Table 8.4: Scenario 2—The Normalized Expected Profit in homogeneous market

		B.com		
	h_{nj}	*Very Low Rejection*	*Rejection Neutral*	*Very High Rejection*
A.com	*Very Low Rejection*	(0.56,0.56)	(0.57,0.59)	(0.65,0.73)
	Rejection Neutral	(0.59,0.57)	(0.73,0.73)	(0.80,0.80)
	Very High Rejection	(0.73,0.65)	(0.80,0.80)	(1.00,1.00) √√

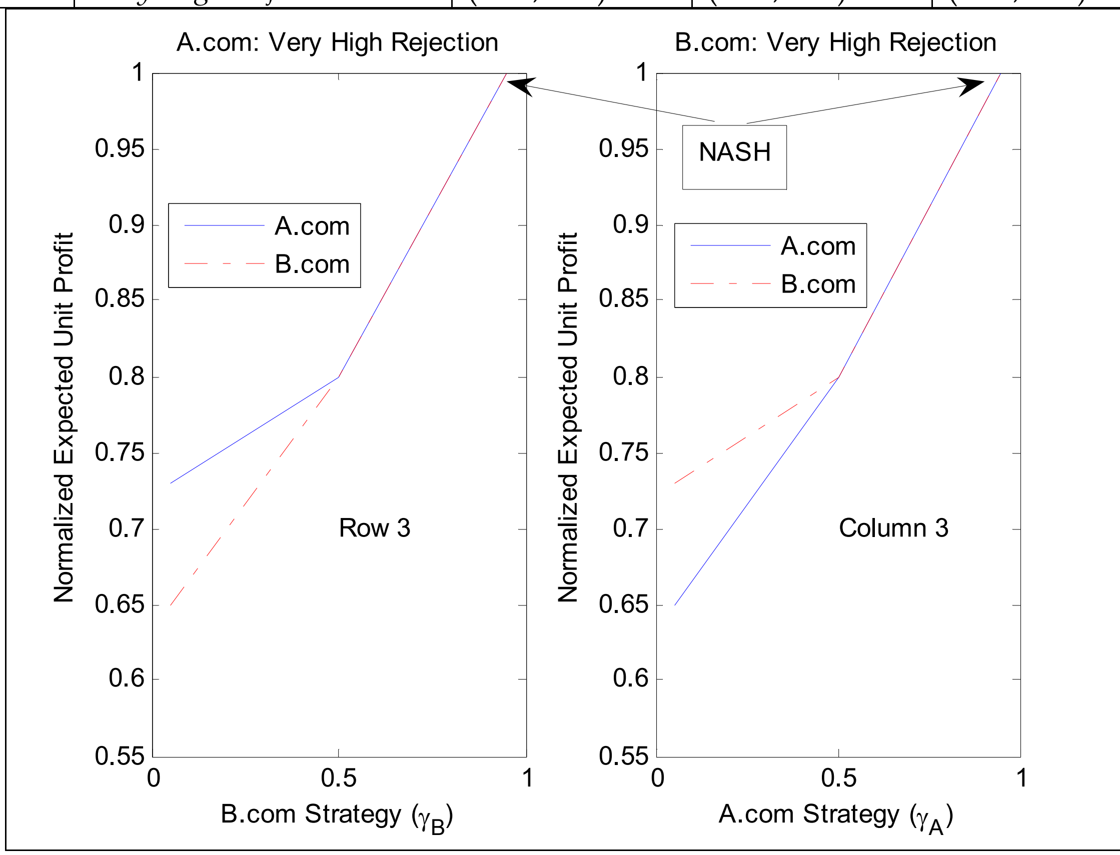


Figure 8.8: 2D Plot- Simulated Bayesian Nash Equilibrium in Homogeneous Market (Scenario 2)

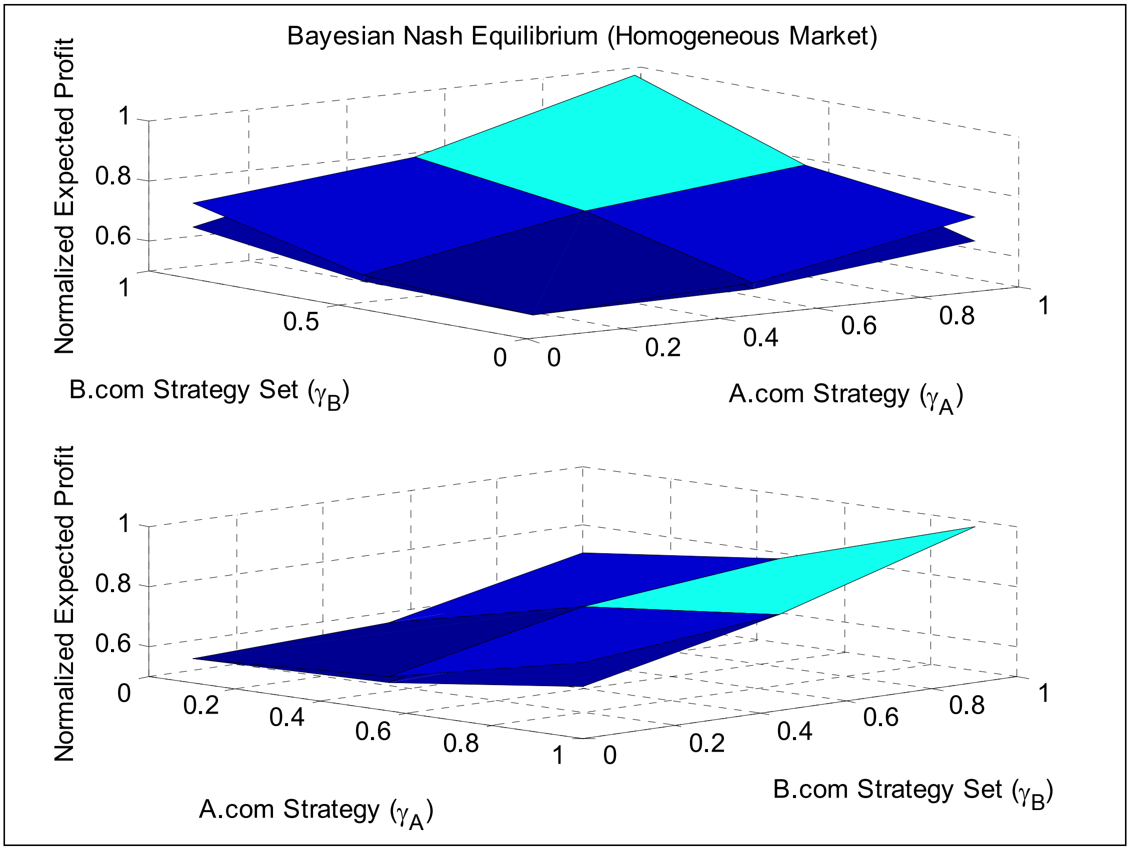


Figure 8.9: 3D Plot-Simulated Bayesian Nash Equilibrium in Homogeneous Market (Scenario 2)

Like the explanation of Chapter 7, Table 8.3 and Table 8.4 show that the *Very High Rejection* strategy is the dominant strategy for both scenarios and:

$$E[u_A(h^*_{A\_Very\_high\_Rejection}, h^*_{B\_Very\_High\_Rejection})] \geq E[u_A(h_{Aj}, h^*_{B\_Very\_High\_Rejection})].$$

This implies that the Bayesian-Nash equilibrium strategy set for ***both providers*** is {*Very High Rejection*, *Very High Rejection}*, which is marked by √√ in the above tables. This strategy set is also the Unique Bayesian Nash Equilibrium among these three strategies.

In Table 8.3 and Table 8.4, the strategy set {*Very High Rejection*, *Very High* Rejection} is a Pareto efficient outcome strategy set because there is no other strategy set (α) to meet the following criterion with strict inequality for at least one strategy (*j*):

$$u_j(\alpha) > u_j(a = \{Very\_High\_Rejection, Very\_High\_Rejection\}) \quad \forall j \qquad (8.7)$$

In 2D plots of Figure 8.6 and Figure 8.8, the x-axis identifies providers strategy set {VLR, RN, VHR} as {1, 2, 3} and the y-axis identifies providers' profit. Each plot is drawn keeping strategy of one provider fixed and varying strategy of other provider. The both sets of plots show that the strategy set {VHR, VHR} is the Bayesian Nash equilibrium strategy set of this game.

The 3D surface plots of Figure 8.7 and Figure 8.9 also show that for each scenario, there is only one peak representing the unique Bayesian Nash equilibrium and the Pareto-efficient outcome strategy set.

Like the analytical method, this strategy set yields the best strategy (the Pareto-efficient outcome) when averaged across the market demand profiles of Figure 8.5. However, this set is not safe to adopt because a provider can change its strategy to *Low Rejection* strategy in low market demand to obtain higher profit as described in Section 8.1.4.1, where the safe strategy set is identified as {*Rejection Neutral, Rejection Neutral*}.

8.1.4.3 The Routing Scheme

The optimization problem requires specifying an initial feasible point. When a session initiation request arrives, the simulator specifies the route preference of the session as the initial feasible point of the nonlinear programming. We performed session level Monte-Carlo simulations in two types of routing schemes.

Minimum-Hop Routing Scheme: The providers first preferred to route a session in the one-hop route, then the two-hop route, and finally the three-hop route.

Maximum-Hop Routing Scheme: The providers first preferred to route a session in the three-hop route, then the two-hop route, and finally the one-hop route.

The price, marginal cost, and profit of a Maximum-Hop routing scheme in steady state are oscillatory compared to those of the Minimum-Hop Routing scheme; therefore, their standard deviations from the means were larger.

Nevertheless, the mean price obtained by both routing schemes at the same load in a steady state is close to each other. In Maximum-Hop routing scheme, a session propagates though larger number of queues; thus, change in the mean packet count in the queuing scheme for each session arrival was higher. This attributes to the higher mean marginal cost and higher standard deviation (std) from the mean for the Maximum-Hop routing scheme. Consequently, Maximum-Hop routing scheme yielded lower Unity profit. Table 8.5 illustrates a set of results.

Table 8.5: Comparison of Results: Minimum-Hop vs. Maximum-Hop

Routing Scenario		**Min-Hop**		**Max-Hop**	
Strategy		Risk Neutral		Risk Neutral	
Market Price ($)	Mean Std	89.96 2.66		90.1 3.18	
		A.com	**B.com**	**A.com**	**B.com**
Marginal Cost ($)	Mean Std	39.5 14	37.1 14	46.8 16	46.3 17
Unit Profit ($)	Mean Std	4.83e4 1.11e4	5.14e4 1.12e4	4.43e4 1.20e4	4.64e4 1.28e4
Network Load	Mean Std	0.771 0.014	0.771 0.012	0.771 0.01	0.774 0.01

8.1.4.4 Traffic Load Adjustment

Traffic load adjustment is commonly known as “load balancing” in the telecommunication industry. We will interchangeably use the term “load balancing” and “traffic load adjustment”. Let us assume that a provider has two large disjoint IP networks (Core A.com and Core B.com) ; i.e. A.com and B.com are not directly connected to each other. Traffic from different enterprises propagates through these networks. Enterprises are dual homed to both the core networks. Providers’ want to maintain desired load levels in each core network. The traditional method to accomplish this is by having routing link weight on the access links from each enterprise to the provider to load balance traffic between the dual home links. Many enterprises connect to each core network. If the provider wants to change network load level in the core network, it has to adjust all the link weights in all the access links. This requires changing link weights of all the access links, which is cumbersome and may cause customer outage.

By implementing our mechanism, providers can adjust core network loads by changing strategies in the analysts of each core. For example, if a provider wants to maintain equal network loads in both the core networks, it can accomplish this by assigning same strategy to both the core networks: e.g. the *{Rejection Neutral, Rejection Neutral}.*

Providers can adjust their network load by selecting an appropriate strategy. When a provider wishes to maintain high load in a network, it should assign lower rejection strategy. When a provider wishes to maintain low load in a network it should assign higher rejection strategies. In Figure 8.10, B.com maintains a high network utilization by adopting the *Very Low Rejection* strategy, and A.com maintains a low network utilization by adopting the *Very High Rejection* strategy.

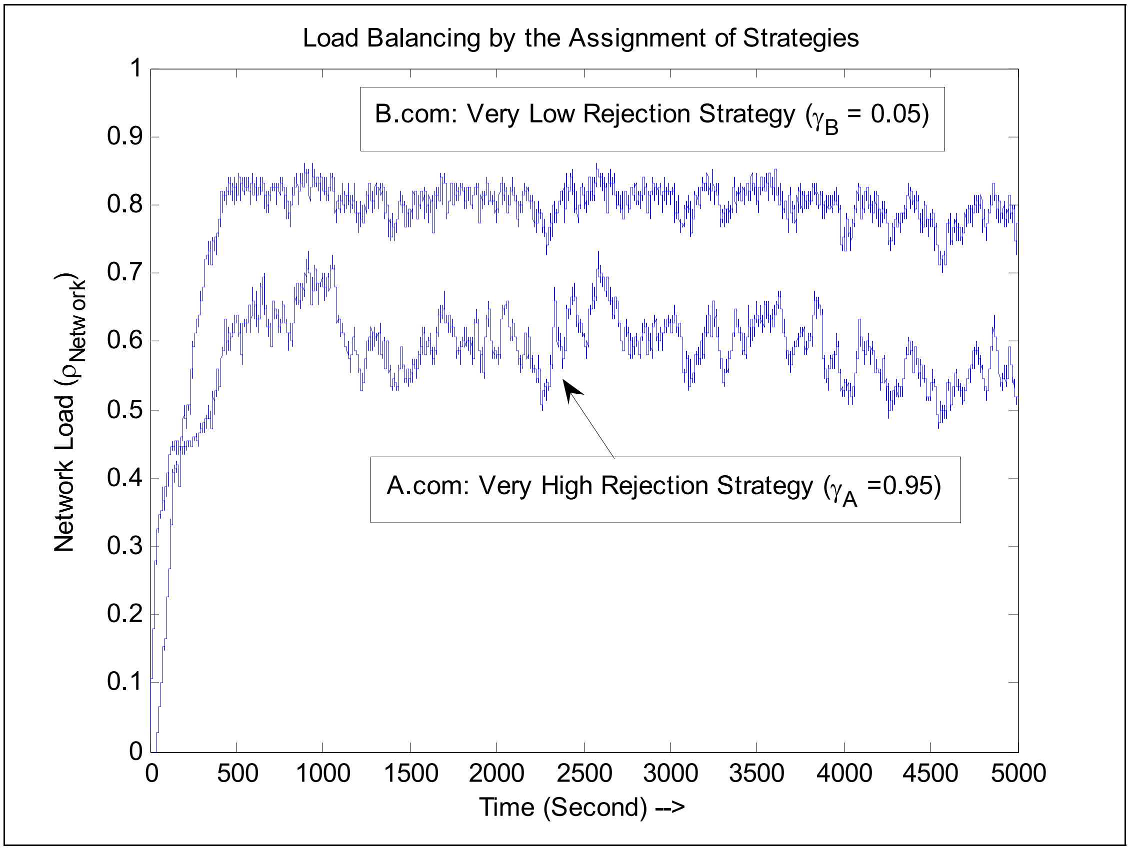


Figure 8.10: Load balancing by strategy assignment

This traffic engineering technique by assigning game strategy can also be used when a large provider has two or more disjoint core networks that transport long distance traffic for its many access networks. Here, access networks of this provider can be viewed as *enterprises* and core networks can be viewed as *providers* in our model. In this scenario, our model behaves similar to a flow controller.

Figure 8.11 shows analytical results of network load for different market load, when a provider assigns the VHR strategy to core A.com and the VLR strategy to B.com. This assignment ensures that core B.com will have higher load than A.com in all market demand.

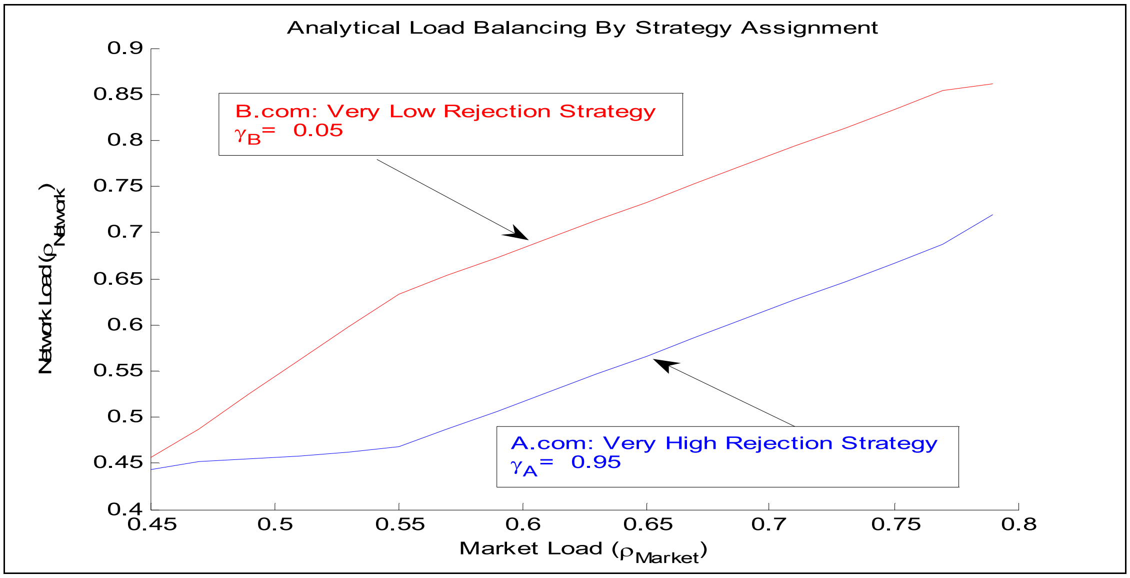


Figure 8.11: Analytical Load adjustment by Strategy Assignment

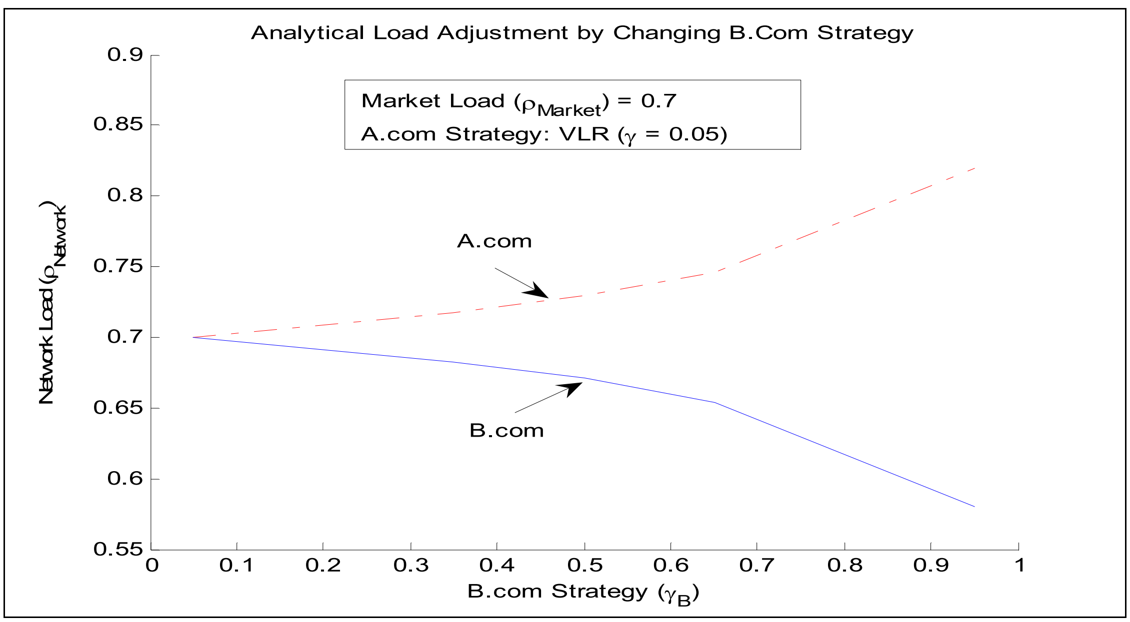


Figure 8.12: Analytical Network load for adjusting B.com strategy

In Figure 8.12, a provider knows the market load. It wants to adjust network load. It assigns the strategy VLR to core A.com. Then it changes strategy of B.com from VLR to VHR to find the appropriate network load of both A.com and B.com.

8.1.5 *Advantage of the Model*

This section describes one of our major findings of this research. This finding is that our model provides relative advantage over classical Bertrand Model of price in the Internet market.

In Table 7.4, the providers' mean prices of service were about $89-91, which was less than the enterprise reservation price of $100. Therefore, the benefit to an enterprise was about (100-90)/100 = 10%.

In the classical Bertrand model without consumer loyalty, the Nash equilibrium price settles to the common marginal cost of two providers. The Nash equilibrium price (p^*) equates to marginal cost (ω) as follows:

$$p_1^* = p_2^* = \omega \qquad (8.8)$$

In our model, both the providers' mean marginal cost were around $31.6 as shown in Table 7.4 and the mean market price turned out to be $89.0. This shows that our model had a ($89-$31.6)/$31.6 = 176% relative markup in market power as compared to the classical Bertrand model of price in infinite capacity. Comparison of the market price and the marginal cost (in both analytical and simulation studies) shows that the market price was always above the marginal cost. We also found the same trend in all combinations of strategies adopted by the two providers. We also observed that the Bayesian-Nash equilibrium market price was above the marginal cost:

$$p^* > \omega \qquad (8.9)$$

This implies that in our model providers obtain positive profit. In contrast, in classical Bertrand model Bayesian-Nash equilibrium market price is equal to the marginal cost ($p^* = \omega$). As a result, in classical Bertrand model, providers will obtain zero profit.

This proves that our model has relative advantage over the Bertrand model. This advantage is one of our major findings of this research. This advantage spawned from our implementations that synthesize game theory and traffic engineering techniques. The following aspects of our model notably influence this advantage:

- **Enforcing capacity constraint**: We promote the idea that providers refrain from the "throw bandwidth" traffic engineering practice because it adds capacity in the market. Adding capacity, similar to the Internet bubble period of late 1990s, is detrimental to the profit of all the providers. In our implementation, providers do not add capacity until the market demand of the optimized operating point (throughput) is achieved. In addition, each provider should maintain market capacity somewhat below the market demand; i.e. capacity is not underutilized. Providers should add capacity only after the optimum operating point is exceeded. Our capacity restriction according to the market demand ensures that marginal cost stays below price. On the other hand, in the classical Bertrand model capacity is underutilized.
- **Competitive bidding**: Classical Bertrand game is a one shot-game: the game ends when the player selects a price; thus, it is not an established market practice [1]. In our implementation, the game is a bidding process for each session arrival.
- **Enforcing Traffic Engineering Rule**: If we do not apply traffic-engineering rules, the mean packet count in the queue system will increase without bound. Since our marginal cost is a function of the mean packet count in a network, the marginal cost will also increase without bound. This will force the price to be close to the marginal cost. Our traffic engineering rules ensure that marginal cost remains lower.
- **Optimum Routing**: Our optimum routing techniques ensure that the traffic is well balanced across the network so that there are no congestion hot spots. A network free from congestion hot spots ensures that marginal cost remains low.

8.2 *Heterogeneous Service-based Market*

This section presents the results obtained from the experiments for the heterogeneous service-based market. This section has the same format of the last section.

8.2.1 *Experiment Objectives*

- **Validation of the model**
  - **Functional validation:** One method of functional validation is to perform qualitative evaluation of simulated results with the functional assumptions of the model. In section 8.2.3.1, we validate the model functions with the simulated results in a heterogeneous market.
- **Applications**
  - **Finding the Best Strategy set:** In section 8.2.4.1, we will find the best strategy set that optimizes providers' profit for the heterogeneous service-based market. We accomplish this by exploring the Bayesian-Nash equilibrium strategy sets and the Pareto-efficient outcome strategy set.
  - **Finding a Preferred Strategy:** Not all the Bayesian-Nash equilibrium strategies are desirable. We will select a preferred Bayesian-Nash equilibrium strategy in section 8.2.4.2.
- **Advantages of the model:** In section 8.2.5, we will discuss whether our model performs better than the classical Bertrand model.

8.2.2 *Parameters*

In the homogeneous service-based network study of Section 8.1, we explained that the *Very High Rejection*, the *Rejection Neutral*, and the *Very Low Rejection* strategies were our research interest. We concentrate on the same in a heterogeneous service-based market. These strategies are shown in Figure 8.13.

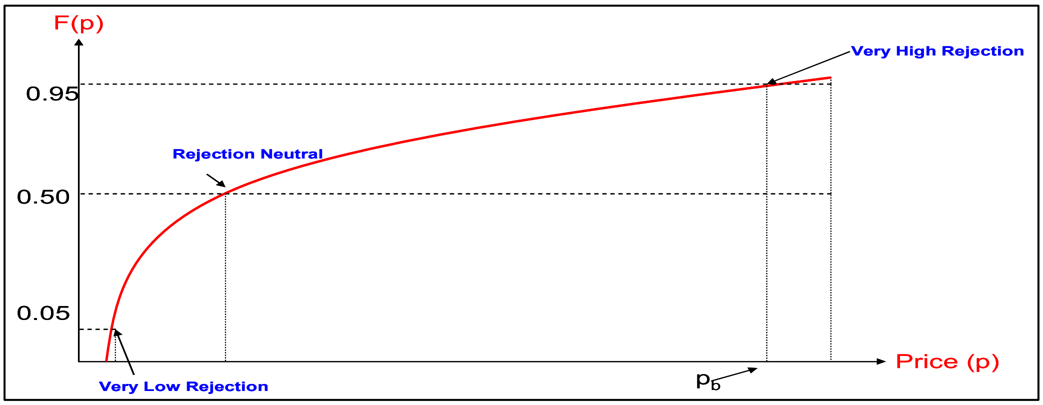


Figure 8.13: A Strategy set of heterogeneous service market

The following table summarizes main parameters of the analytical studies:

Table 8.6: Summary of Parameter for Heterogeneous services

The Class of Service	Heterogeneous: Blue, Green, Red		
Market	Duopoly		
Strategy	Strategy set of Figure 8.13		
Network Topology and TE Rules	The topology and Rules of Chapter 5		
Reservation Price (Ω)	Blue = \$160.00	Green = \$100.00	Red = \$70.00
Service cost coefficients (δ_s)	Blue = 1.0	Green = 0.10	Red = 0.01
Product Rule	Service cannot be switched. For example, an application requiring Blue security cannot switch to Green security.		
Provider fixed cost coefficient (θ)	10.0		

8.2.3 *Validation of the model*

8.2.3.1 Functional Validation

8.2.3.1.1 Experiment 1

In this experiment, we examine the validity of the model when both the providers adopt the *Rejection Neutral* strategy for all three services as shown in the following table.

Table 8.7: Heterogeneous strategies for functional validation experiment 1

Provider	Service Class	Pricing strategy	Pricing Equation
A.com	Blue, Green, Red	*Rejection Neutral*	$p_{A,s,t} = h(.) = Mean(F_{A,s,t}(p))$
B.com	Blue, Green, Red	*Rejection Neutral*	$p_{B,s,t} = h(.) = Mean(F_{B,s,t}(p))$

Figure 8.14 illustrates the simulation results. Plots a and b depict the market price and the marginal cost of A.com for the market load, ρ_{Market} =0.71. Plots c and d illustrate the mean market price and the mean marginal cost of A.com for market loads ρ_{Market} from 0.40 to 0.71.

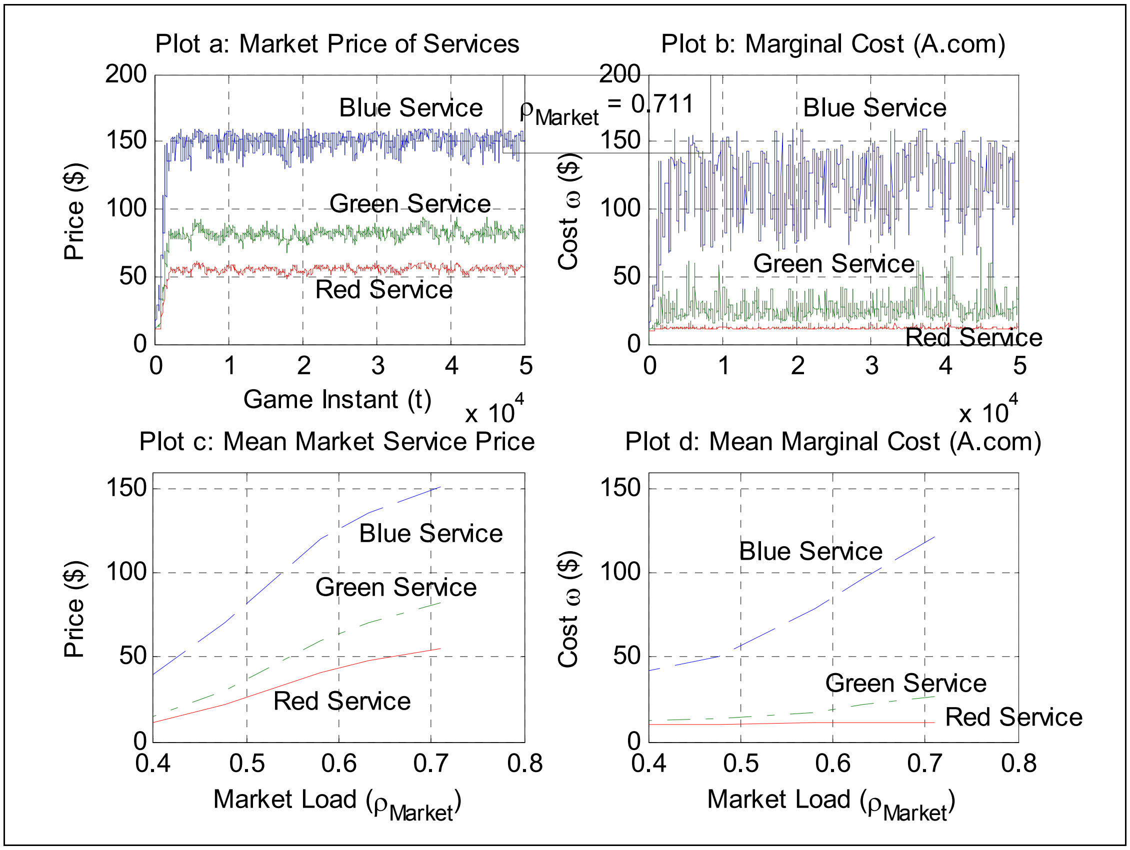


Figure 8.14: Heterogeneous Results: Price and Cost for Rejection Neutral Strategies

The Plot a of Figure 8.14 illustrates that Blue market price was higher than Green price, and Green Price was higher than Red Price at a market load of 0.71. Comparison of Plot a and Plot b shows that market price of each service class was higher than the marginal cost of each respective service class. Comparison of Plot c and Plot d shows that our oligopoly model assumptions were satisfied because each price of service was lower than the respective reservation price and was higher than the respective marginal cost in all Market Load. In addition, the price of Blue service was higher than that of Green, and Green was greater than Red. For example, Plot c shows that at the market load of 0.711, mean market prices (p_s) of Blue, Green, and Red service classes are \$151.0, \$81.9, and \$55.7, which are less than their respective reservation prices (Ω_s) of \$160.0, \$100.0, and \$70.0. Plot d illustrates that at a market load of 0.771, A.com's mean marginal costs (ω_s) of Blue, Green, and Red service class, respectively, are \$122.0, \$26.5, \$11.7. These marginal costs are less than the corresponding mean maker prices of \$151.0, \$81.9, and \$55.7. Thus, in all market demand levels, the following results are true:

$$\bar{\omega}_s < \bar{p}_s < \Omega_s \qquad (8.10)$$
$$p_r < p_g < p_b$$

These results satisfied the oligopoly assumptions stated in Chapter 3.

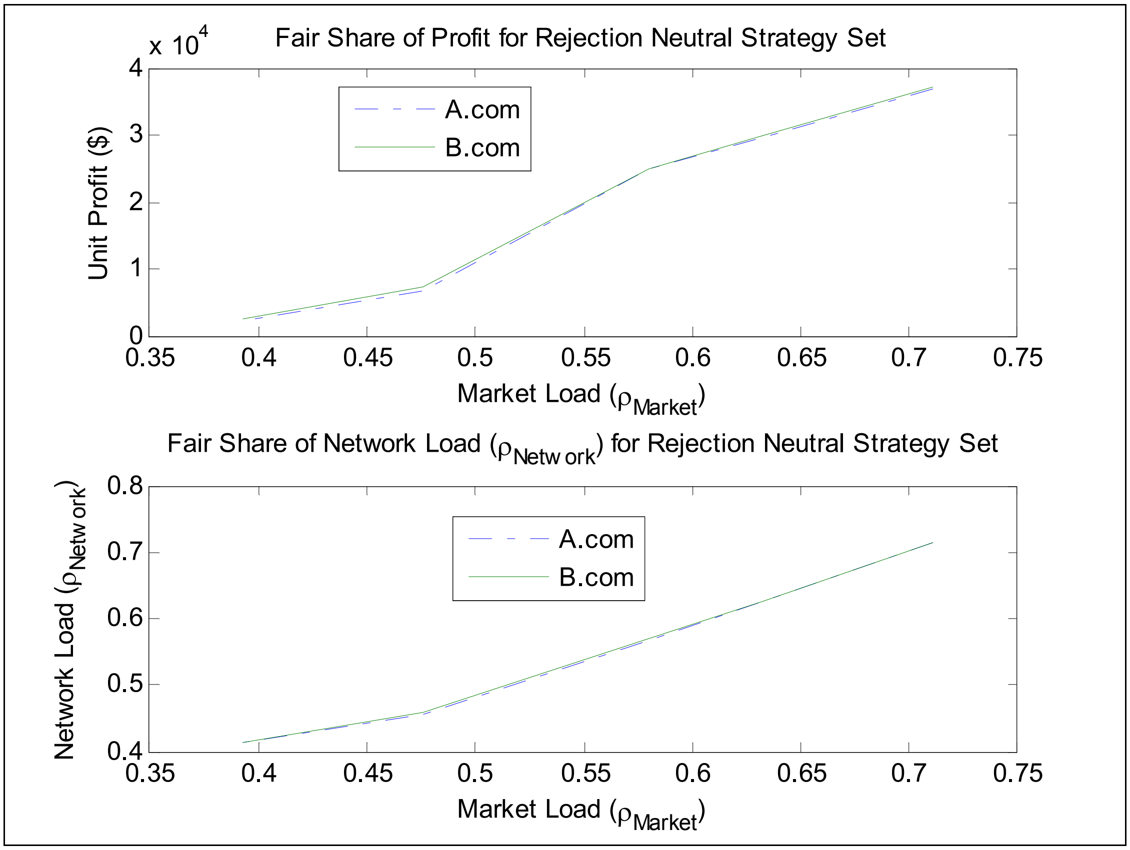


Figure 8.15: Comparison of Profit and Throughput

As shown in Figure 8.15, both providers also achieved a fair share of bandwidth and profit:

$$U_A(h(.)\text{: Rejection Neutral}) = U_B(h(.)\text{: Rejection Neutral})$$

The above results validated the anticipated behavior of the *Rejection Neutral* strategies of our model in the heterogeneous market network.

8.2.3.1.2 *Experiment 2*

The objective of this study was to observe the effect of increasing the rejection probability of the most expensive service while decreasing the rejection probability of the least expensive service. The following table summarizes the strategy set.

Table 8.8: Heterogeneous strategies for functional validation Experiment 2

<table>
<tr><th>Provider</th><th>Service Class</th><th colspan="2">Pricing strategy</th></tr>
<tr><td rowspan="3">A.com</td><td rowspan="3">Blue, Green, Red</td><td>Blue</td><td>Very High Rejection (VHR)</td></tr>
<tr><td>Green</td><td>Rejection Neutral (RN)</td></tr>
<tr><td>Red</td><td>Very Low Rejection (VLR)</td></tr>
<tr><td>B.com</td><td>Blue, Green, Red</td><td colspan="2">Rejection Neutral (RN)</td></tr>
</table>

According to the model assumptions, we expect that A.com will win almost all the Red sessions because its bid price obtained by the *Very Low Rejection* strategy is lower than the bid price of B.com obtained by the *Rejection Neutral* strategy. As a result, A.com's network load will be higher. This will cause the marginal cost of Blue service in A.com to be higher than that of B.com. As a result, A.com's profit margin (difference of price and marginal cost) from the Blue service will be lower than that of B.com. In addition, the A.com's belief function will shift to the right more than B.com in each instant of the game. A.com's bids for Blue service will be comparatively higher than those of B.com for the majority of the sessions. A.com will lose the majority of the Blue sessions; therefore, B.com's profit from Blue service will be higher than that of A.com.

Similarly, A.com's bids for Green service will be comparatively higher than those of A.com for the majority of the sessions. B.com will win the majority of the Green sessions; thus, B.com's profit from Green service will be higher than A.com.

Figure 8.16 compares the simulation results of this experiment. Plots in the left column represent A.com, and plots in the right column represent B.com. Plots in the top row illustrate the difference between the price and the marginal cost per Mbps, which we define as surplus ($\hat{p} - \hat{\omega}$). Plots in the center row depict the traffic load of each service class. The plots on the bottom row show the unit profit

($(\hat{p}-\hat{\omega})\hat{Y}$) of the providers. All these plots are drawn for the market load (x-axis) from 0.4 to 0.75.

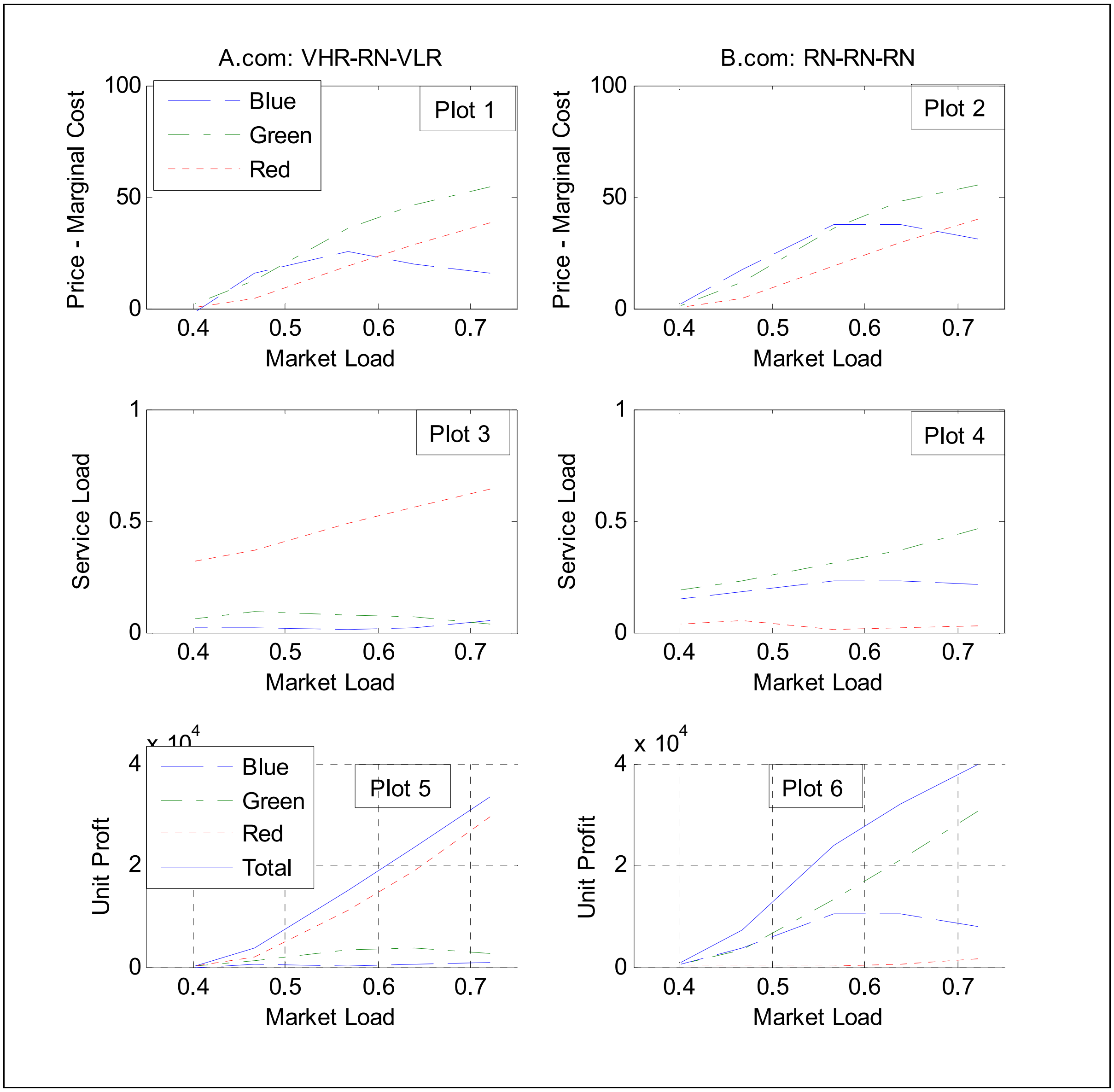


Figure 8.16: Heterogeneous Results of strategies: VHR-RN-VLR vs. RN-RN-RN

In Figure 8.16, the dotted lines in Plot 3 and Plot 4 depict the network loads of Red service class in A.com and B.com, respectively. A.com's Red bid prices were lower because A.com's *Very Low Rejection* strategy and B.com's *Rejection Neutral* strategy; thus, A.com's Red load was higher. B.com lost almost all Red bids and its Red load was very low. This also caused A.com to operate in a higher network load than B.com. Plot 5 shows that the major source of the A.com unit profit was from Red service class. On the other hand, B.com obtained a tiny portion of the profit from the Red service.

Because A.com operated in a higher network load than B.com, the Green service bid prices of A.com was higher than B.com, although both assigned the *Rejection Neutral* strategy to Green service class. Consequently, B.com won the majority of Green service class. The dashed-dotted lines in Plot 3 and Plot 4 of Figure 8.16 depict the resulting higher Green service load for B.com. The dashed-dotted lines in Plot 1 and Plot 2 show that the surplus ($\hat{p}-\hat{\omega}$) from the Green service was the highest in the higher load market compared to Blue and Red services. The unit profit curves in Plot 5 and Plot 6 show that the significant source of B.com's profit was attributed to Green service. On the other hand, A.com obtained a tiny portion of the profit from the Green service.

Similarly, higher network load and higher rejection strategy (A.com: *Very High Rejection*. B.com: *Rejection Neutral*) of A.com caused B.com to win majority of Blue services. Hence, a source of significant profit of B.com was Blue service as shown as dashed lines. For a closer validation of the above arguments, Table 8.9 presents simulation results at a market load of 57%.

Table 8.9: Results at a Market Load of 57%

Provider	A.com			B.com		
Service Class	Blue	Green	Red	Blue	Green	Red
Strategy	**VHR**	**RN**	**VLR**	**RN**	**RN**	**RN**
Mean Market Price ($\hat{p}$)	\$114.4	\$53.6	\$30.0	\$114.0	\$53.6	\$30.0
Mean Marginal Cost ($\hat{\omega}$)	\$88.7	\$18.0	\$11.0	\$76.4	\$17.5	\$10.7
$\hat{p}-\hat{\omega}$ (per Mbps)	\$25.7	\$35.6	\$18.9	\$38.0	\$36.1	\$19.3
Mean Network Load	0.01	0.08	0.49	0.24	0.31	0.01
Mean Throughput ($\hat{Y}$) Mbps	14.4	97.2	588	288	372	12
Unit Profit ($(\hat{p}-\hat{\omega})\hat{Y}$)	\$0.04e4	\$0.35e4	\$1.11e4	\$1.09e4	\$1.34e4	\$0.02e4
Provider Unit Profit	**\$1.50e4**			**\$2.45e4**		
Provider Network Load	**58.3%**			**56.0%**		

Simulation results presented in this section validated the anticipated functional behavior of the model.

8.2.3.1.3 *Experiment 3*

The objective of this study was to observe the effect of decreasing the rejection probability of the most expensive service while increasing the rejection probability of the least expensive service. We conduct this experiment to observe the opposite effect of experiment 2. The following table summarizes the strategy set.

Table 8.10: Heterogeneous strategies for functional validation experiment 3

Provider	Service Class	Pricing strategy	
A.com	Blue, Green, Red	Blue	Very Low Rejection (VLR)
		Green	Rejection Neutral (RN)
		Red	Very High Rejection (VHR)
B.com	Blue, Green, Red	Rejection Neutral (RN)	

In this experiment, we assigned A.com the *Very High Rejection* strategy for Red and the *Very Low Rejection* strategy for Blue. We conduct this experiment to observe the opposite effect of experiment 2. Our intention was to observe the effect of decreasing the rejection probability of the most expensive service while sacrificing the probability of winning the least expensive service. In this experiment, we expect that A.com' VHR strategy for Red will cause it to bid very high for Red service; thus, B.com will win the majority of the Red bids. Consequently, its traffic load will be higher. This high traffic load will cause B.com to bid comparatively higher over A.com for Blue and Green. As a result, it will lose Blue and Green services. Taking advantage of this situation, A.com will attain majority of the Blue and Green services loads.

Figure 8.17 depicts simulation results of this experiment.

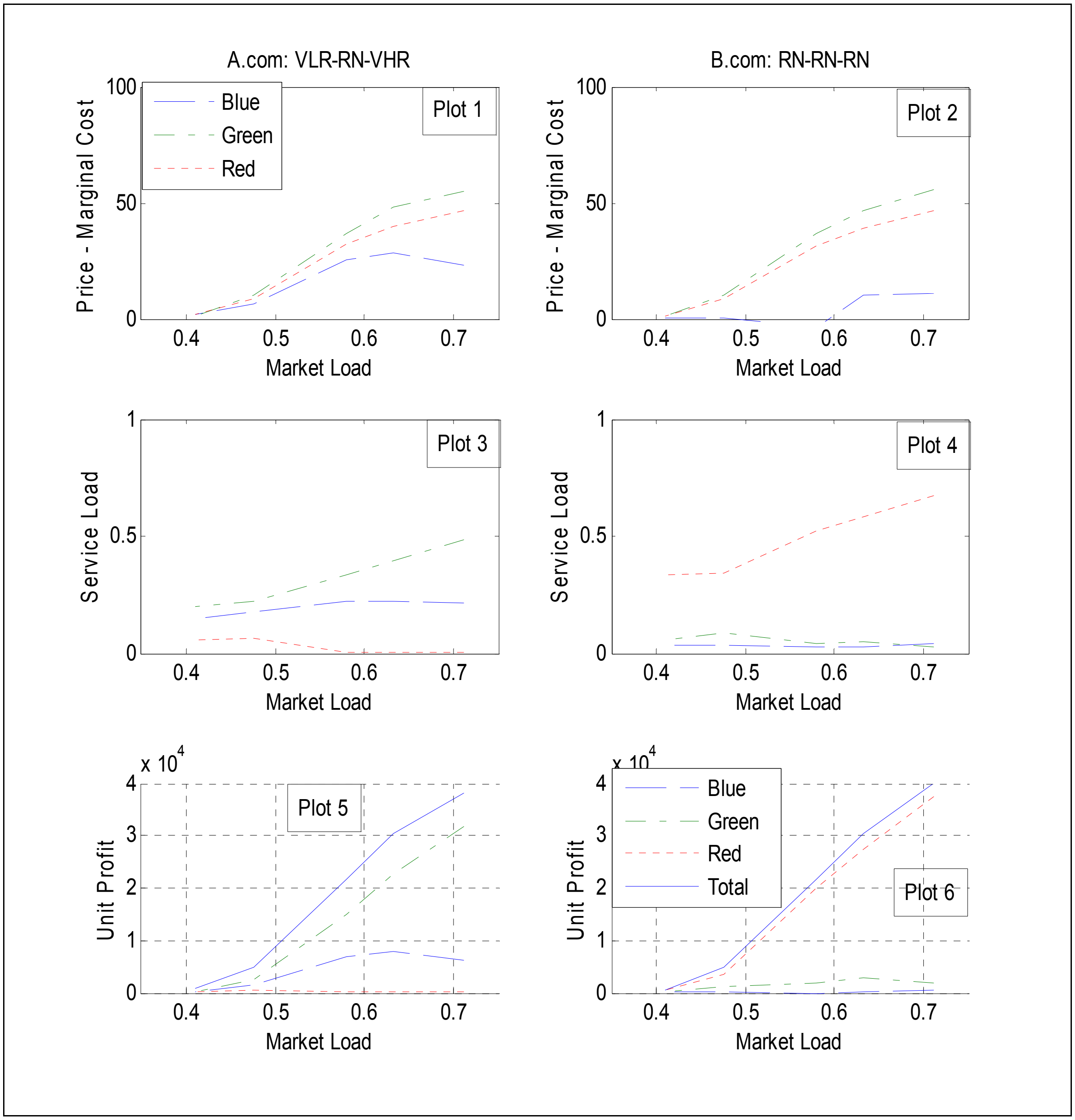


Figure 8.17: Heterogeneous Results of strategies: VLR-RN-VHR vs. RN-RN-RN

The comparison of Plot 3 and Plot 4 of Figure 8.17 shows that by having VHR strategy for Red service, A.com managed to operate in a very low Red service load. One the other hand, by having RN strategy B.com operated in high Red load. For example, at a 60% market load, A.com operated at around 0% of Red load, however B.com's Red load was at around 58% of the network load. Consequently, B.com's Blue and Green loads were less than 5% of market load. High network load of B.com also caused its Bid prices of Green service higher than those of A.com although both adopted RN strategy for Green service. Thus, A.com obtained majority of the Green service as shown in the Green load curve of Plot 3. By having a VLR strategy, A.com managed to obtain majority of the Blue traffic.

As shown in the Plot 6 of Figure 8.17, the almost all the source of B.com profit was Red service. On the other hand, Plot 5 illustrates that A.com's profit source was Blue and Green service.

Plot 1 and Plot 2 show that surplus (price – marginal cost) obtained from Green and Red services were almost the same for both providers. Surplus obtained from Blue service was less than those for Green and Red services. Thus, A.com did not achieve any advantage of profit although it tried to maximize load of the Blue service.

The lesson learned from this experiment is that although the price of the highest security providing service is the highest, its surplus was lower than the other services (a consequence of high service cost coefficient ($\delta_b = 1.0$) of Blue service). Thus, if a provider increases the winning probability of the most expensive service while its production cost is high, it will not achieve favorable outcome. A provider should decrease the rejection probability (increase the winning probability) of a service that yields higher surplus to optimize profit.

Notice that in both the experiment 2 and 3, the {RN, RN, RN} strategy set performed either equal or better than the strategy sets {VHR, RN, VLR} and {VLR, RN, VHR} in this mix of traffic and service surplus.

8.2.4 *Application*

8.2.4.1 *Finding the Pareto-Efficient Outcome Strategy Set*

As explained in Section 7.4.2, we cannot analytically determine providers' profit for dissimilar strategy sets in a heterogeneous market because each service throughput of each provider is unknown. In simulation, providers' keep records of each service throughput. Thus, we will determine Nash equilibrium by simulation. In this section, we locate the Bayesian-Nash equilibrium and the Pareto-efficient strategy set of the heterogeneous service-based network by applying the same procedure of Section 8.1.4.2.

Mapping three strategies {*Very High Rejection*, *Rejection Neutral,* and *Very Low Rejection*} and three services {Blue, Green, and Red} creates a set of 27 combinations. Strategic interaction between two providers requires conducting simulation for 27x27 combinations. This is not feasible due to the logistical limitation of this research. In addition, due to the limitations of the computing resources, providers may only select a limited set of strategies. Thus, we reduce strategies and the classes of service combinations into 3 tuples as in the following table to determine the Bayesian-Nash equilibrium. We anticipate that the providers will likely implement these strategies.

Table 8.11: Heterogeneous strategies to determine Bayesian-Nash Equilibrium

	Blue	Green	Red
VHR-RN-VLR	*Very High Rejection*	*Rejection Neutral*	*Very Low Rejection*
VLR-RN-VHR	*Very Low Rejection*	*Rejection Neutral*	*Very High Rejection*
RN-RN-RN	*Rejection-Neutral*	*Rejection-Neutral*	*Rejection-Neutral*

Scenario 1:

The simulation yields the following normalized expected profits for scenarios 1 and 2 of Section 8.1.4.2.

Table 8.12: Scenario 1--The normalized Expected profit in Heterogeneous market

	B.com			
	h_{nj}	VHR-RN-VLR	RN-RN-RN	VLR-RN-VHR
A.com	VHR-RN-VLR	(0.84,0.84) √	(0.61,0.84)	(0.63, 0.75)
	RN-RN-RN	(0.84,0.61)	(0.87,0.87) √	(0.82, 0.75)
	VLR-RN-VHR	(0.75,0.63)	(0.75,0.82)	(1.00, 1.00) √√

For Scenario 1 Table 8.12 shows that there were three Bayesian-Nash equilibriums for these strategy sets. The Bayesian-Nash equilibriums were {VHR-RN-VLR, VHR-RN-VLR}, {RN-RN-RN, RN-RN-RN}, and {VLR-RN-VHR, VLR-RN-VHR} and are marked with symbol √. The results also show that the strategy set {VLR-RN-VHR, VLR-RN-VHR} provided the Pareto-efficient outcome and is marked by the symbol √√.

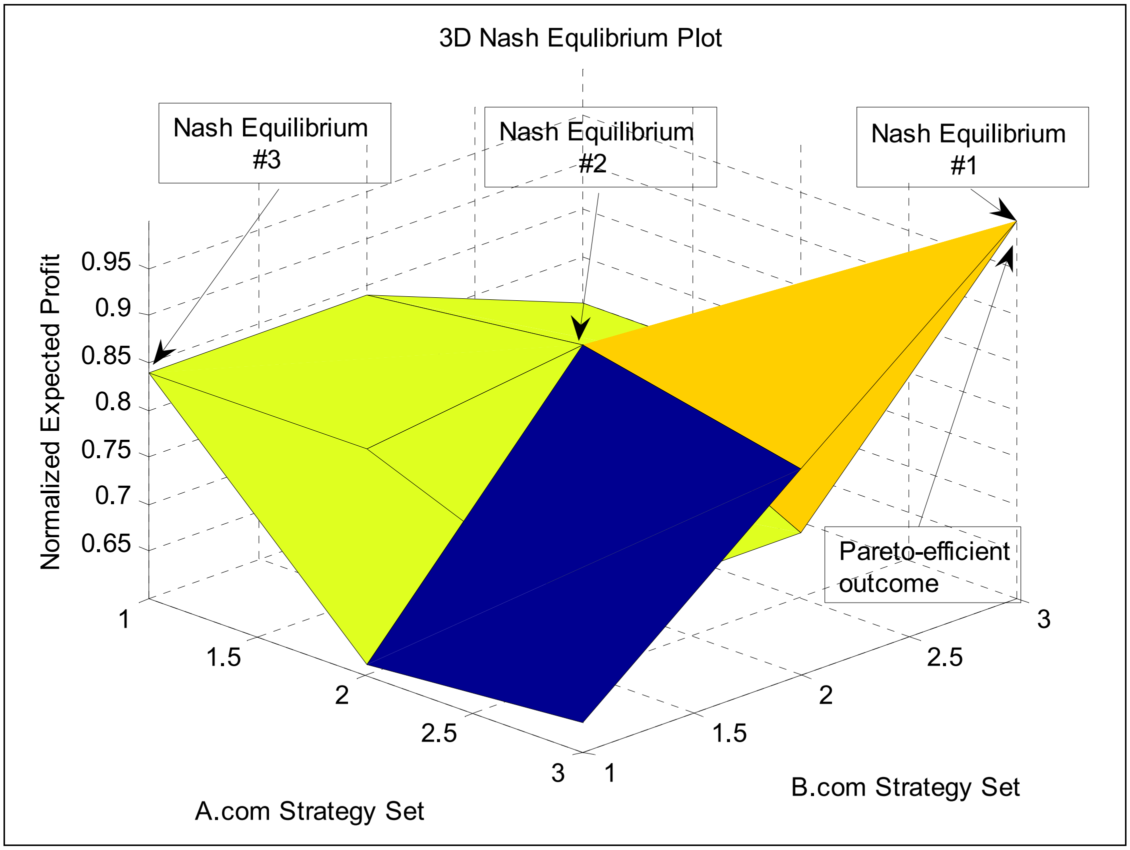


Figure 8.18: 3D Plot Simulated Bayesian Nash Equilibrium in Heterogeneous Market (Scenario 1)

The 3D surface plot of Figure 8.18 shows that there are three peaks representing three Bayesian Nash equilibrium strategy sets. The highest peak represents the Pareto-efficient outcome strategy set. Note that the x-axis and the y-axis represent the three strategy sets of A.com and B.com as {1, 2, 3}.

In 2D plots of this section, the x-axis identifies providers strategy set {VHR-RN-VLR, RN-RN-RN, VLR-RN-VLR} as {1, 2, 3} and the y-axis identifies providers' profit. Each plot is drawn by keeping the strategy of one provider fixed and by varying strategies of the other provider. The both sets of plots show that the strategy set {VLR-RN-VLR, VLR-RN-VLR} is the Bayesian Nash equilibrium strategy set of this game.

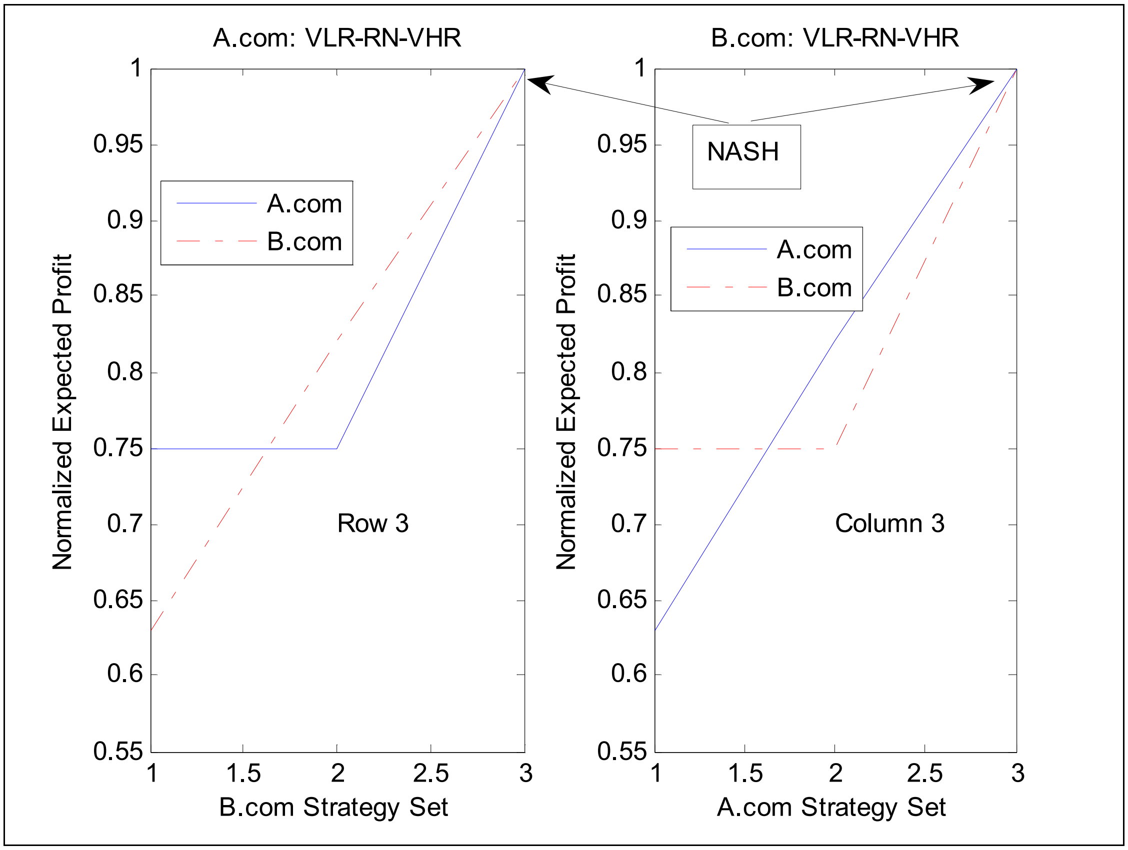


Figure 8.19: 2D Plot--Simulated #1 Bayesian Nash Equilibrium in Heterogeneous Market (Scenario 1)

Figure 8.19 shows the Nash equilibrium #1 in 2D view. This Nash equilibrium corresponds to the Row 3 and Column 3 of Table 8.12

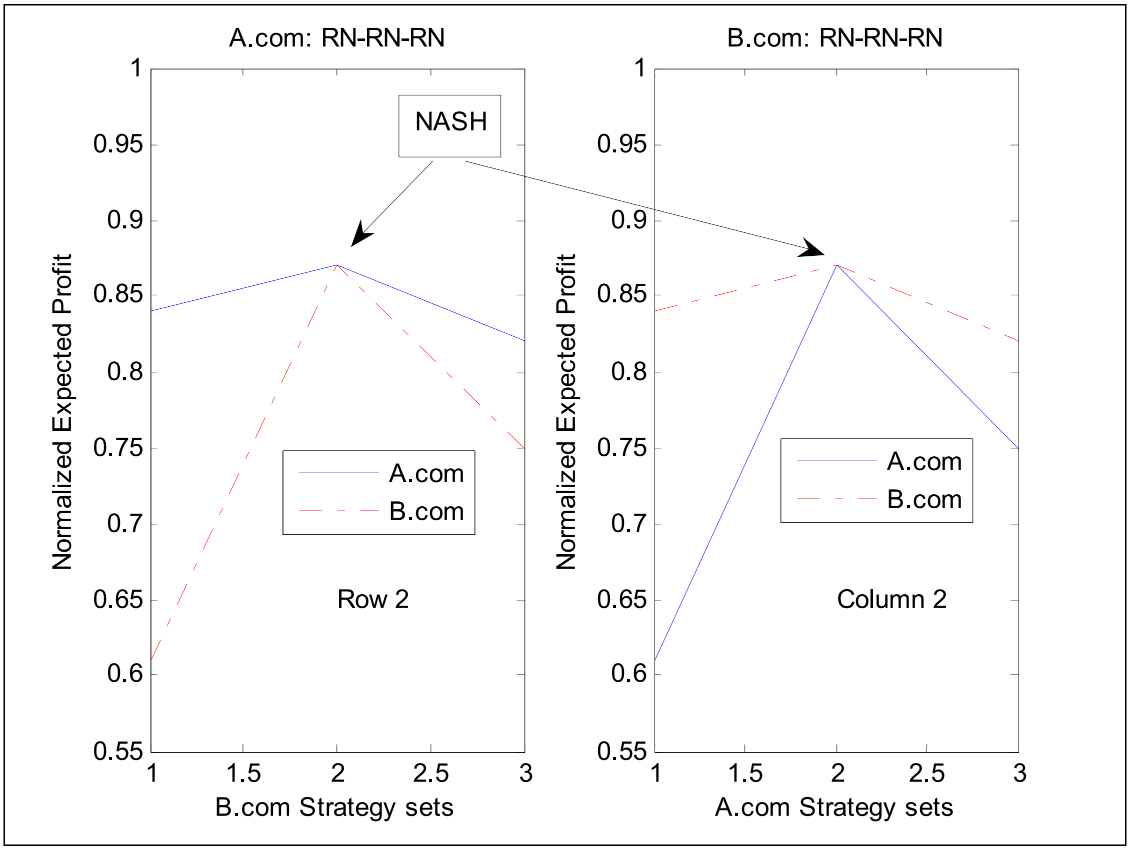


Figure 8.20: 2D Plot—Simulated #2 Bayesian-Nash Equilibrium in Heterogeneous Market (Scenario 1)

Figure 8.20 shows the Nash equilibrium #2 in 2D view. This Nash equilibrium corresponds to the Row 2 and Column 2 of Table 8.12.

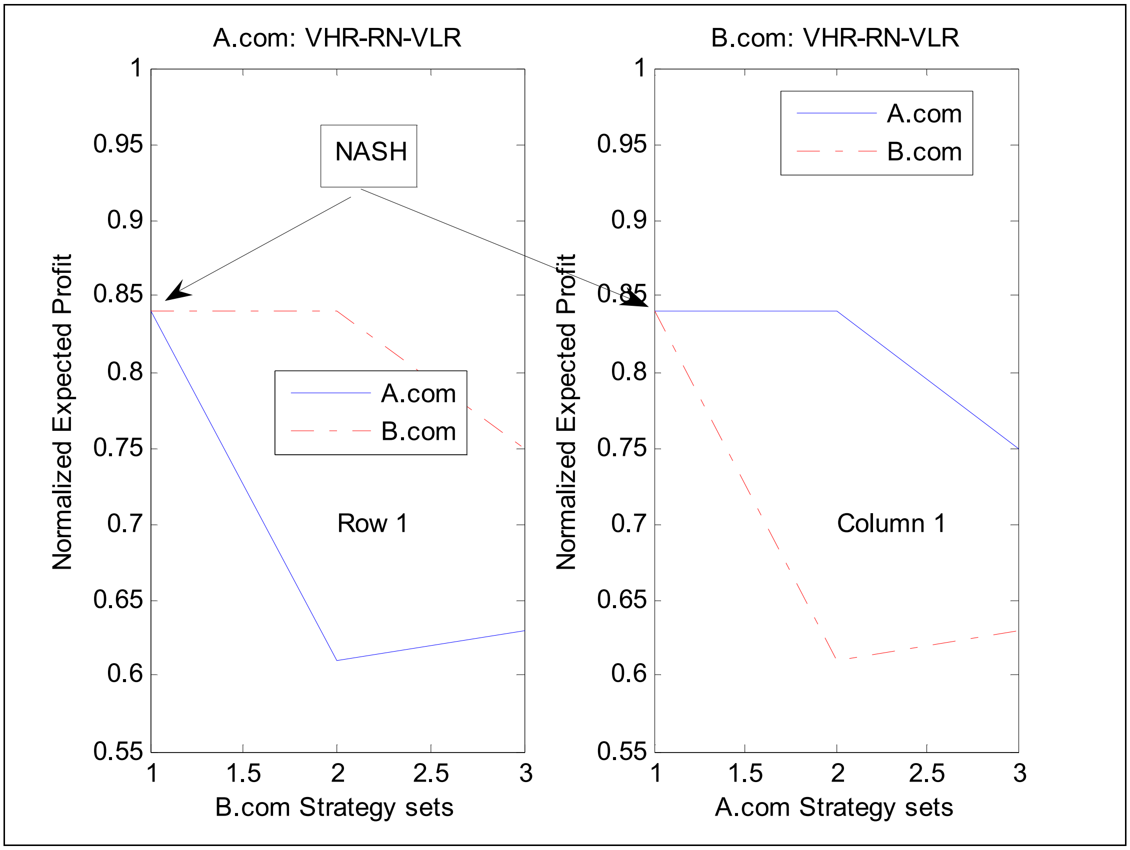


Figure 8.21: 2D Plot--Simulated #3 Bayesian-Nash Equilibrium in Heterogeneous Market (Scenario 1)

Figure 8.21 shows the Nash equilibrium #3 in 2D view. This Nash equilibrium corresponds to the Row 1 and Column 1 of Table 8.12.

Scenario 2:

Table 8.13: Scenario 2--The normalized Expected profit in Heterogeneous market

	B.com			
	h_{nj}	VHR-RN-VLR	RN-RN-RN	VLR-RN-VHR
A.com	VHR-RN-VLR	(0.84,0.84)	(0.65,0.86)	(0.66,0.77)
	RN-RN-RN	(0.86,0.65)	(0.87,0.87) √	(0.86,0.78)
	VLR-RN-VHR	(0.77,0.66)	(0.78,0.86)	(1.00,1.00) √√

For Scenario 2 Table 8.13 shows that there were two Bayesian-Nash equilibriums for these strategy sets. The Bayesian-Nash equilibriums were {RN-RN-RN, RN-RN-RN} and {VLR-RN-VHR, VLR-RN-VHR} and are marked with symbol √. The results also show that the strategy set {VLR-RN-VHR, VLR-RN-VHR} provided the Pareto-efficient outcome and is marked by symbol √√.

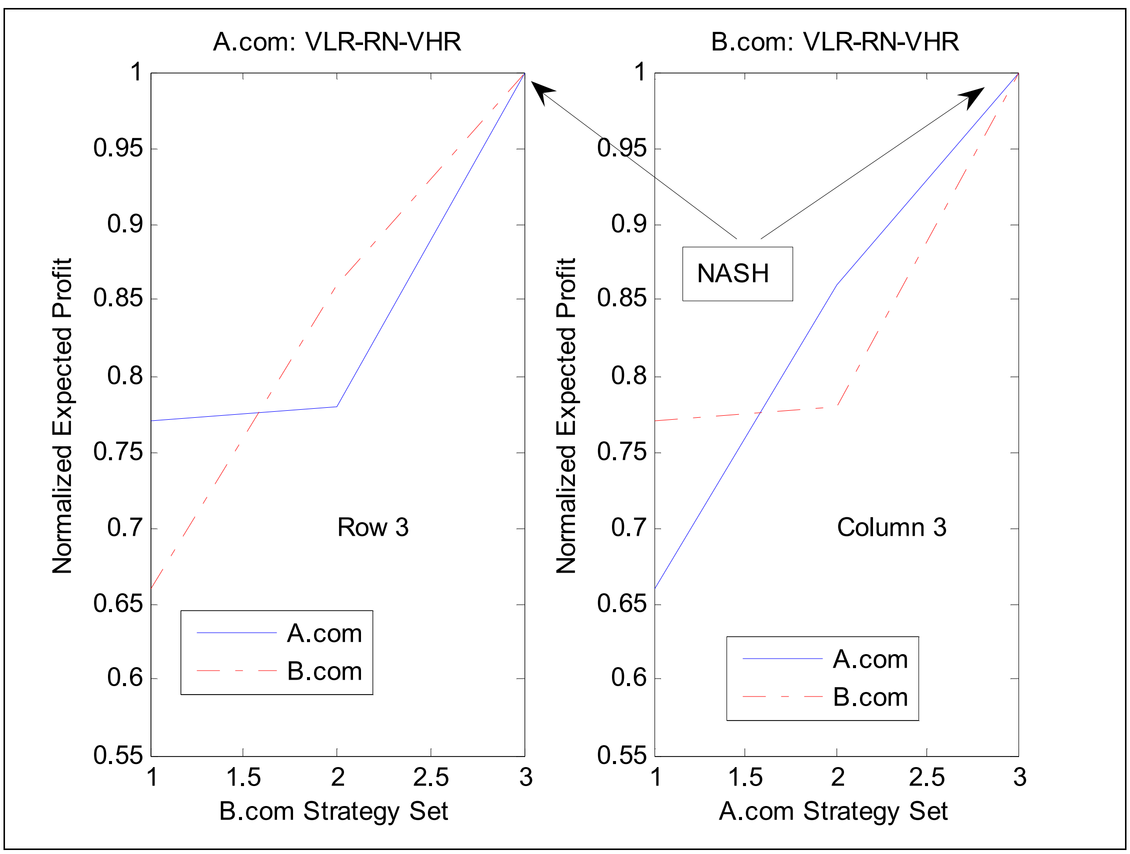


Figure 8.22: 2D Plot—Simulated #1 Bayesian-Nash Equilibrium in Heterogeneous Market (Scenario 2)

Figure 8.22 shows the Nash equilibrium #1 in 2D view. This Nash equilibrium corresponds to the Row 3 and Column 3 of Table 8.13.

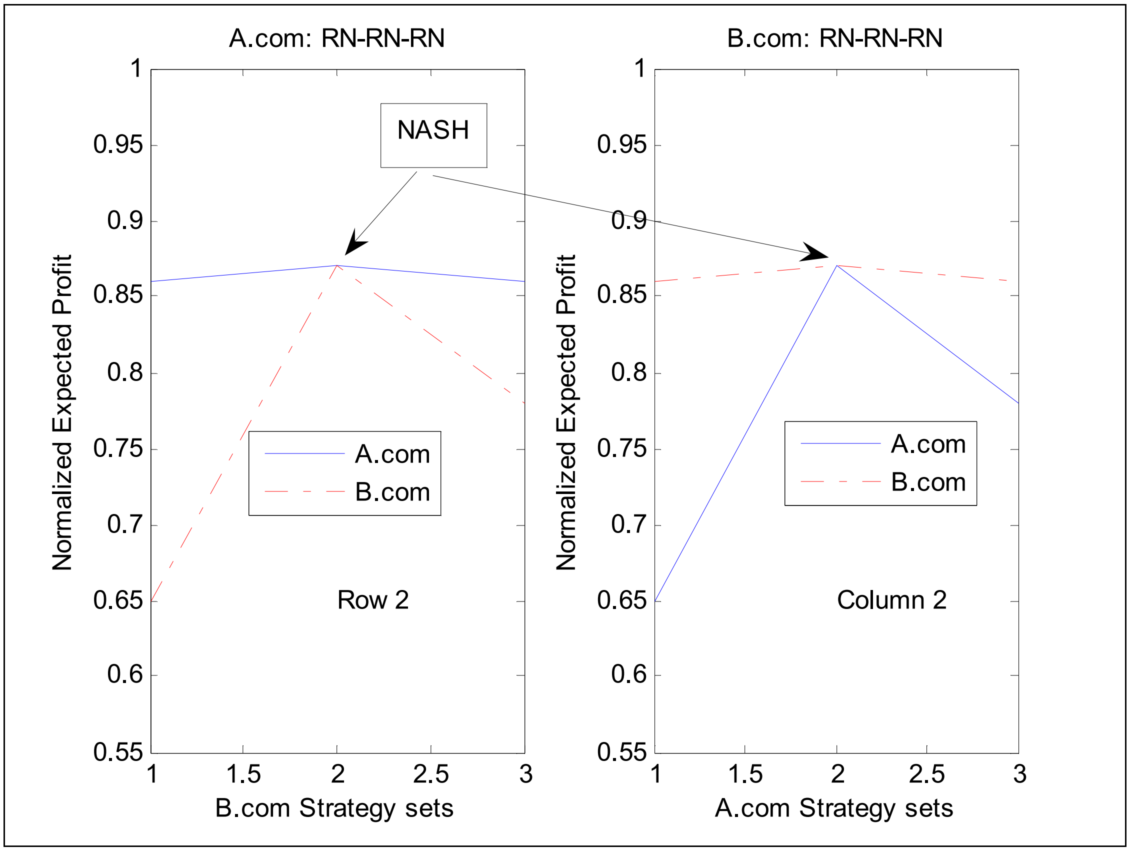


Figure 8.23: 2D Plot--Simulated #2 Bayesian Nash Equilibrium in Heterogeneous Market (Scenario 2)

Figure 8.23 shows the Nash equilibrium #2 in 2D view. This Nash equilibrium corresponds to the Row 2 and Column 2 of Table 8.13.

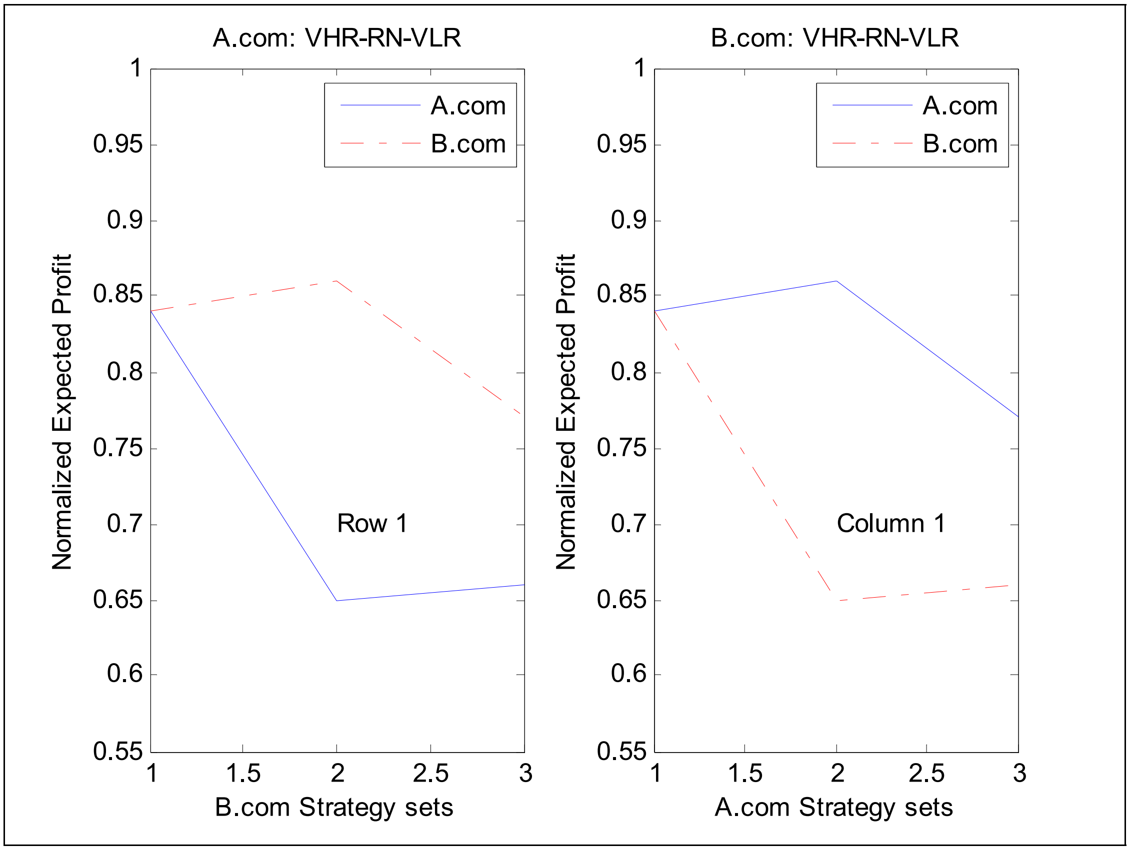


Figure 8.24: Example of No Bayesian Nash Equilibrium

Figure 8.24 shows the Nash equilibrium #2 in 2D view. This Nash equilibrium corresponds to the Row 1 and Column 1 of Table 8.13. We can see that if one provider can improve profit by changing strategy in expense of other provider's profit; thus, there is no Nash equilibrium in Row 1 and Column in scenario 2.

8.2.4.2 Preferred Strategy

According to the transitive preference properties of the enterprises as stated in section 3.2, the market price of services should satisfy the following equation:

$$p_b > p_g > p_r \tag{8.11}$$

This equation implies that the price of Blue service should be strictly higher than that of Green service. Similarly, the price of Green service should be higher than that of Red service.

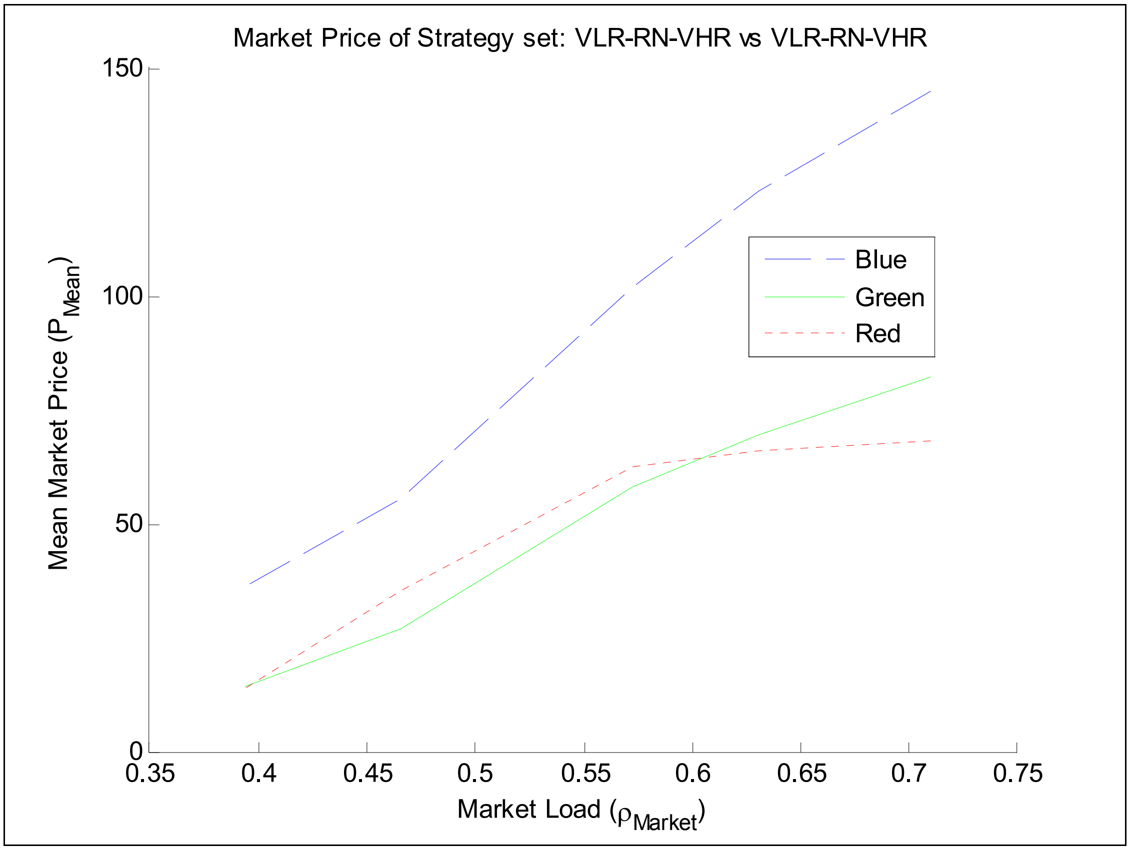


Figure 8.25: Price of Services: VLR-RN-VHR vs. VLR-RN-VHR

Figure 8.25 depicts simulated mean price of services at different market load levels for the Bayesian-Nash strategy set {VLR-RN-VHR, VLR-RN-VHR}, where A.com and B.com, respectively, adopt VLR-RN-VHR and VLR-RN-VHR strategies. In some market demand, the price of Red was higher than Green.

Figure 8.26 illustrates a cause of this situation. The Belief function (F(p)) of Red service is shown as a solid line. The Belief function (F(p)) of Green service is shown as a dash-dotted line. In high load, the Belief function of Red service moves to the right and comes close to that of Green service. The *Very High Rejection* strategy of Red class yields higher price from Red Belief function than that of the *Rejection Neutral* strategy of Green class from the Green Belief function.

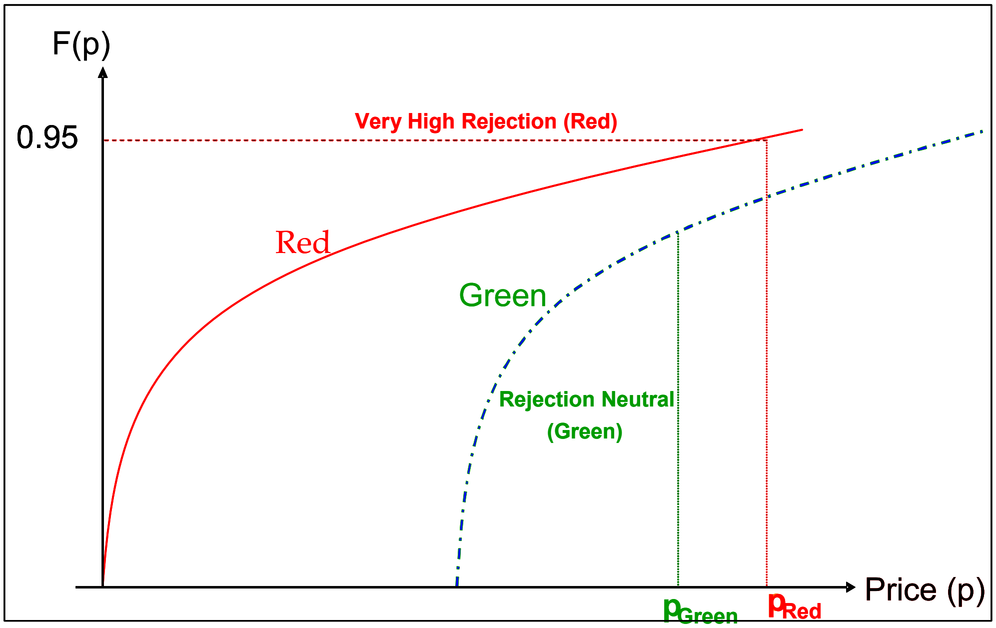


Figure 8.26: Cause of Red Price higher than Green

Applications' security requirements do not change; thus, the consumers' do not switch their product preferences. The price of Red higher than Green violates the preference properties of equation (8.11). Most importantly, customers will not make agreement to subscribe Red services; instead, they will select higher security providing and cheaper Green service if they know that providers' will deploy VLR-RN-VHR, VLR-RN-VHR} strategy set. Hence, the strategy set {VLR-RN-VHR, VLR-RN-VHR} is not desirable.

As shown in Table 8.12 and Table 8.13, the next Bayesian-Nash equilibrium strategy set is {RN-RN-RN, RN-RN-RN}. As depicted in the Plot 3 of Figure 8.14, the prices of service satisfy equation (8.11) for the strategy set {RN-RN-RN, RN-RN-RN}. Therefore, we recommend the {RN-RN-RN, RN-RN-RN} strategy set as the best preferred set for a heterogeneous service-based market. Note that this is similar to the recommended strategy set for a homogeneous market.

8.2.5 *Advantage of the Model*

This section describes one of our major findings of this research. This is finding is that our model provides relative advantage over classical Bertrand model of price in the heterogeneous service-based Internet market.

The heterogeneous market analytical and session level Monte-Carlo simulation results in sections 7.4 and 8.2.3 show that price is above marginal cost and providers received positive profit. The Bayesian-Nash equilibrium price was higher than the marginal cost of each class of service as follows:

$$\begin{aligned} p_b^* &> \omega_b \\ p_g^* &> \omega_g \\ p_r^* &> \omega_r \end{aligned} \tag{8.12}$$

This implies that our model ensures positive profit for the providers. In contrast, in classical Bertrand model Bayesian-Nash equilibrium market price is equal to the marginal cost ($p^* = \omega$) when the consumers do not switch services. As a result, in classical Bertrand model, providers will obtain zero profit.

Thus, the novel model for the heterogeneous market yields relative advantage over the classical Bertrand model without service switch. The implementation aspects of our model that combined to yield this advantage were presented in Section 8.1.5.

Note that in Bertrand model by using microeconomic service differentiation, providers can achieve a mark-up advantage (i.e. positive profit) over the classical Bertrand model when consumers have option to switch services. However, in our model consumers do not switch service class because it is based on the preference of application security requirements. Chapter 3 describes this preference.

8.3 *Chapter Summary*

We conducted analytical and session level Monte-Carlo simulation studies in homogeneous and heterogeneous service-based networks. Simulation results adequately validated analytical results. Simulation results also verified the functional behavior of the model. The unit profit curves obtained by our model satisfied the properties of the profit function. Thus, the model allowed providers to determine the optimum network load that maximized their profit.

Our optimized routing method shows that the Minimum-Hops routing scheme yields slightly higher profit compared to the Maximum-Hops routing scheme.

Our framework determined the dominant strategy, the Bayesian-Nash equilibrium strategies, the Pareto-efficient strategy, and the preferred strategies. In the homogeneous service-based market, a unique Bayesian-Nash equilibrium existed for the {*Very High Risk, Very High Risk}* strategy set. This strategy set also provided the Pareto-efficient outcome. In contrast, Bayesian-Nash equilibriums existed in the heterogeneous service-based market for strategy sets: {VHR-RN-VLR, VHR-RN-VLR}, {RN-RN-RN, RN-RN-RN}, and {VLR-RN-VHR, VLR-RN-VHR}. The Pareto-efficient outcome was {VLR-RN-VHR, VLR-RN-VHR}. We observed, however, that not all Bayesian-Nash equilibriums were preferable in maintaining service price order. The best-preferred strategy was the *Rejection Neutral* strategy for all classes of service.

Our model provided relative advantage over the classical Bertrand model, which is one method to determine prices of services in the Internet today. Our model illustrated the relative mark up in providers' market power compared to the Bertrand model in both heterogeneous and homogeneous service-based markets when consumers do not switch services. In the Bertrand model, the Nash equilibrium price converged to the marginal cost; thus, providers earned zero profit.

On the other hand, the Bayesian-Nash equilibrium market prices in our model were much higher than the marginal cost; therefore, providers gained positive profit.

Our model also benefited enterprises and wireless customers because the market price was always less than the enterprise/customers budget even though providers optimized their profits.

In some strategic markets, competitors randomly select price bids [13][14]. The *Rejection Neutral* strategy provides the same mean results as the *Random Rejection* strategy and both strategies result in a fair profit share and bandwidth. Therefore, the *Rejection Neutral* strategy can be used to complement the *Random Rejection* strategy.

Another key lesson is that the change in market demand changes the winning provider and affects their relative revenues when two providers adopt dissimilar pricing strategies in a homogeneous service-based market. At higher market demand levels providers earn a higher profit share by playing high rejection strategies. At lower market demand levels, providers earn a higher profit share for low rejection strategies. For example, a provider acquires a larger profit share at market load levels above 0.70 for the *Very High Rejection* strategy if the other provider adopts the *Risk Neutral* strategy. At low market load levels (ρ <0.70), the *Very High Rejection* strategy results in smaller profit shares relative to a *Risk Neutral* competitor. Thus, providers may not always enjoy a higher profit share due to the dynamic nature of Internet traffic if they cannot accurately forecast market demand levels and interactively adjust strategies. As mentioned earlier, the *Rejection Neutral* strategy profit share is indifferent to the change in market demand and is the preferred safe strategy.

Our model allows a provider to increase or decrease profit shares by appropriately assigning strategies in a heterogeneous service-based market. A provider's strategy should be to bid high for Red service, and consequently allow opponents to win the majority of Red bids exhausting their network capacity. The provider's strategy should be neutral or low rejection for high valued services.

However, assigning very low rejection strategy to high valued services and very high rejection strategy to valued services may break customers' price preferences.

We also learned that if all providers adopt the same strategy, they gain fair shares of profit. For example, the *Rejection Neutral* strategy ensures that the providers enjoy a fair share of profit and load at all market demand levels. When all providers adopt the *High Rejection* strategy, their profit and market price increase. Note, however, that the high market price has a detrimental effect on market demand according to microeconomics [1][2]; this effect was not studied (or modeled) here.

9 *Conclusion*

9.1 *Summary of Contributions*

9.1.1 *A Novel Automatic Price Transaction Architecture*

We introduced the novel *Automatic Price Transaction-based One-to-Many Peer Network* architecture that automates price negotiation between customers and multiple providers prior to the session establishment request. A customer can simultaneously request a service price from multiple providers and subscribe with the provider that offers the lowest price.

The architecture includes an *Analyst* module in each provider network and *Price Broker* modules in both the customer and the provider peer interfaces. The *Price Broker* module of each customer performs price negotiations with the *Price Broker* modules of all the providers in a one-to-many peer network. The protocol to perform this price negotiation is analogous to the sealed-bid-reverse-auction. The *Analyst* of each provider computes a competitive service price and feeds the price to the providers' *Price Booker.* The *Analyst* computes the price based on the *Providers Optimized Game in Internet Traffic* model.

The architecture will help small Internet Service Providers (ISPs) to broadcast their budget and instantaneously subscribe from the large ISP of their choice based on the lowest service price. Similarly, this architecture will allow wireless customers to negotiate price interactively with multiple wireless providers to subscribe to services from the provider that offer the lowest price. This architecture will also help provider's to select a price instantaneously in synchrony with the network congestion and the dynamic Internet traffic demand.

9.1.2 *An Extension of the Current ATIS and 3GPP Architecture*

The current Alliance for Telecommunication Industry Solutions (ATIS) standard [68] supports one-to-one peer network architecture. This standard neither includes any price negotiation nor charging components. Our architecture extends the ATIS peer network architecture to support the automatic price transaction based on one-to-many peer architecture.

The current 3rd Generation Partnership Project (3GPP) standard specifies an on-line charging method for wireless consumers. However, this standard does not specify automatic price negotiation components, enable a wireless user to shop from multiple wireless operators at the same time, nor provide any function to compute price based on game theory. Our architecture extends the 3GPP charging architecture to support all these options that 3GPP standard does not support.

9.1.3 *Session Initiation Protocol based Price Transaction Protocol*

Currently, the Internet Engineering Task Force (IETF) recommended SIP extensions and SIP components allow the introduction of a diverse range of applications and services. In addition, the RFC 3455 [67] specifies two header fields (*P-Charging-Vector, P-Charging-Function-Addresses*) to transport pricing information for the 3GPP charging mechanism. However, the IETF SIP standard does not specify a price transaction mechanism or price-based SIP call flow.

Although our architecture is protocol agnostic, we present an architecture that supports SIP entities as the *Price Broker* and the *Analyst* for the automatic price transactions. Our proposal also includes a SIP call flow to implement the price transaction protocol.

9.1.4 *The Providers Optimized Game in Internet Traffic*

We developed the new *Providers Optimized Game in Internet Traffic* model that is a viable approach in optimizing providers' profit in peer or wireless networks synchronized with dynamic Internet traffic demand. The model allows providers to offer competitive service price within customers' budget. Providers can exploit the agility of game theory to synthesize economic theories and Internet traffic engineering techniques, maximize their profit, and engineer networks' optimum performance. The model optimizes profit in two methods: Selecting a strategically appropriate price and minimizing congestion sensitive costs.

A provider can predict how other providers will strategically interact in a competitive market. This prediction is a belief function or a mixed strategy profile extended from the previous work based on the Bertrand oligopoly model of price. Our proposed belief function is sensitive to the dynamic Internet traffic demand, the network congestion, the service class, and the providers' strategies. Providers can optimize profit by adopting our recommended strategies to determine service prices from the belief functions.

Unlike dynamic game, our game does not keep or rely on the total history. However, in each game time, the game computes the change in cost from one game time to the next game time and uses this change in cost as a game parameter. Since the game looks into a *one-step* history and forgets all other history, the strategic interaction corresponds to a *myopic*[12] Markovian-Bayesian [4] static game of incomplete information.

We perform cost optimization by minimizing network congestion. The model associates the congestion indicator—the mean IP-packet count in the network queue system—with the service cost. M/M/1 queuing analysis determines the mean packet count. Our model applies two well-known non-linear programming techniques, the Gradient Projection algorithm and the Golden section line search, to minimize the mean packet count by performing optimal routing of [85].

9.1.5 *An Analytical Model, a Network Model, and a Session Level Monte-Carlo Simulator*

We designed a network, formulated an algorithm, and developed both the session level Monte-Carlo simulation and analytical models in a duopoly market. We created a session level Monte-Carlo simulation model in MATLAB that performs automatic price transactions, call set up, optimum routing, and providers' games.

[12] The meaning of the word "myopic" is nearsighted, unable to see future moves clearly.

Simulation results in various scenarios validated the Mathematical model. The simulation results showed that this network architecture optimized the profit of providers close to the analytical optimized profit.

9.1.6 *A Framework to Determine the Best Preferred Strategy*

In our model, providers can use our strategy framework to determine price from the belief function. These strategies reflect the probability of a customer rejecting a certain price of service. This new approach determines the dominant strategy, the Bayesian-Nash equilibrium strategy, the Pareto efficient outcome strategy, and the best-preferred strategy optimizing providers' profit in both the homogenous and heterogeneous service-based networks. The session level Monte-Carlo simulation results show that not all Bayesian Nash equilibrium and Pareto optimum outcome strategies are preferred strategies.

Adopting the same strategy set allows providers to obtain a fair profit share and network load. In a homogeneous service-based network, both simulation and analytical experiments illustrate that: if providers adopt the *Very High Rejection* strategy, then the Bayesian Nash equilibrium and the Pareto efficient outcome occur. However, espousing the *Very High Rejection* strategy is not a safe strategy since a provider can switch to a lower rejection strategy in low traffic demand and can obtain a higher profit share than a competitor that adopts the *Very High Rejection* strategy. A lower rejection strategy is not safe to assign because a competitor can switch to a higher rejection strategy in high market demand to maximize profit shares. At higher market demand levels, providers earn a higher profit share by playing high rejection strategies. Providers can earn a higher profit share for low rejection strategies at lower market demand levels. Thus, providers may not always enjoy a higher profit share due to the dynamic nature of Internet traffic if they cannot accurately forecast market demand levels and interactively adjust strategies. The Internet traffic demand level is unpredictable. Selecting a higher or lower rejection strategy suitable to the Internet traffic demand level is complex and impractical. The *Rejection Neutral* strategy profit share is indifferent to the change in market demand and is our recommended safe strategy.

In heterogeneous service-based network experiments, the following strategy sets yielded Bayesian Nash equilibriums: {VLR-RN-VHR, VLR-RN-VHR}, {RN-RN-RN, RN-RN-RN}, and {VHR-RN-VLR, VHR-RN-VLR}. The strategy set {VLR-RN-VHR, VLR-RN-VHR} resulted in the Pareto efficient outcome. However, the {VLR-RN-VHR, VLR-RN-VHR} set demonstrated a potential of breaking transitive preference properties by endorsing a higher price for the lower service class compared to higher service class in certain market demand levels. Thus, the {VLR-RN-VHR, VLR-RN-VHR} set was not considered as a preferred strategy. The next Bayesian Nash equilibrium set {RN-RN-RN, RN-RN-RN} maintained a price of service according to customers' transitive preference property; thus, this was weighed as the best strategy set.

9.2 *Limitations*

9.2.1 *Traffic Distribution Pattern*

The traffic distribution pattern used in this study was based on an empirical model developed prior to the incorporation of VoIP and IMS services in the internet.

9.2.2 *The Cost Function*

The cost functions of providers are proprietary information; thus, we did not have access to the cost function of any provider. We developed a cost function based on network congestion and hypothetical parameters: the service cost coefficient and the provider fixed cost coefficient. Note that these parameters are commonly assigned to both providers in our analyses for fair comparison; thus, they do not influence the comparative results of providers' strategies.

9.2.3 *Network Queue Model*

Our objective is to synthesize the game theory with the well-established queuing theory to optimize provider's profit and profit. The M/M/1 system [59] is a well-established traffic analysis method for a FIFO based queuing and scheduling system in academic fields that allows for Poisson distributed packet arrival and exponentially distributed packet length. When traffic with Poisson distributed arrival rate aggregates into an integrated FIFO queue, the aggregate arrival distribution continues to be Poisson. When traffic with Exponential distributed packet lengths merges into an integrated queue, the aggregate packet distribution is hyper-exponential. We should thus adopt the M/G/1 model for computing the mean packet count in the queue system. However, in order to use results from the theory of networks of queues, we approximate with M/M/1 model. This is one of our limitations of this research.

9.3 *Advantage*

9.3.1 *Improvement on Classical Models*

Our approach has a relative advantage over the classical Bertrand oligopoly model of price when consumers do not switch services. The classical Bertrand model of price causes the Nash-equilibrium market price of service to converge to the marginal cost of production. Our proposed model allows the market price of service to converge above marginal cost; thus, providers gain positive profit as opposed to the zero profit in the Bertrand Nash equilibrium [1]-[5]. In our model, the market price of service is always less than the customer's budget. As a result, the customers also gain positive profit by deploying the proposed price transaction architecture. Further, by implementing suitable strategies, providers can obtain a fair share of profit and desired load.

9.3.2 *Automation of Pricing and Billing*

Our proposal will eliminate the reactive time of price computation. It will take into account the dynamic nature of internet traffic while keeping the price of services within the budget of the customers. Since the price transaction mechanism is based on sealed bid reverse auction, customers are ensured to be charged less than their budget contrary to the criticism of dynamic pricing that customers may run out of budget.

9.3.3 *Synthesis of Game Theory and Traffic Engineering Techniques*

The current network architectures only meet the technological and service needs. The economic aspects are not often taken into account in network design. For example, one of the existing traffic engineering methods is the addition of bandwidth. The addition of extra capacities in an oligopoly market may cause significant unutilized capacity if the demand is lower.

The classical Bertrand model the market price settles to the marginal cost in underutilized capacity [1]; thus, providers earn zero profit. As the number of providers increases and they bring capacity in the market, a gradual reduction of market power occurs according to the Cournot model [2] and may cause providers to earn zero profit.

One the other hand, our model allows providers to obtain positive profit. We recommend that providers refrain from the "throw bandwidth" traffic engineering practice because it adds capacity in the market. Adding capacity, similar to the Internet bubble period of late 1990s, is detrimental to the profit of all the providers. In our implementation, providers do not add capacity until the market demand of the optimized operating point (throughput) is achieved. In addition, each provider should maintain market capacity somewhat below the market demand; i.e. capacity is not underutilized. Providers should add capacity only after the optimum operating point (throughput) is exceeded. Our capacity restriction according to the market demand ensures that marginal cost stays below price. Thus, providers' earn positive profit.

Another common current traffic engineering practice is to perform load adjustment by parameters such as link weights. For example, in the Border Gateway Protocol (BGP) or in the Private Network Network Interface (PNNI) implementations, the link weights are often computed inversely proportional to propagation delay without considering the economically competitive advantage of other similar routes in the Internet market. Prior to the explosive growth of the Internet, the expensive Public Switched Telephone Network's (PSTN) price of service was a function of the distance traveled (e.g. long-distance or international) by a call. Massive deployment of fiber-optic cables around the globe reduced the distance specific cost for Internet services. In addition, these methods are static, do not account the dynamic nature of internet traffic, and do not optimize provider profit. Thus, we do not implement these traditional methods.

Our proposed network architecture and algorithms performs automatic traffic engineering while maintaining the required QoS in dynamic Internet environment. In addition, we optimize the profit of participants in terms of the technology and microeconomics such as providers' strategic competition, application specific service differentiation, and network congestion sensitive cost.

9.3.4 *Implementation of Strategies*

Our method provides an advantage of strategy implementations over the current method. Currently, in some oligopoly markets, price randomization is providers' common practice [13][14]]. Since belief function is continuous, the price randomization requires an infinite number of points in the price interval.

For example, if we implement the *Random Rejection* strategy in a network, for *each call*, an *Analyst* of each provider will have to develop a *discrete belief function* for a selected number of prices, pick a random number that will indicate the rejection probability, and find a price from the belief function that corresponds to this number. This mechanism will be hard to implement because it will require extra processing and memory to develop and store the belief function.

On the other hand, in our model, an *Analyst* of each provider is not required to develop a belief function during each call because proposed strategies are *algebraic functions* of network and market parameters (See Chapter 7). For example, The *Rejection Neutral* strategy or the *Random Rejection* strategy algorithms (See Section 8.1.3) can be implemented using algebraic functions.

9.4 *Practical Applications*

9.4.1 *Automatic Price-based Services*

The main application of this proposed method is to enable an automatic system to instantaneously compute strategic congestion-sensitive prices of Internet services in a competitive market and to optimize providers' market share of profit.

9.4.2 *Profit Optimization and Determining Optimum Throughput*

We have shown by mathematical analyses and session level Monte-Carlo simulation that our method is a new approach to perform profit optimization and to determine optimum operating load in the network subject to the network architecture, traffic pattern, service class mix, and strategies available.

9.4.3 *Traffic Load Distribution*

Assume that a provider has two disjoint core networks and requires distributing access networks' traffic load between these core networks. By implementing appropriate strategies, a provider can distribute the access traffic according to the desired load levels of the core networks. For example, assume the provider has two disjoint core networks: Core X and Core Y. The provider also has many access networks. The access networks' traffic propagates through the core networks. The provider intends to maintain an operating load close to the maximum traffic engineered load in Core X and a lower load in Core Y. The provider can accomplish this by assigning the *Very Low Rejection* strategy to Core X and the *Very High Rejection* strategy to Core Y. If the provider plans balancing traffic load equally between Core X and Core, it should assign same strategy to both core networks.

9.4.4 *Least Price Routing*

Similar to the method of traffic load distribution, our approach can also perform the least price routing. Assume that the enterprise networks are Edge-Label Switch Routers (E-LSRs) and the providers are either disjoint networks or the autonomous systems of Border Gateway Protocol (BGP). The E-LSR wishes to select an autonomous system with least price routing where the routing parameters are the price in addition to QoS attributes. By implementing our method, the E-LSR can select the route through the lowest priced autonomous systems.

9.4.5 *Forecasting and Capacity Planning*

Due to the rapid growth in the Internet savvy population and emerging multi-media applications that consume high bandwidth, Internet market demand is rapidly increasing. To maximize profit at all market demand levels, providers need to accurately maintain optimum network load. Our analytical approach allows providers to predict this load.

Traffic load in the network depends on the market demand and network capacity. If the network load increases beyond the optimum load due to increased market demand, a provider can maintain a desired load by proactively planning capacity to add capacity and enforcing traffic-engineering rules. For example, by implementing our approach, a provider could optimize profit at an operating load of 0.7704 under certain traffic engineering rules for homogeneous service-based market. Thus, implementation of our model allows a provider to forecast when a new capacity needs to be added.

9.4.6 *Service Provisioning*

By using the proposed model, a provider can compute which class of service earns better profit. Based on this information, a provider can assign higher bandwidth for the higher profiting service.

9.4.7 *Innovation Disclosure*

We submitted an invention disclosure of the model: Sprint Docket #2857, 2004.

9.5 *Future work*

9.5.1 *Variable Reservation Price*

Not all customers may value Internet services in the same way. In addition, customers' wealth may be different. Therefore, one customer's budget for a given Internet service may be different from another customer's budget for the same service. Our research was based on a fixed reservation price. A future research could vary the reservation price to observe the profit of both customer and provider.

9.5.2 *Experiment on 3GPP Network*

We conducted analytical and session level Monte-Carlo simulation studies in the proposed one-to-many enterprise-provider peer network. We proposed an extension of the 3GPP wireless network; however, we did not conduct analytical and simulation studies due to time limitations. Although in our model, both the ATIS extension and the 3GPP extension employ the same price transaction protocol and architecture, the cost computation model will be different depending on the 3GPP charging function used: session, event, and bearer. A provider will most likely implement the charging functions, which yield most profit. Thus, a future simulation and laboratory analysis to compare the performance of these three functions in our model could show advantages over the current pricing methods in the wireless network.

9.5.3 *Priority based Queue system*

We conducted research in a network that supported M/M/1 queue systems because integrated queue and FIFO scheduling is currently most prevalent. However, in the future, providers will most likely implement segregated queues and priority scheduling in their networks. Therefore, evaluating performance of the proposed model using priority based queue systems should indicate better results because in a priority scheduling system congestion sensitive costs of higher valued services will be lower.

Bibliography

1. Hal R. Varian, *Microeconomic Analysis*, Third Edition. NY, London: W.W. Norton & Company, 1992

2. A. Mas-Colell, M.D. Whinston, and J.R. Green, *Microeconomic Theory*. Oxford, United Kingdom: Oxford University Press, 1995.

3. R. Gibbons, *A Primer in Game Theory*, NY, London, Harvester Wheatsheaf, 1992

4. D. Fudenberg and J Tirole, *Game Theory,* The MIT Press, Cambridge, 1991

5. Theodore Turocy, Bernhard Stengel, “Game Thoery”, CDAM Research Report, LSE-CDAM-2001-09, http://www.cdam.lse.ac.uk/Reports/Files/cdam-2001-09.pdf

6. J. Rosen, “Existence and uniqueness of equilibrium points for concave n-person games,” *Econometrica*, vol.33, no. 3, pp. 520-534, 1965.

7. F. P. Kelly, “Charging and rate control for elastic traffic,” *European Transaction on Telecommunications,* vol. 8, pp. 33-37, 1997.

8. Johari, R., and Tsitsiklis, J.N. (2004). Efficiency loss in a network resource allocation game, *Mathematics of Operations Research* 29 (3): 407-435.

9. L. DaSilva, “Static Pricing in Multiple-Service Networks: A Game-Theoretic Analysis,” PhD Dissertation, University of Kansas, July 1998.

10. Richard J. La, V. Anantharam, “Network Pricing Using Game Theoretic Approach.” Proceedings of 38rd IEEE Conference on Decision and Control, Phoenix, AZ, Dec. 1999, IEEE Press, Piscataway, NJ, pp. 4008-4013.

11. Linhai He, J Warland, “Pricing Internet Services With Multiple Providers,” 41st Annual Allerton Conference on Communication, Control and Computing, Monticello, IL, Oct. 2003.

12. X. Cao, H. Shen, R. Milito, P. Wirth, “Internet Pricing with a Game Theoretical Approach: Concepts and Examples,” IEEE/ACM Transactions on Networking, Vol. 10, No 2, April 2002.

13. S. Bandyopadhyay, J. Baron, A. Chaturvedi, “Competition Among Sellers in Online Exchanges,” *Information Systems Research*, Vol. 16, Issue 1, pp. 47-60

14. S. Bandyopadhyay et al., “Simulating Sellers’ Behavior in a Reverse Auction B2B Exchange”, Proceedings of the International Conference on Computational Science 2003: 365-374. (Also in Decision Support Systems Journal).

15. Zhangxi Lin, Peng Si Ow, Dale O. Stahl, and Andrew B. Whinston, "Exploring Traffic Pricing for the Virtual Private Network", Information Technology and Management, No.3, October 2002, 301-327.

16. A. Gupta, D. Stahl, A. Whinston, “A Stochastic Equilibrium Model of Internet Pricing”, *Journal of Economic Dynamics and Control*, 21, 1997, pp697-722.

17. D. Stahl, R. Dai, A. Whinston, "An economic Analysis of Multiple Internet QoS Channels," in the *Economics of the Internet and E-Commerce*, by M.R. Baye, Indiana University, Bloomington; Elsevier online publication, 2002

18. X. Geng, A. Whinston, "Profiting from Value-Added Wireless Services", IEEE Communications, August 2001, pp: 85-87

19. A. B. MacKenzie, S. B. Wicker, "Game Theory in Communications: Motivation, Explanation, and Application to Power Control," IEEE *GlobeCom 2001*, November 25-29, 2001.

20. R. La and V. Anantharam, "A Game-theoretic Look at the Gaussian Multiaccess Channel," *Advances in Network Information* Theory, Vol. 66, proceedings of the March 2003 Discrete Mathematics and Theoretical Computer Science workshop.

21. M. Bowling, M. Veloso, "An Analysis of Stochastic Game Theory for Multiagent Reinforcement Learning," Michael Bowling and Manuela M., Technical report CMU-CS-00-165, 2000, Available: citeseer.ist.psu.edu/hu99multiagent.html

22. Erfu Yang, Dongbing Gu, "Multiagent Reinforcement Learning for Multi-Robot System: A Survey", Technical Report, Department of Computer Science, University of Essex, Available:http://citeseer.ist.psu.edu/2970.html

23. C. Papadimitriou, "Algorithms, games, and the Internet," in *Processing of the 33rd Annual ACM Symposium on the Theory of Computing,* 2001, pp. 749-753.

24. A.S. Schulz and N. Stier Moses, "On the performance of user equilibria in traffic networks," MIT Sloan School of Management, Working Paper 4274-02, 2002.

25. R.T. Maheswaran and T.Basar, "Nash equilibrium and decentralized negotiation in auctioning divisible resources," *Group Decision and Negotiation*, 2003.

26. L. Breker, "A Survey of Network Pricing Schemes", In *Proc. Of the 8th Symposium on Computer Science*, University of Saskatchewan, April 1996.

27. L. A. DaSilva, "Pricing for QoS-Enabled Networks: A Survey", *IEEE Communications Review*, Vol. 3, No. 2, 2000.

28. L.W. McKnight et al., "Pricing Internet Services: What Comes After Flat Rate?", *The MIT/Tufts Internet Service Quality Economics Workshop*, Cambridge, MA, December, 1999.

29. M. Yuksel, Internet Pricing Literature, Rensselaer Polytechnic Institute, NY, http://networks.ecse.rpi.edu/~yuksem/internetpricing/literature/pricing_papers.html.

30. R. Singh, M. Yuksel, S. Kalyanaraman, and T. Ravichandran, "A comparative evaluation of Internet pricing models: Smart market and dynamic capacity contracting," *Proceedings of Workshop on Information Technologies and Systems (WITS)*, 2000.

31. M. Caesar; S. Balaraman; D. Ghosal, "A comparative study of pricing strategies for IP telephony." *IEEE Global Telecommunications Conference*, vol.1, 2000. p.344-9.

32. X. Chang; D. Petr. "A survey of pricing for integrated service networks." *Computer Communications*, vol.24, (no.18), Dec. 2001. p.1808-18.

33. S. D. Patek, E. C-Nanez, "Pricing of Dialup Services: an Example of Congestion-Dependent Pricing in the Internet", *Proceedings of the 39th IEEE Conference on Decision and Control*, vol.3, 2000. p.2296-301.

34. A. Ganesh, K. Laevens, and R. Steinberg. Congestion Pricing and User Adaptation. In *Proceedings of IEEE INFOCOM*, pages 959–965, Anchorage, AK, April 2001.

35. I. Ch. Paschalidis and J. N. Tsitsiklis. Congestion-Dependent Pricing of Network Services. *IEEE/ACM Transactions on Networking*, 8(2):171–184, April 2000.

36. X. Wang, H. Schulzrinne,"Pricing Network Resources for Adaptive Applications in a Differentiated Services Network", In Proceeding of INFOCOM'2001 , Anchorage, Alaska, Apr. 2001.

37. S. Shenker, "Fundamental design issues for the future Internet", *IEEE Journal on Selected Areas in Communications,* vol. 13, pp. 1176-1188, 1995.

38. J. N. Tsitsiklis, B. V. Roy, "Regression Methods for Pricing Complex American-Style Options," *IEEE Transactions on Neural Networks*, Vol. 12, No. 4, July 2001

39. Enrique C-Nanez, Stephen D. Patek, "On Improving the Performance of Simulation-Based Algorithms for Average Reward Processes with Application to Network Pricing," *Proceedings of the 2001 Winter Simulation Conference.*

40. A. O'Donnell, H. Sethu, "Congestion Control, Differentiated Services, and Efficient Capacity Management Through a Novel Pricing Strategy," in *Computer Communications*, vol. 26, no. 13, pp. 1457-1469, Aug. 2003.

41. R. Arnott and K. Small, "The Economics of Traffic Congestion", *American Scientist,* 82:446-445, Septermber/October 1994.

42. Semret, N.; Liao, R.R.-F.; Campbell, A.T.; Lazar, A.A. "Peering and Provisioning of Differentiated Internet Services." *Proceedings of IEEE INFOCOM 2000. Conference on Computer Communications*, vol.2, 2000. p.414-20.

43. V. Krishna, "Auction Theory", Academic Press, 2002

44. P. Klemperer, "Auctions: Theory and Practice", Princeton Press

45. C. Day, B Hobbs, J. Pang, "Olgopolistic Competition in Power Networks: A Conjectured Supply Function Approach", IEEE Transactions on Power Systems, 17(3), Aug. 2002, 597-607

46. D. Kincaid, W. Cheney, "*Numerical Analysis: Mathematics of Scientific Computation,*" Brooks/Cole, 3rd Edition, 2002

47. E. D. Gaughan, Introduction to Analysis, Fifth Edition. Brooks/Cole, Thomson Learning, 1998.

48. D. P. Bertsekas, *Nonlinear Programming,* 2nd ed. Cambridge, Massachusetts: Athena Scientific, 1999.

49. D. Bertsimas and J.N. Tsitsiklis, *Introduction to Linear Optimization*. Cambridge, Massachusetts: Athena Scientific, 1997.

50. David G. Luenberger, *Linear and Nonlinear Programming*, Second Edition, 1984, Addison-Wesley Publishing Company

51. R. Fourer, D. Gay, and B. Kernighan, *AMPL*, Second Edition, Thomson Brooks/Cole, 2003.

52. S. Floyd, and V. Paxon, "Difficulties in Simulating the Internet", *IEEE/ACM Transaction on Networking*, vol 9, no. 4, pp. 392-403, 2001.

53. L. Claffy, "Recent Traffic Measurement from an Internet Backbone", http://www.caida.org/outreach/papers/1998/Inet98/Inet98.pdf, Cooperative Association for Internet Data Analysis, www.caida.org

54. W. E. Leland, M.S. Taqqu, W. Willinger, and D.V. Wilson. "On the Self-Similar Nature of Ethernet Traffic." In IEEE/ACM Transactions on Networking, 1994.

55. V. Paxon and S. Floys. Wide Area Traffic: The Failure of Poisson Modeling. IEEE/ACM Transactions on Networking, 1995.

56. I. Norros. On the Use of Fractional Brownian Motion in the Theory of Connectionless Networks. In IEEE J. Select. Areas Commun., volume 13, pages 953-962, 1995

57. J. Cao, W. S. Cleveland, D. Ling, D. X. Sun. Internet Traffic Tends *Toward* Poisson and Independent as the Load Increases, in *Nonlinear Estimation and Classification,* eds. C. Holmes et al., Springer, New York, 2002. pp 83-109.

58. T. Karagiannis et al., "A Nonstationary Poisson View of Internet Traffic", ACM SIGMETRICS, pp. 166-177, 2000.

59. T.G. Robertazzi. *Computer Networks and Systems: Queuing Theory and Performance,* Springer, Third Edition, 2002.

60. M. Tanner, *Practical Queueing Analysis*, The IBM McGraw-Hill Series, McGraw-Hill, London, 1995.

61. Mikka Poikselka et al., *The IMS – IP Multimedia Concepts and Services in Mobile Domain,* John Wiley & Sons Ltd, England, 2004.

62. Gonzalo Camarillo et al., *The 3G IP Multimedia Subsystem*, John Wiley & Sons Ltd, England, 2004.

63. Alan Johnston, *SIP: Understanding Session Initiation Protocol,* Second Edition, Artech House, Boston, 2004.

64. Gonzalo Camarillo, *SIP Demystified*, McGraw-Hill, 2002.

65. Henry Sinnreich et al., *Internet Communications Using SIP*, John Wiley & Sons, New York, 2001.

66. J. Rosenberg et al., "SIP: Session Initiation Protocol", RFC 3261, The Internet Engineering Task Force, www.ietf.org, The Internet Society, 2002.

67. M. Garcia-Martin et al., "Private Header (P-Header) Extensions to the Session Initiation Protocol (SIP) for the 3rd-Generation Partnership Project (3GPP)," RFC 3455, The Internet Engineering Task Force, www.ietf.org, The Internet Society, 2003.

68. ATIS PTSC IP NNI Document, Work on Progress, Scheduled to be completed in January, 2006, www.atis.org.

69. 3GPP, "Service Principles", Third Generation Partnership Project, Technical Standard 22.101

70. 3GPP, "Charging Implications of IMS Architecture", Third Generation Partnership Project, Technical Standard 23.815

71. 3GPP, "Telecommunication Management; Charging Management; Charging Principles", Third Generation Partnership Project, Technical Standard 32.200, June 2003.

72. 3GPP, "Telecommunication Management; Charging Management; Charging data description for the`IMS", Third Generation Partnership Project, Technical Standard 32.225, June 2003.

73. 3GPP, “Charging Data Description for IMS”, Third Generation Partnership Project, Technical Standard 32.235.

74. Thomas Robertazzi, *Computer Networks and Systems: Queueing Theory and Performance Evalutaion*, Second Edition, Springer-Verlag, New York 1994

75. A. Leon-Garcia, *Probability and Random Processes for Electrical Engineering*, Addision-Wesley, New York, 1989

76. G. Casella, *Statistical Inference*, Second Edition, Duxbury, 2002

77. The Yankee Group, “*VoIP Peering: Bridging the Gap Between Public and Private Network”,* Communication Network Infrastructure, May 2002

78. D. Klien, “*Peering into the Future”,* Telecommunications Online, February, 2005.

79. The Yankee Group, “*VoIP is Heating Up the Need for Native IP Peering*”, MarketResearch.com, June 2004.

80. T. Szigeti, C. Hattigh, *End-to-End QoS Network Design: Quality of Service in LANS. WANS, and VPNs*, 1st Edition, Cisco Press, November, 2004.

81. M. Caesar, D. Ghosal, and R. Katz, “Resource Management for IP Telephony Networks”, *International Workshop on QoS,* May 2002.

82. Aloizio Pereira da Silva, Geraldo Robson Mateus. "Performance Analysis for Data Service in Third Generation Mobile Telecommunication Networks," *ss*, vol.00, p. 0227, 35th 2002.

83. M. Gerla and J. Tsai, "Multicluster, mobile, multimedia radio network," *ACM/Baltzer Journal of Wireless Networks*, vol. 1, no. 3, pp. 255–265, 1995.

84. Xiaoyan Fang, Dipak Ghosal, "Performance Modeling and QoS Evaluation of MAC/RLC Layer in GSM/GPRS Networks", ICC 2003 General Conference—Networking, May 2003.

85. Spragins, John. et al. *Telecommunications Protocols and Design*, Addison-Wesley, August, 1994.

Appendix A: Mathematical Optimization Technique

The Necessary and Sufficient Conditions

The Kuhn-Tucker condition for the constrained non-linear programming is defined in [50] as follows.

The First Order Necessary Condition: *Let **x*** be a relative minimum point for the problem*

$$\begin{aligned} &\textit{Minimize}: && f(\mathbf{x}) \\ &\textit{subject to}: && \mathbf{h}(\mathbf{x}) = 0 \\ & && \mathbf{g}(\mathbf{x}) \le 0 \end{aligned}$$

*and suppose **x*** is a regular point for the constraints. Then there is a vector* $\lambda \in E^m$ *and a vector* $\mu \in E^p$ *with* $\mu \ge 0$ *such that*

$$\nabla f(\mathbf{x}^*) + \lambda^T \nabla \mathbf{h}(\mathbf{x}^*) + \mu^T \nabla \mathbf{g}(\mathbf{x}^*) = \mathbf{0}$$

$$\mu^T g(\mathbf{x}^*) = 0$$

The Second Order Necessary and Sufficiency conditions for the constrained non-linear programming is defined in [50] as follows:

Second-Order Necessary Condtions*. Suppose the functions* $f, g, h \in C^2$ *and x* is a regular point. If x* is a relative minimum point for problem, there there is a* $\lambda \in E$, $\mu \in E$, $\mu \ge 0$ *such that*
$L(x^*) = F(x^*) + \lambda^T H(x^*) + \mu^T G(x^*)$ *is positive semidefinite on the tangent subspace of the active constraints at x*.*

Second-Order Sufficiency Condtions: *Suppose there is a point x* satisfying h(x*)=0, and a* $\lambda \in E^m$ *such that* $\Delta f(x^*) + \lambda^T \Delta h(x^*) = 0$.
Suppose also that the matrix $L(x^*) = F(x^*) + \lambda^T \Delta h(x^*) = 0$ *is positive definite on* $M = \{y : \Delta h(x^*) = 0\}$ *, that is, for* $y \in M, y \ne 0$ *there holds* $y^T L(x^*) y > 0$ *. Then x* is a strict local minimum of f subject to h(x)=0.*

The Gradient Projection Algorithm

The following Gradient Projection Algorithm is reproduced from [50]. To optimize (minimize) function f(**x**) for a given feasible point **x**, one step of the Gradient Projection Algorithm is as follows:

1. Find the subspace of active constraints *M*, and form $\mathbf{A}_q$, W(**x**).
2. Calculate $\mathbf{P} = \mathbf{I} - \mathbf{A_q^T}(\mathbf{A_q}\mathbf{A_q^T})^{-1}\mathbf{A_q}$ and $\mathbf{d} = -\mathbf{P}\nabla f(\mathbf{x})^T$.
3. If $\mathbf{d} \neq \mathbf{0}$, find α_1 and α_2 achieving, respectively,

   Max $\{\alpha_1: \mathbf{x} + \alpha_1 \mathbf{d}$ is feasible$\}$

   Min $\{f(\mathbf{x} + \alpha_2 \mathbf{d}): 0 \leq \alpha_2 \leq \alpha_1\}$

   $\mathbf{x}_{k+1} = \mathbf{x}_k + \alpha_2 \mathbf{d}$ and return to 1.
4. If **d = 0**, find $\lambda = -(\mathbf{A_q}\mathbf{A}_q^T)^{-1}\mathbf{A}_q \nabla f(\mathbf{X})^T$

   a) If $\lambda_j \geq 0$, for all *j* corresponding to active inequalities, stop;

      X satisfies the Karush-Khun-Tucker condition.

   b) Otherwise, delete the row from $\mathbf{A_q}$ corresponding to the inequality with most negative component of $\boldsymbol{\lambda}$ (and drop the corresponding constraint from W(**x**)) and return to 2.

The Golden Section Line Search

The algorithm of the Golden Section line search method is described in [50]. We implement the Golden Section Line search method in MATLAB to find minimum of unimodal (single minimum) function $f(\mathbf{X}+\alpha\mathbf{D})$ over a closed interval ($0<\alpha<\alpha_{Max}$). Here, ***X*** is an initial point vector, ***D*** is a directon vector, and α_{Max} is a maximum distance to move during optimization, and α is a scaling factor.

This line search method uses the Golden section ratio that is derived from the Fibonacci ratio by allowing Fibonacci search N measurement point to approach infinity.

$$\lim_{N\to\infty}\frac{F_{N-1}}{F_N}=\frac{1}{\tau}=0.618$$

The following is our Golden Section Line Search Algorithm:

$$\varepsilon \leftarrow 1e^{-12}$$
$$X \leftarrow Initial\ \ position$$
$$D \leftarrow Direction$$
$$\alpha_{Max} \leftarrow Max\ Dist.$$
$$\tau \leftarrow \frac{1+\sqrt{5}}{2}$$
$$a \leftarrow 0; b \leftarrow \alpha_{Max};$$
$$x_1 \leftarrow a+(b-a)(1-\frac{1}{\tau})$$
$$x_2 \leftarrow a+(b-a)\frac{1}{\tau}$$
$$f_1 \leftarrow f(X+x_1D)$$
$$f_2 \leftarrow f(X+x_2D)$$

$$while\ (b-a) > \varepsilon$$
$$\quad if\ \ f_1 > f_2$$
$$\quad a \leftarrow x_1; x_1 \leftarrow x_2;\ f_1 \leftarrow f_2;$$
$$\quad x_2 \leftarrow a+(b-a)\frac{1}{\tau}$$
$$\quad f_2 \leftarrow f(X+x_2D)$$
$$\quad else$$
$$\quad b \leftarrow x_2; x_2 \leftarrow x_1;\ f_2 \leftarrow f_1;$$
$$\quad x_1 \leftarrow a+(b-a)(1-\frac{1}{\tau})$$
$$\quad f_1 \leftarrow f(X+x_1D)$$
$$\quad end$$
$$\text{minimum\_point} \leftarrow \frac{\text{a+b}}{2}$$
$$end$$

Appendix B:List of Acronyms

3GPP	Third Generation Partnership Project
ATIS	Alliance for Telecommunications and Industry Solutions
ATM	Asynchronous Transfer Mode
B2BUA	Back-to-back User Agent
BCF	Bearer Charging Function
BFE	Bearer Functional Entity
BGF	Border Gateway Function
BGP	Border Gateway Protocol
BICC	Bearer Independent Call Control
CCFE	Call Control Functional Entity
CMS	Cable Management Server
CRFE	Call Routing Functional Entity
CR-LDP	Constrained-based Label Distribution Protocol
CMSS	Cable Management Server Signaling
ECF	Event Charging Function
E-LSR	Edge-Label Switch Router
FIFO	First-In-First-Out
FONC	First Order Necessary Condition
GMPLS	Generalized Multi Protocol Label Switching
HR	High Rejection
IETF	Internet Engineering Task Force
I-CSCF	Interrogating-Call Session Control Function
IP	Internet Protocol
ISP	Internet Service Provider
IMS	Internet Multimedia Subsystem
ITU	International Telecommunication Union
LSP	Label Switch Paths
LR	Low Rejection
PNNI	Private Network-to-Network Interface
MG	Media Gateway
MGC	Media Gateway Controller
MPLS	Multi Protocol Label Switching
MPλS	Multi Protocol Lambda Switching
MR	Media Relay
P-CSCF	Proxy-Call Session Control Function
PDA	Personal Digital Assistants
PTSC	Packet-Technology and System Committee
QoS	Quality of Service
S-CSCF	Serving-Call Session Control Function

SBC	Session Border Controller
SCF	Session Charging Function
SONC	Second Order Necessary Condition
SOSC	Second Order Sufficient Condition
RFC	Request For Comment
RFP	Request For Purchase
RL	Reinforcement Learning
RN	Rejection Neutral
RR	Round Robin
RSVP-TE	Resource Reservation Protocol Trafic Extension
SIP	Session Initiation Protocol
UA	User Agent
VHR	Very High Rejection
VLR	Very Low Rejection
VoIP	Voice over Internet Protocol
VP	Virtual Path
VPC	Virtual Path Connection
VPN	Virtual Private Network
WRR	Weighted Round Robin