Model Based Signal Processing for GPR Data Inversion

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Committee

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OUTLINE

- **Introduction**
  - GPR Applications
  - Thesis Objectives
- **The Inverse Problem**
  - Forward Modeling – FMCW Radar
  - Layer Stripping Approach
  - The Model Based Approach
- **Model Based Parameter Estimation**
  - MMSE based (Gauss-Newton)
  - Spectral Estimation based (MUSIC)
- **Inversion on actual radar data**
  - Tests on Antarctic snow radar data
  - Tests at the Sandbox lab
  - Tests on Greenland Plane wave data
  - GUI for data inversion algorithm
- **Conclusions & Future Work**

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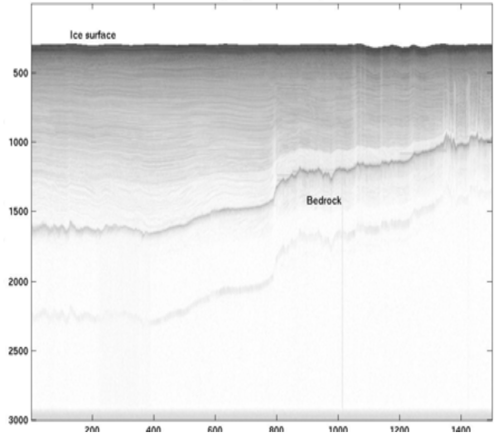
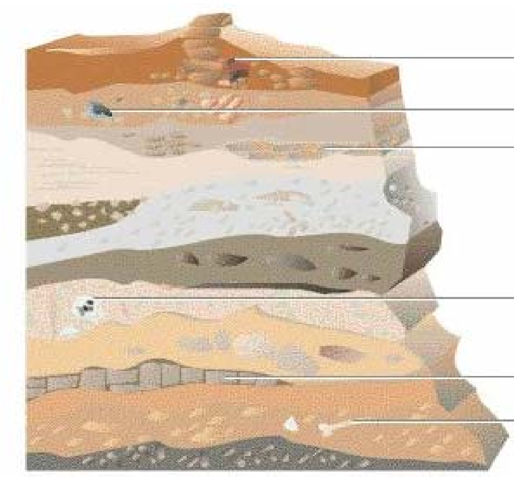
Ground **P**enetrating **R**adar Applications:



- Ice-sheet thickness measurements, bedrock mapping (Global Warming problem)
- Target detection (Landmines)
- Non-destructive testing of engineering structures
- **Sub-surface Characterization (Earth, Martian Surface)**



INTRODUCTION

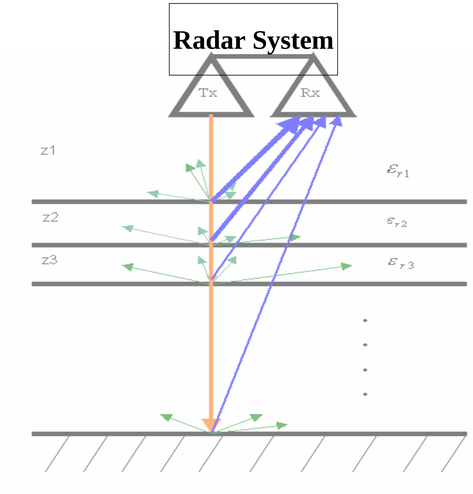
Concepts

Characterization : Determining the permittivity profile of a multi-layered media

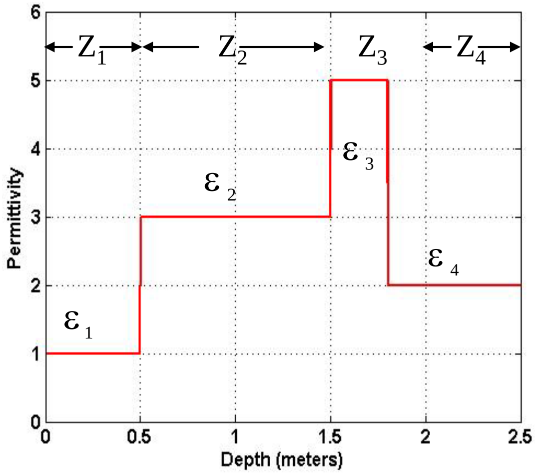
Permittivity (Dielectric Constant) : A quantity that describes the ability of a material to store electric charge.

Material	Relative Permittivity
Air	1
Water	80
Ice	3.14
Dry Snow	1.5-3
Wet Snow	Depends on moisture, particle size
Dry Soil	2 - 4
Dry Sand	3 – 5

Multi-layered structure



Permittivity Profile



THESIS OBJECTIVES

Thesis Objectives

Develop a signal processing algorithm to

1. Enhance features of radar data (reflectivity profiles with improved resolution)
2. Estimate the permittivity profile from recorded GPR data

→ ***Electro-Magnetic (EM) Inversion***

Principle

Permittivity contrast in layered media causes reflection of incident EM Wave

Challenges

- Radar return is corrupted by noise & clutter
- Unwanted effects due to radar system (Eg: non-linearities)
- Needs good understanding of EM propagation phenomenon

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THE GENERAL INVERSE PROBLEM

Inverse Problem: Estimation of unknown parameters given an observation

Steps for the study of an inverse problem

- System Parameterization:

  Identify set of model parameters **(m)** which characterize the phenomenon (observation)

  Observation – Radar return

  Model parameters – Permittivity values

- Forward Modeling:

  Deduce a mathematical relationship **F(m)** between model parameters **(m)** and actual observations **(Y)**

- Inverse Modeling:

  Use forward model and observed data to infer actual values of model parameters

$$Y = F(m) + \text{Noise} + \text{System effects} + \text{Clutter}$$

Estimate **m** given **Y**

FORWARD MODELING

→ Mathematical relationship between permittivities & observed radar return signal

Wave propagation Phenomena (1-D Plane wave approximation)

- **Reflection –** Reflection Coefficient Γ_k

$$\Gamma_k = \left(\sqrt{\varepsilon_{k+1}} - \sqrt{\varepsilon_k}\right) \Big/ \left(\sqrt{\varepsilon_{k+1}} + \sqrt{\varepsilon_k}\right)$$

- **Transmission –** Transmission Coefficient T_k

$$T_k = \sqrt{4\sqrt{\varepsilon_k \varepsilon_{k+1}} \Big/ \left[\sqrt{\varepsilon_{k+1}} + \sqrt{\varepsilon_k}\right]^2}$$

- **Attenuation –** Attenuation Coefficient B_k
  - Spreading — $\left(\frac{1}{4\pi R^2}\right)$ factor
  - Absorption — Conductivity, particle distribution need to be known
  - Scattering — Neglected in our analysis

Effective amplitude of reflected signal at layer ***K***
(combined effect of Γ_k, T_k, B_k)

$$A_k = B_k \Gamma_k \prod_{j=1}^{k} T_j \quad — (1)$$

2-way time delay experienced by signal reflected from layer ***K***

$$\tau_k = \frac{2}{c}\left[z_1 + \sum_{k=1}^{L}(z_k - z_{k-1})\sqrt{\varepsilon_k}\right] \quad — (2)$$

C = 3x 10^8 m/s
Z_1 – Surface height

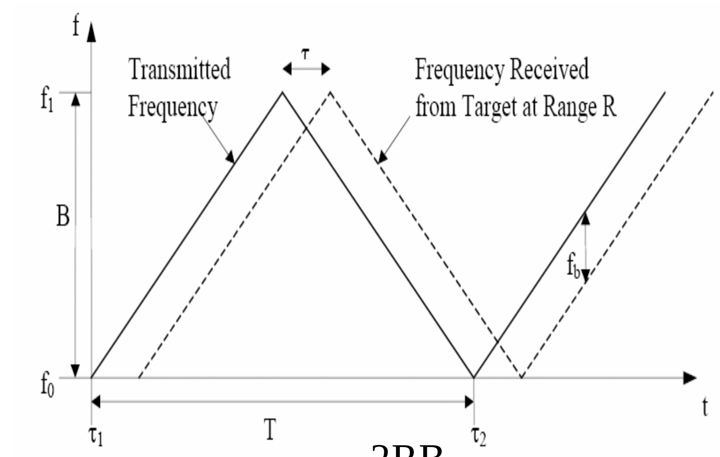
Estimate ε using (1) and (2) recursively

FORWARD MODELING

Illustration - FMCW Radar

- FMCW - Frequency Modulated Continuous Wave Radar
- Transmits a frequency sweep – *Chirp signal*

$$V_t(t) = A_t \, Cos\left(2\pi\left[f_0 t + \alpha t^2\right] + \theta_0\right)$$

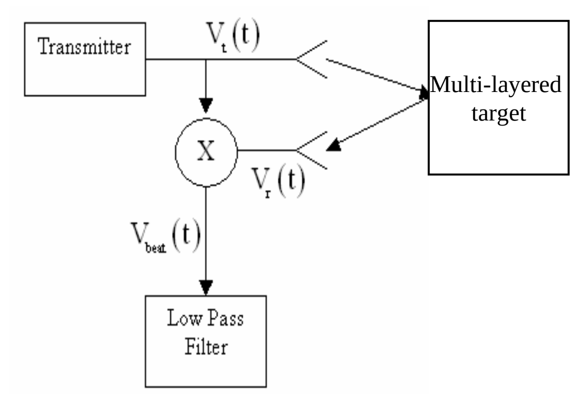


- Reflected signal is mixed with a copy of the transmitted signal to generate *Beat Signal (IF Signal).*
- Beat signal is a function of time delay $\tau \propto f_b$ (beat frequency) ; $f_b = \dfrac{2RB}{Tc}$

For multiple targets,

$$V_{beat}(\tau) = \sum_{k=0}^{L-1} A_k \Gamma_k \prod_{j=1}^{k-1} T_j \cos(2\pi\{f_0 \tau_k + \alpha \tau_k (2t - \tau_k)\} + n$$

$V_{beat}(\tau)$ is the forward model **F(m)**



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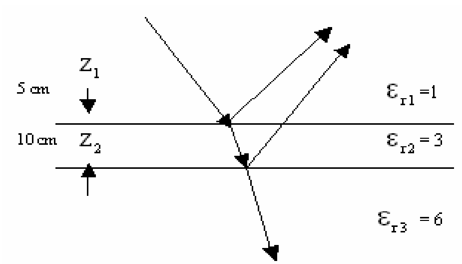
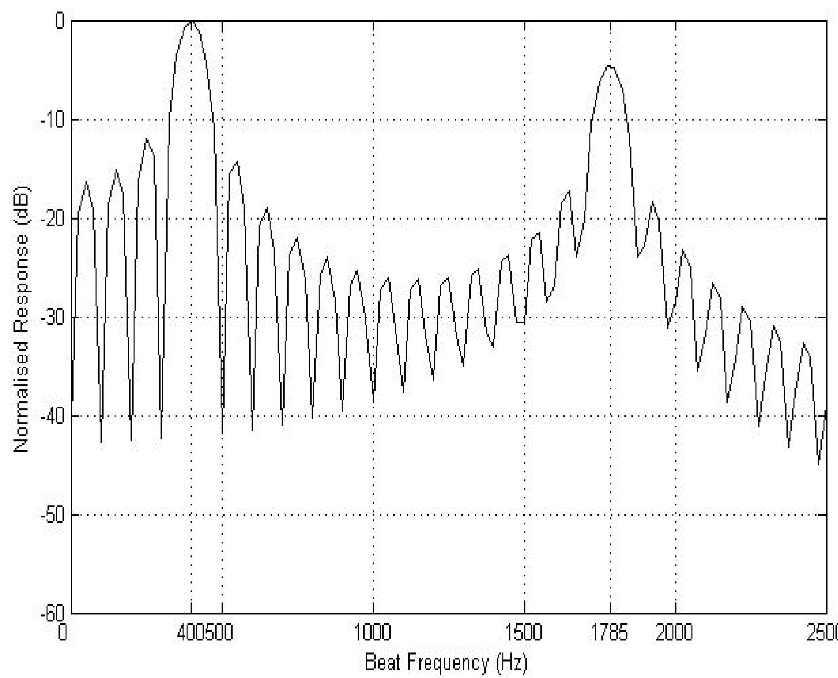
FORWARD MODELING

FMCW Radar

- Fast Fourier Transform (FFT) of $V_{beat}(\tau)$ gives frequency response of the target
- Plot of signal spectrum Vs distance – *Range Profile*



Bandwidth	6 GHz
Start Frequency	2 GHz
Sweep time	10 ms
Chirp rate	300 GHz/s
Permittivity vector ε_r	[1 3 6]
Depth vector Z (cm)	[5 10]
Beat frequency vector $f_b = \frac{2RB}{Tc}$ (Hz)	[400 1785.6]



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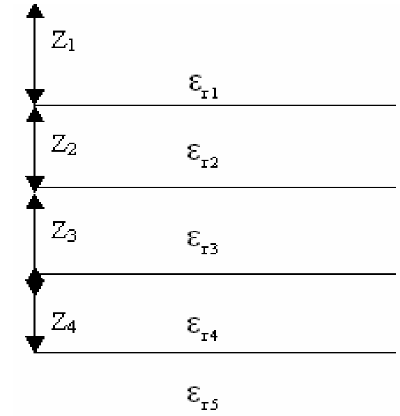
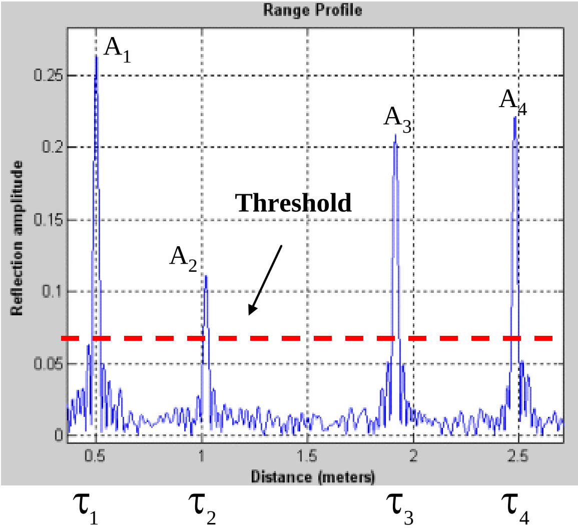
INVERSION

LAYER STRIPPING APPROACH

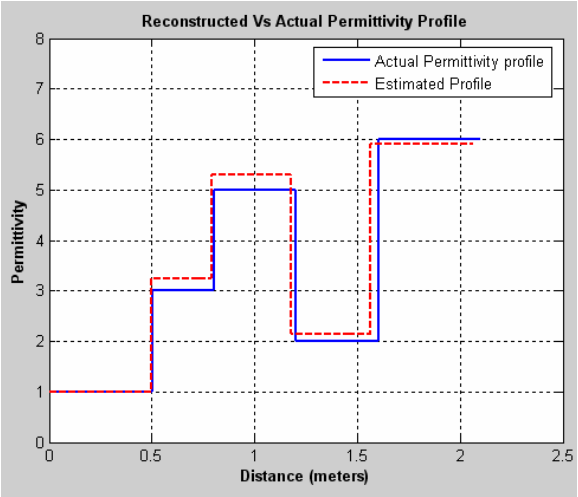
- An elementary approach to inversion
- Plot signal spectrum (Range Profile) using Fast Fourier Transform (FFT)
- Set threshold on amplitudes
- Locate Amplitudes (A_k's) and Time delays (τ_k's) from range profile



Permittivity vector ε_r	[1 3 5 2 6]
Depth-vector(m) **Z**	[0.5 0.3 0.4 0.4]



Recursively use (1) and (2) from the forward model to estimate the permittivity of every layer



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LAYER STRIPPING APPROACH

Limitations

- Missed Peaks
- False Alarms

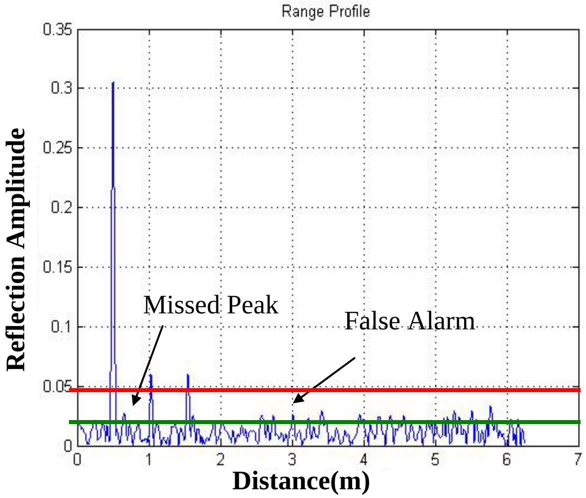
} Inappropriate thresholds distort reconstructed profile

- The side-lobe masking problem

- Weaker returns masked by side-lobes of stronger returns

- Windowing functions attenuate the lower frequencies that contain most of the information about the deeper structure

- Layer Stripping is not very reliable to detect subtle variations in permittivity



Solution:

Incorporate the underlying phenomenon into the inversion process

→ **The Model Based Approach**

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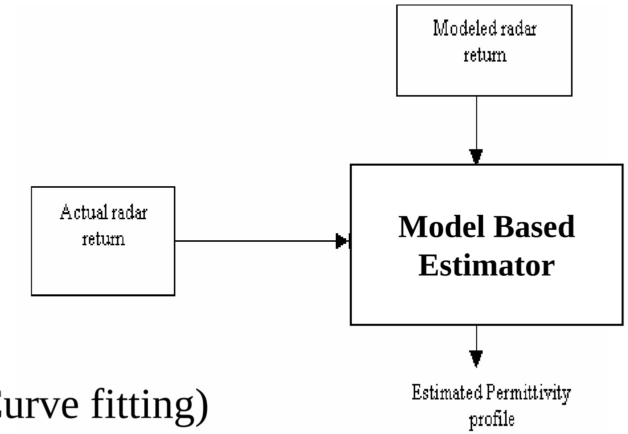
THE MODEL BASED ESTIMATION

Model Based Estimator

An estimator which incorporates the mathematical model **F(m)** to estimate unknown parameters **(m)**.



Regression Estimators (Data fitting or Curve fitting)

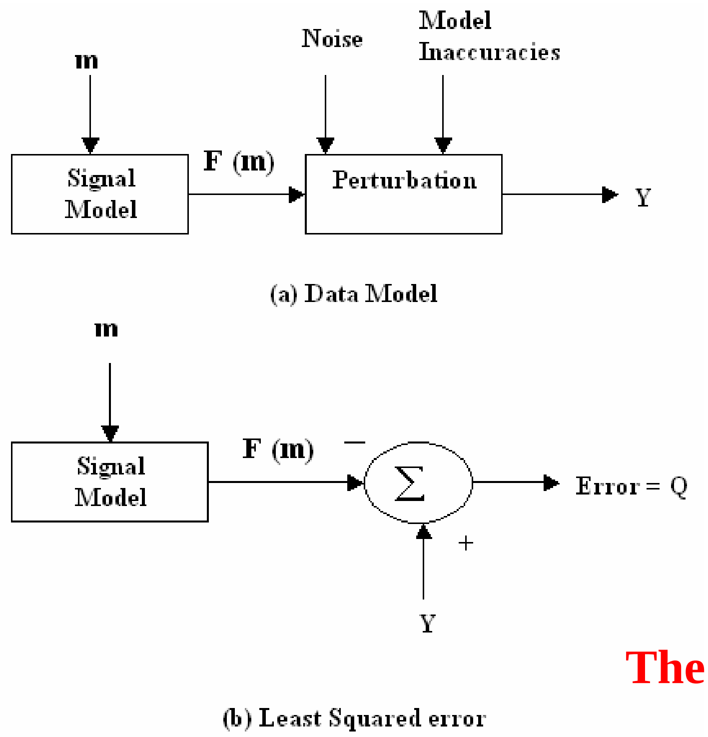
- Fit parameters to the observation (data) - based on some criterion
- Given an observed data set $Y = \{ y[0], y[1], ..., y[N-1] \}$, forward model F(m)
- Fit **m** to **Y**
- F(m) is non-linear , hence **Non-linear Regression**

Least Squares Estimation

- Estimate parameters based on the approach of minimizing the Mean Squared Error (MSE) between the observed data **(Y)** and the forward model **F(m)**
- No assumptions are made about the data unlike other regression based estimators
- For non-linear model, use Non-Linear Least Squares

NON-LINEAR LEAST SQUARES ESTIMATION

Based on MMSE (*Minimum Mean Squared Error*)



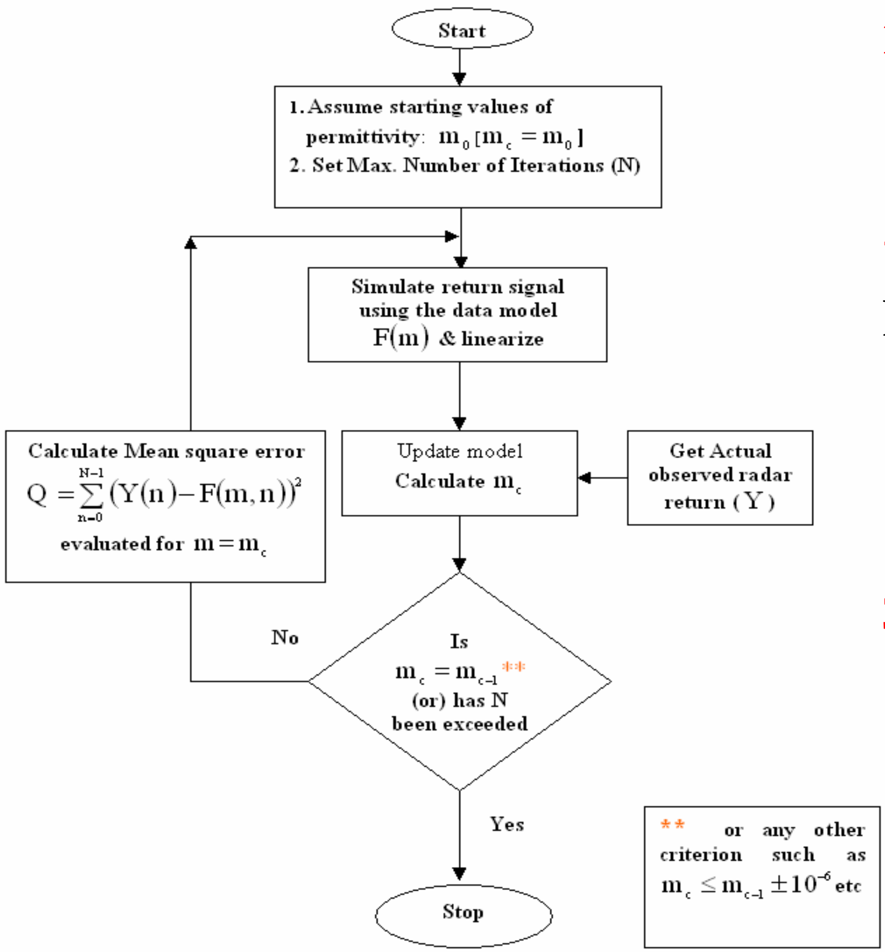
- The Least Squared Error Criterion is

$$Q = \sum_{n=0}^{N-1} \left(Y(n) - F(m,n)\right)^2$$

- Relationship between signal model **F(m)** and **m** is non-linear
- **F(m)** has to be linearized
- How ?

The Gauss Newton Iterative Minimization Algorithm

GAUSS – NEWTON METHOD



1. Initialization

$$m = m_0 \qquad \text{(Starting guess)}$$

2. Linearization

$$F(m) \cong F(m_c) + [\nabla_m F(m_c)](m - m_c)$$

m_c - set of current model parameters

$\nabla_m F(m_c)$ - matrix of partial derivatives of **F(m)** w.r.t **m**

3. Updation

$$m_{k+1} = m_k + \left[H^T(m_k)H(m_k)\right]^{-1} H^T(m_k)\left[Y - F(m_k)\right]$$

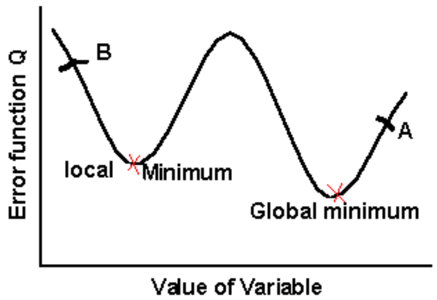
$$[H = \nabla_m F(m_c)]$$

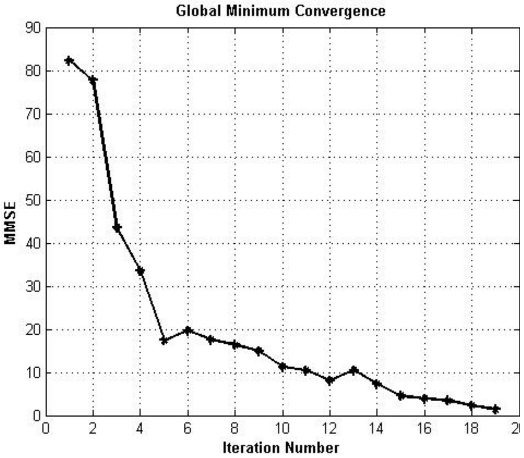
GAUSS – NEWTON METHOD

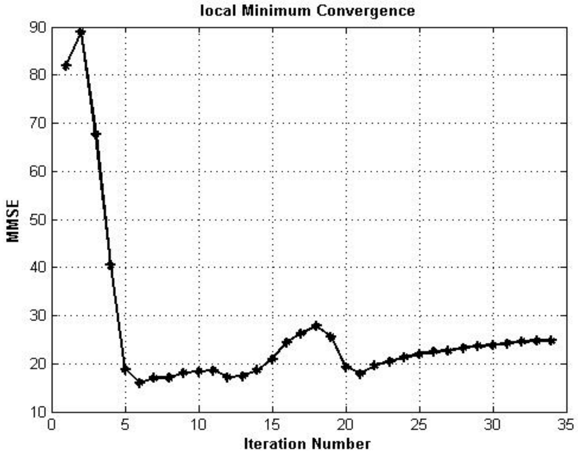
Performance

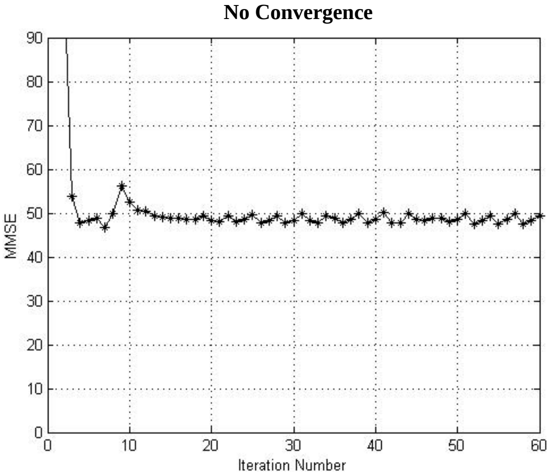
Algorithm may yield :

- Global minimum convergence
- Local minimum convergence
- No convergence







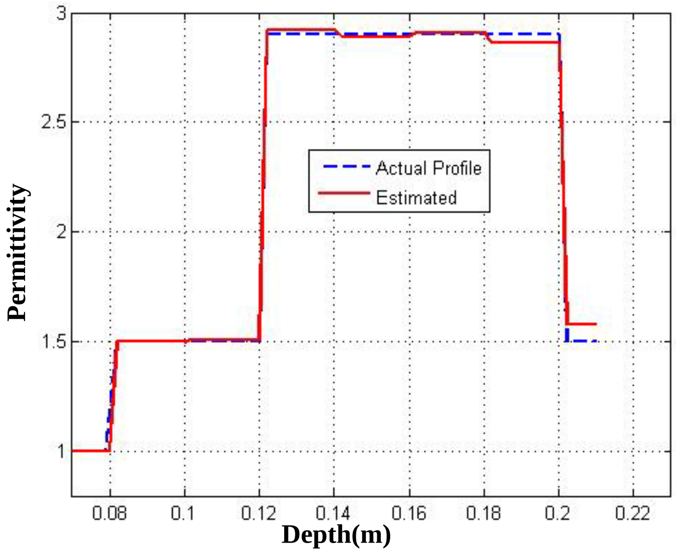


- A good starting guess yields a good estimate (A,B)
- To improve convergence - Run the algorithm with multiple starting guess values

GAUSS – NEWTON METHOD

Performance - Convergence Issues

Permittivity vector ε_r	[1 1.5 2.9 1.5]
Depth vector Z (cm)	[8 4 8]
SNR	20 dB



- Global minimum was reached 2/10 times

- The rest were local, non-convergence cases

- For 10 dB SNR, Global minimum was reached 1/50 times

Limitations

- Convergence is dependent on SNR

- Iterative search method (Computationally inefficient)

- Convergence is not guaranteed (in spite of several starting guesses)

- Large of model parameters (>15) → poor convergence

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GAUSS – NEWTON METHOD

- Cannot be used to invert actual radar data
- Other regression based techniques are also iterative search methods and cannot guarantee global minimum convergence
- Need for a more reliable estimator

⇨ **Model Based Spectral Estimation Techniques**

SPECTRAL ESTIMATION BASED INVERSION

Inversion:

Estimate Frequencies → Estimate Amplitudes→ Permittivity profile

Parametric Spectral Estimation : Using a model to estimate frequency components in a signal

- Suitable for applications in which signals can be represented by complex exponential models
- Radar signals consist of sinusoids embedded in noise

→ MUSIC Algorithm

MUSIC

- MUSIC : MUltiple SIgnal Classification
- High resolution frequency estimation technique
- Exploits Orthogonality of signal and Noise
- Enhances valid returns and suppresses noise peaks

MUSIC

Frequency Estimation

- Signal model can be written as: $x(n)=\sum_{k=1}^{P} A_k\, e^{jn\omega_k} + w(n)$ [Assuming *x(n)* consists of *P* complex exponentials in white noise *w(n)*]
- Form the (*M x M*) autocorrelation matrix (R_x) of *x(n)*
- Decompose R_x into Eigen values λ_i's and Eigen vectors V_i's

Eigen values : $\lambda_1 \geq \lambda_2 \geq ..... \geq \lambda_P \geq \lambda_{P+1} \geq .....\lambda_M$

Eigen vectors : $\underbrace{V_1 \geq V_2 \geq ..... \geq V_P}_{\text{'P' signal eigen vectors}} \geq \underbrace{V_{P+1} \geq .....V_M}_{\text{'M-P' noise eigen vectors}}$

‘*P*’ signal eigen vectors ‘*M-P*’ noise eigen vectors

$$v_i\left(e^{j\omega}\right)=\sum_{k=0}^{M-1} v_i(k)\, e^{-jk\omega} \;;\; i=p+1,\, p+2, ..., M$$

→ Will yield zero at the frequencies of complex exponentials

→ The frequency estimation function $P_{music}\left(e^{j\omega}\right) = \dfrac{1}{V_i\left(e^{jw}\right)}$ Will yield sharp peaks at the frequencies of complex exponentials

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MUSIC
Amplitude Estimation

$$x(n) = \sum_{k=1}^{P} A_k \, e^{jn\omega_k} + w(n)$$

Aim is to estimate A_k's ; ω_k's are known from the peaks of the frequency estimation function of MUSIC

$$\begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ \vdots \\ x(N-1]) \end{bmatrix} = \begin{bmatrix} 1 & \dots & 1 \\ e^{j\omega_1} & \dots & e^{j\omega_k} \\ \vdots & \vdots & \dots \\ \vdots & & \\ e^{j(N-1)\omega_1} & \dots & e^{j(N-1)\omega_k} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ \vdots \\ A_k \end{bmatrix} + \begin{bmatrix} w(0) \\ w(1) \\ \vdots \\ \vdots \\ w(N-1) \end{bmatrix}$$

$$\underbrace{X}_{} = \underbrace{S}_{} \cdot \underbrace{A}_{} + \underbrace{W}_{}$$

$$\hat{A} = \left(\hat{S}^H \hat{S}\right)^{-1} \hat{S}^H . X$$

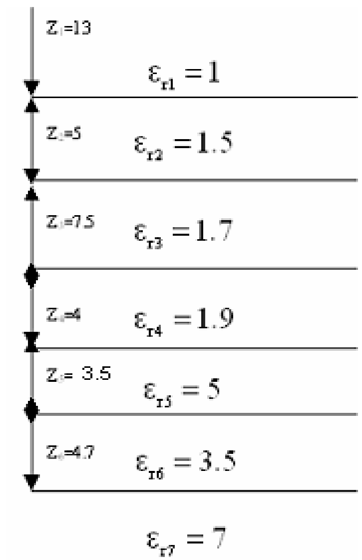
is the Maximum Likelihood Estimator of $\boldsymbol{A}$

(only if $\boldsymbol{W}$ is White Gaussian)

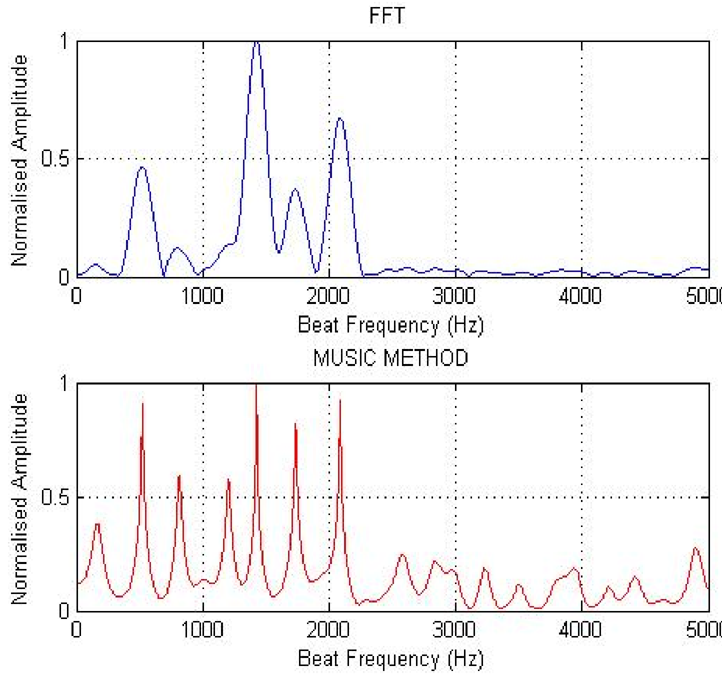
MUSIC

Resolution Capability

Type of radar	FMCW
Range Resolution	2.5 cm (free space)
BW	6 GHz
SNR	10 dB



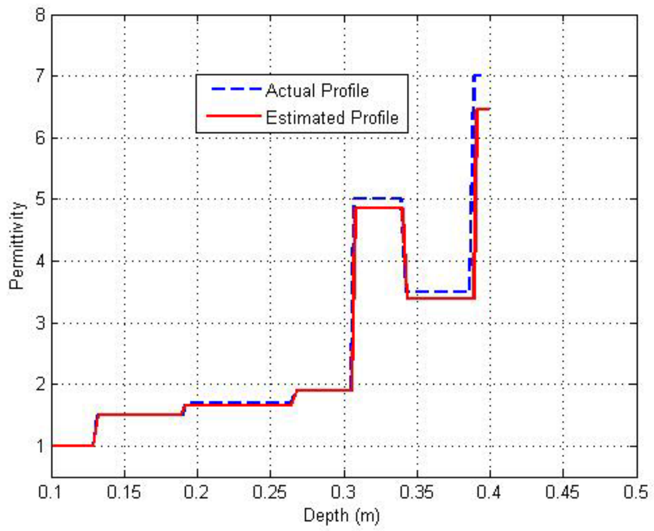
$$x(n) = \sum_{k=1}^{p} \Gamma_k \prod_{j=1}^{k} T_j . e^{\{2\pi(f_0\tau_k + 2t\alpha\tau_k - \alpha\tau_k^{2})\}} + w(n)$$



Range Profiles using FFT and MUSIC

MUSIC

Inversion – Simulation Results



Actual Profile Vs
Reconstructed Profile using MUSIC

- Reconstructed profile matches well with true profile
- Not constrained by layer depths

Impact of SNR

- Good reconstruction results up to 5 dB SNR
- Does not work well below 5 dB

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MUSIC

Performance

- Good simulation results
- Can be applied on actual data (if SNR is good enough)
- Computational cost (Eigen decomposition)
- Good forward model is required
- Gaussian Noise statistics for amplitude estimation

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INVERSION ON ACTUAL DATA

1. Field experiments in Antarctica using FMCW Radar
2. Sandbox tests
3. Plane Wave test in Greenland

FMCW RADAR TEST - ANTARCTICA

Ultra Wideband FMCW Radar – Used to measure snow thickness in Antarctica

Use MUSIC to estimate the permittivity profile from measured radar data

Parameters of FMCW radar

Characteristic	Value	Unit
Radar Type	FM-CW	
Sweep Frequency	2-8	GHz
Range Resolution	≅4	Cm
Sweep Time	10	msec
Transmit Power	13	dBm
PRF	25	Hz
Sampling Rate	5	MHz
Antenna	TEM Horn Antenna	

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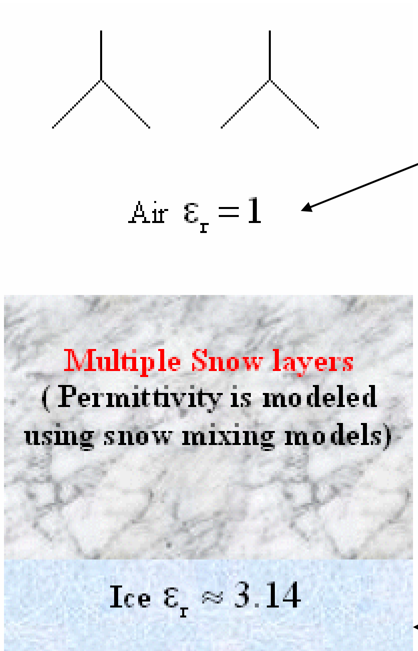
FMCW RADAR TEST – ANTARCTICA

Core Data modeling

Dielectric structure of the test site

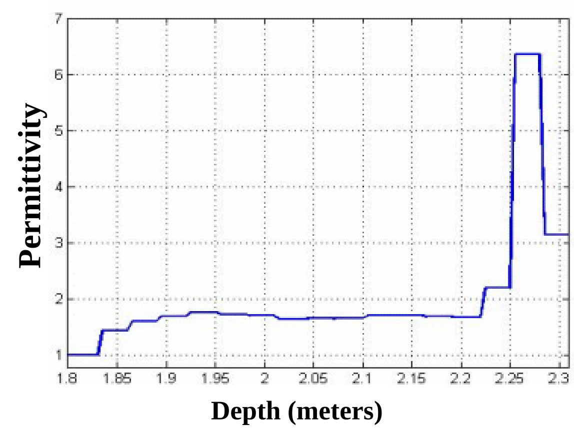


Snow pit data (Pit 1)

Layer Thickness (m)	Density (g/cm^3)	Salinity [‰]	Wetness [Vol %]
1.83	1.40	0	0
0.03	0.191	0.145	-0.46
0.03	0.254	-	0.73
0.03	0.328	0.07	0.09
0.03	0.364	0.31	0.00
0.03	0.355	0.11	0.00
0.03	0.334	0.21	0.00
0.03	0.285	0.20	0.39
0.03	0.293	0.12	0.49
0.03	0.244	0.31	1.41
0.03	0.254	0.34	1.67
0.03	0.245	0.30	1.76
0.03	0.226	0.27	1.96
0.03	0.309	0.05	0.31
0.03	-	2.17	3.00
0.015	-	29.4	3.28

Modeling the true permittivity profile

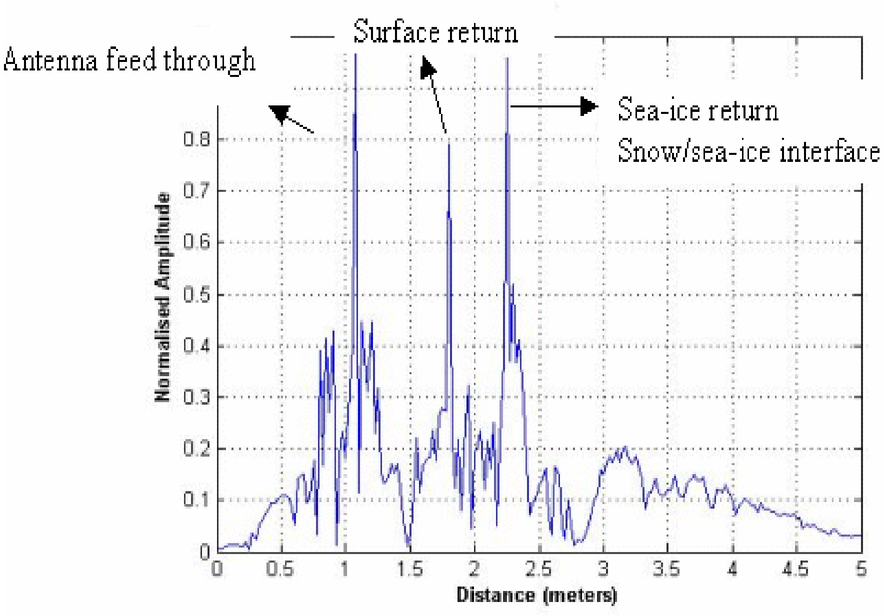
- Mixture of dry snow, water & brine
- Consider brine as an inclusion within a wet snow mixture
- Wet snow permittivity model : *Debye-like model*
- Brine permittivity model : *Stogryn's model*
- Use a mixing model for effective permittivity



FMCW RADAR TEST – ANTARCTICA

Measured Data

FFT Range Profile(Pit 1)



- Remove antenna feed-through
- Remove system effects using calibration data
- Enhance profile using MUSIC
- Estimate unknown frequencies & amplitudes

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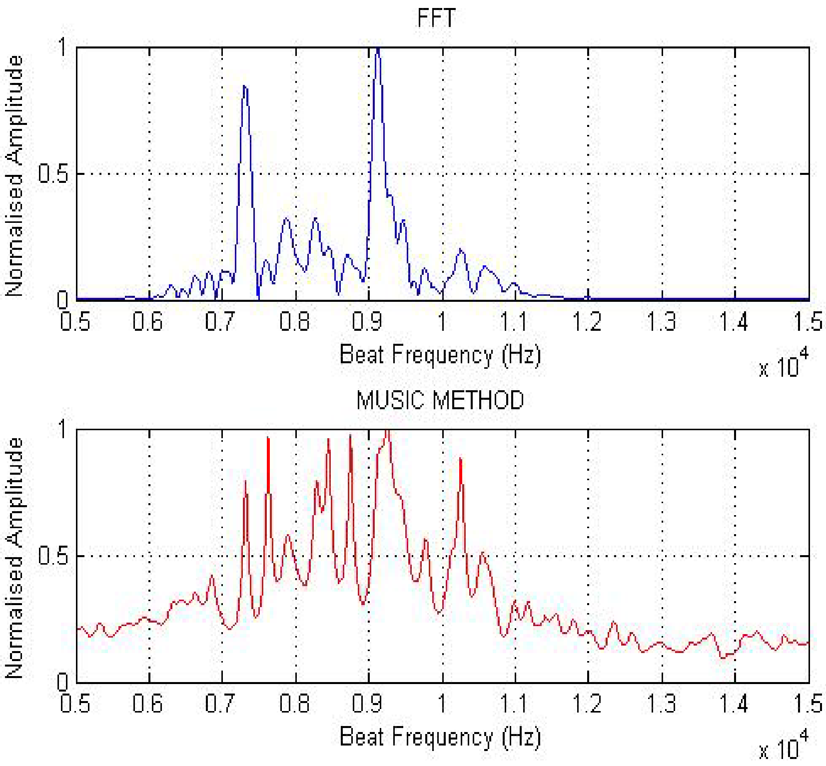
FMCW RADAR TEST – ANTARCTICA

Inversion

Range Profiles obtained using FFT and MUSIC



Comparison of estimated beat frequencies of core with those of FFT and MUSIC

Beat Freq (F_B) ***Core*** (Hz)	*Beat Freq (F_B)* ***MUSIC*** (Hz)	Beat Freq (F_B) ***IFFT*** (Hz)
7320	7317.0	7290
7474	7622.0	7875
7630	7885.0	8275
7781	8290.0	9120
7936	8445.0	
8088	8748.5	
8243	9150.0	
8399	9240.0	
8559	9770.0	
8721		
8884		
9047		
9203		
9390		

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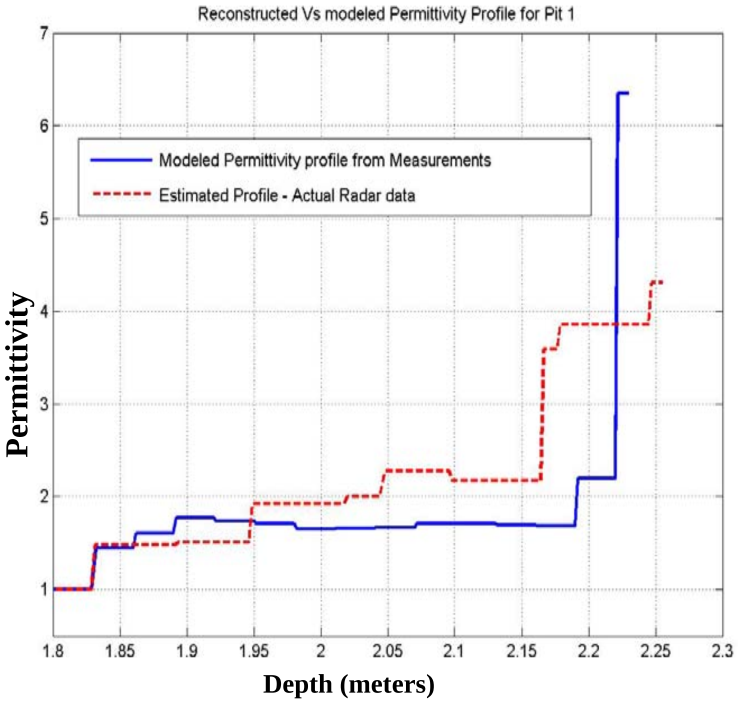
FMCW RADAR TEST – ANTARCTICA

Reconstructed Profile



- Good match up until 2.15 m depth
- Deviations may be due to:

(1) A discrepancy in the model representing the radar return

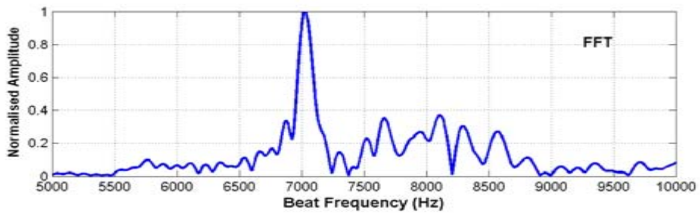
(2) Subtle changes in permittivity that MUSIC is not able to distinguish

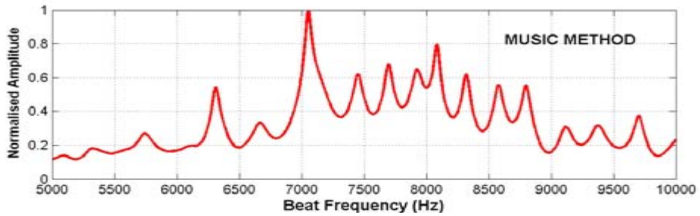
(3) Error in calibration data

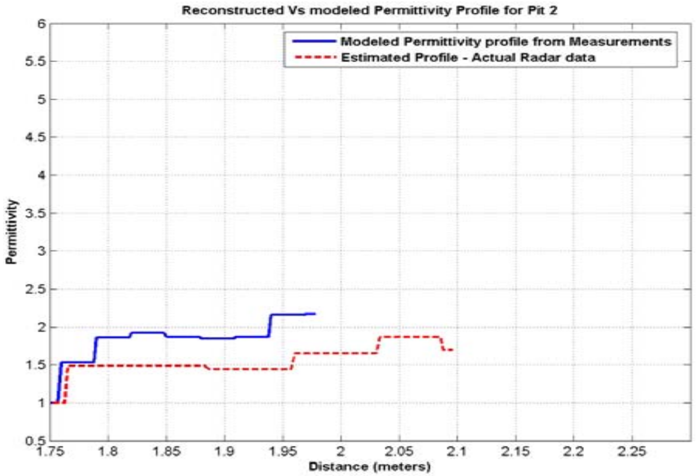
(4) Measurement errors

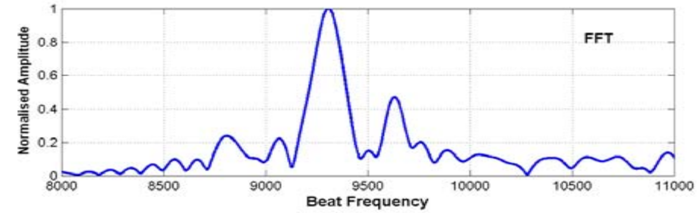
FMCW RADAR TEST – ANTARCTICA

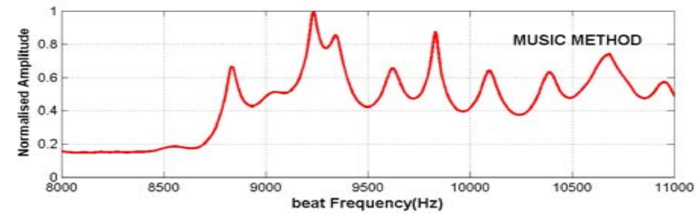
Inversion on other data sets

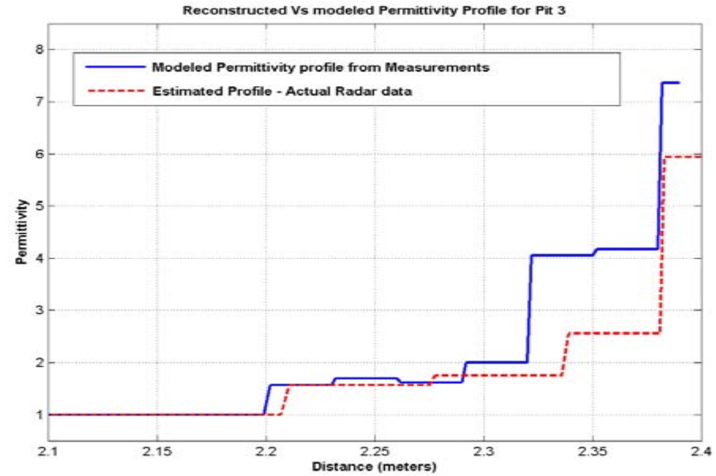












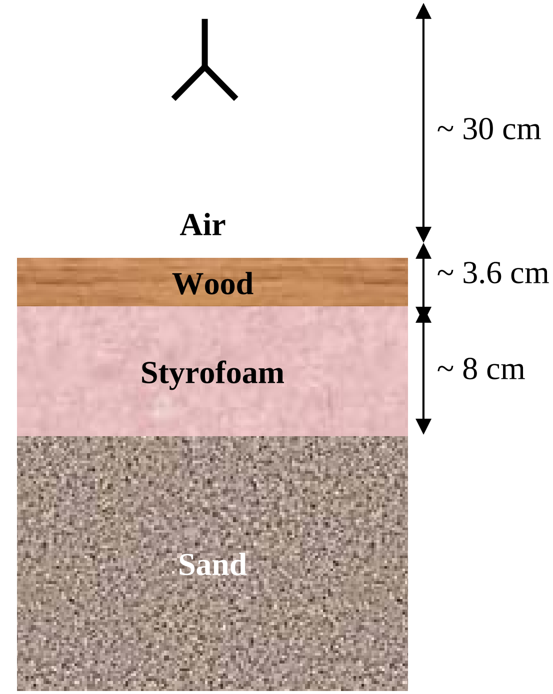
SANDBOX TESTS

Experiment Set-up

RSL Sandbox facility

Start Frequency	2 GHz
Bandwidth	7 GHz
Number of frequency samples	1601
Sweep time	800 ms
Transmit Power	0 dBm
Calibration type	1 port
IF Bandwidth	3000 Hz
Antenna type	TEM Horn
Antenna Gain	10 dB

Network Analyzer Parameters

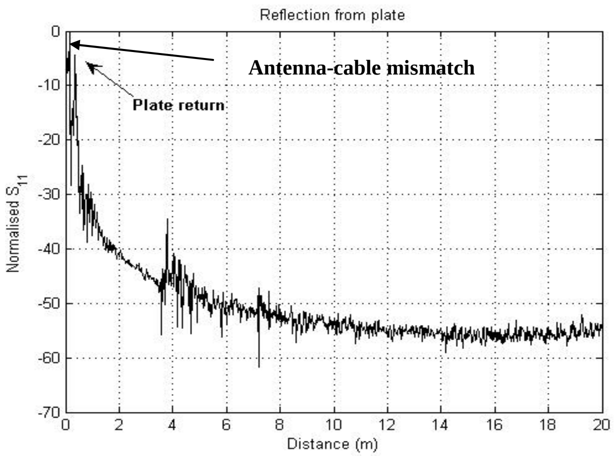


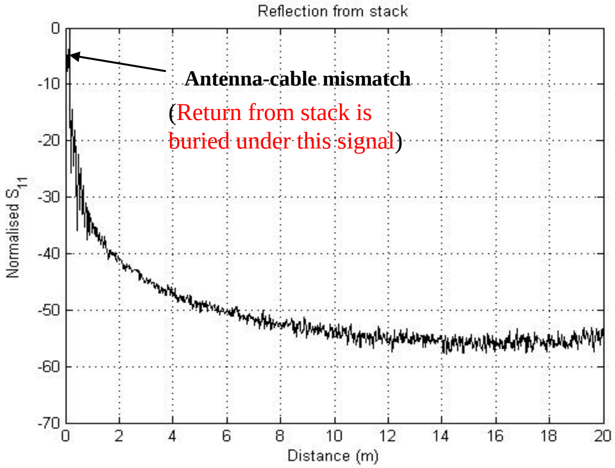
Dielectric stack to test inversion

SANDBOX TESTS

Measurements

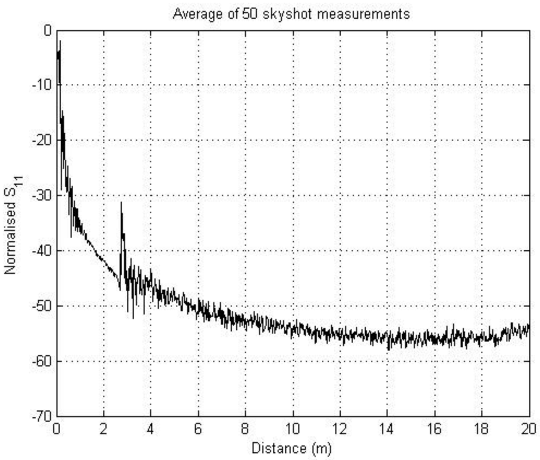
- Calibrate at antenna terminals
- Measure S_{11} with Aluminum plate as target
- Measure S_{11} with multi-layered stack arrangement
- Mismatch between antenna and the cable connecting the Network Analyzer is removed by taking Sky- shot measurements
- Subtract Sky shot from $S_{11}$of target, plate
- Use plate impulse response to remove system effect
- Apply MUSIC to enhance and invert

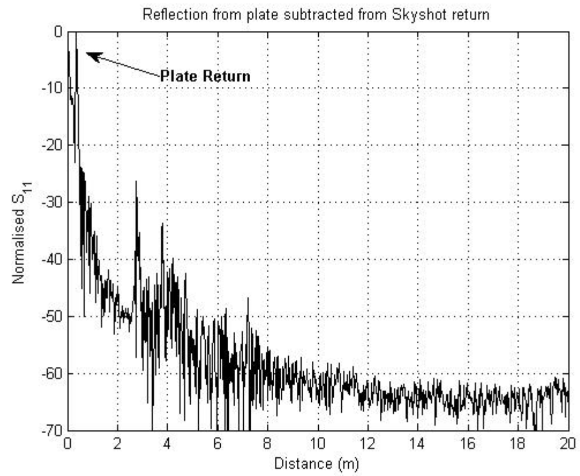


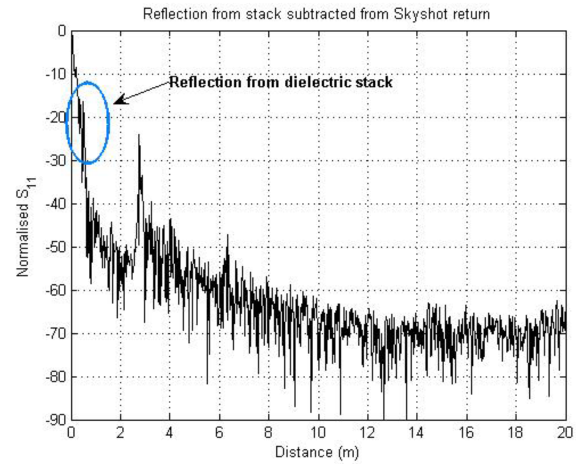


SANDBOX TESTS

Measurements

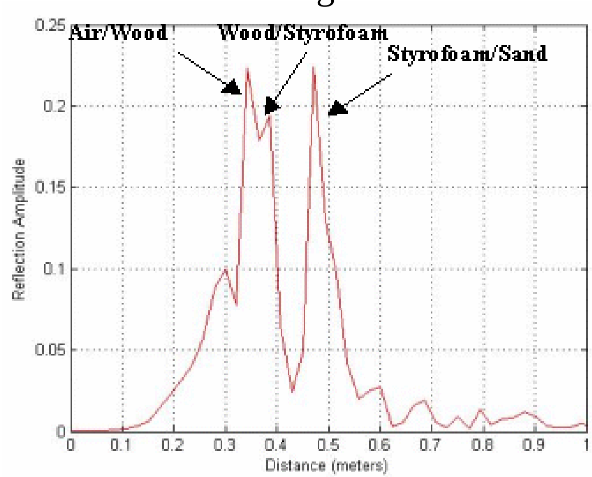






Signal after removing sky shot, system effects

FFT Range Profile

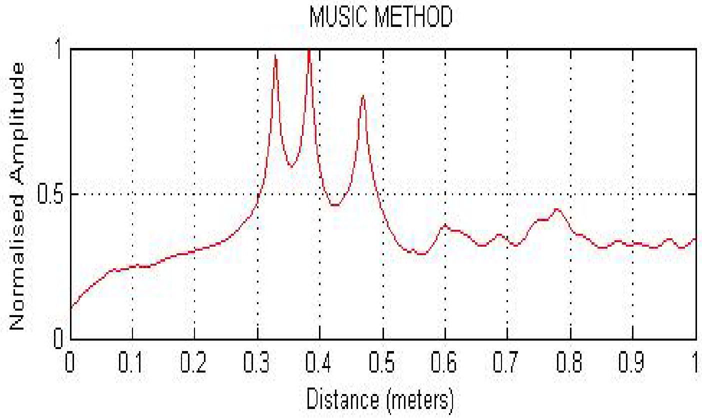


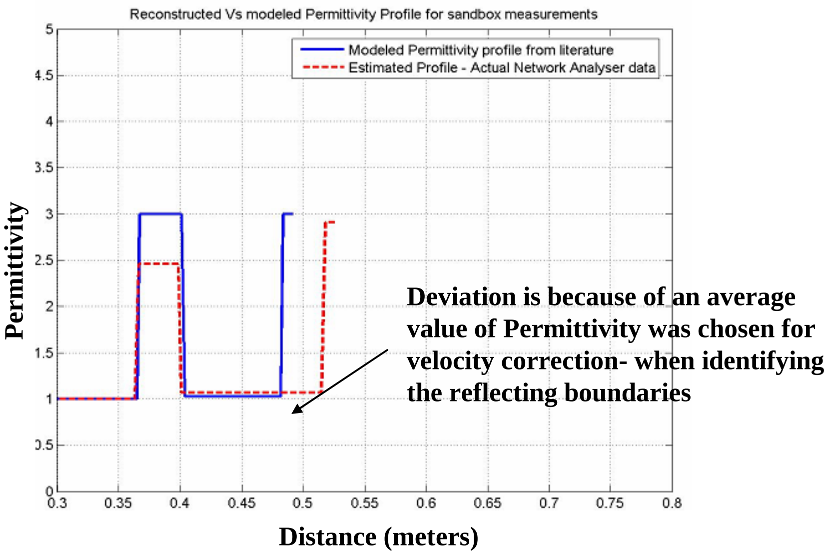
This signal can now be fed into the inversion algorithm

SANDBOX TESTS

Results

Range Profiles using MUSIC





- Problem with reconstruction of Permittivity profile
- Properties of noise could not be confirmed
- Layer Stripping approach was followed

Reference permittivity values

Air : 1

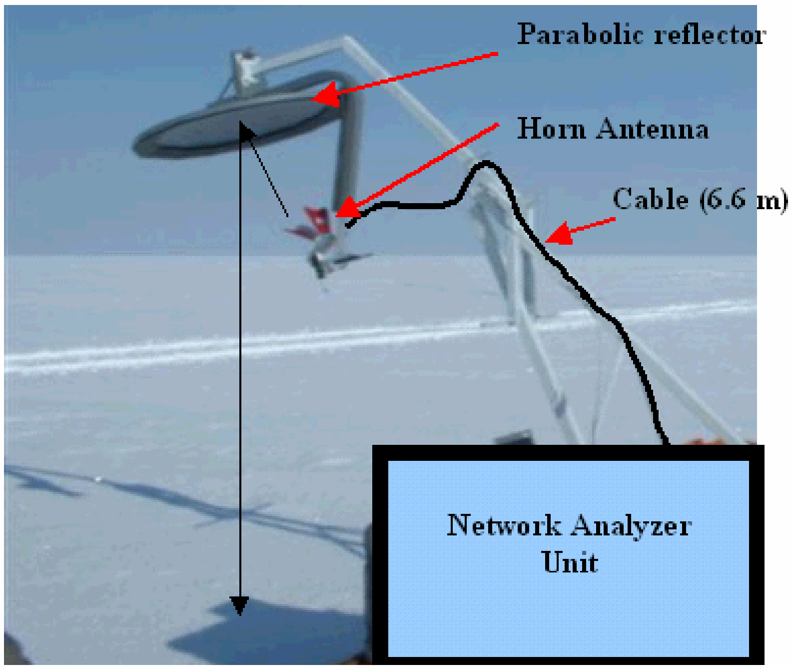
Wood: 2 – 6 (a value of 3 was chosen for modeling)

Styrofoam : 1.03

Sand : 2.5 – 3.5 (a value of 3 was chosen for modeling)

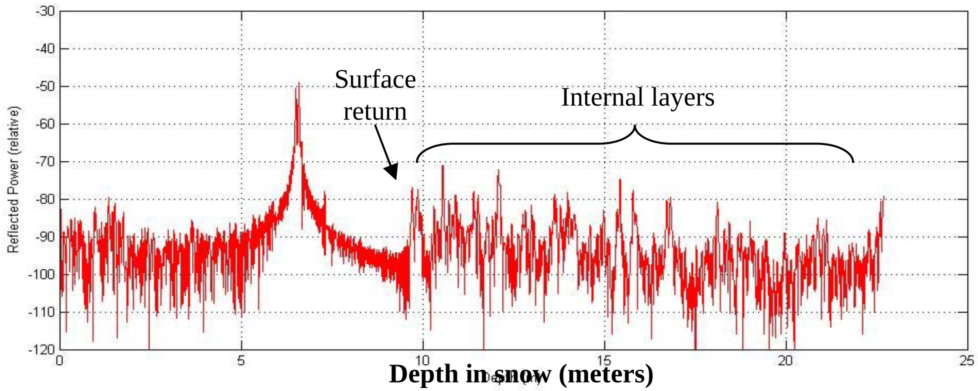
PLANE WAVE DATA INVERSION

Setup - Greenland



Type of radar	Step Frequency
Start Frequency	12 GHz
Bandwidth	6 GHz
Sweep Time	4.72 sec
No. of frequency points	801

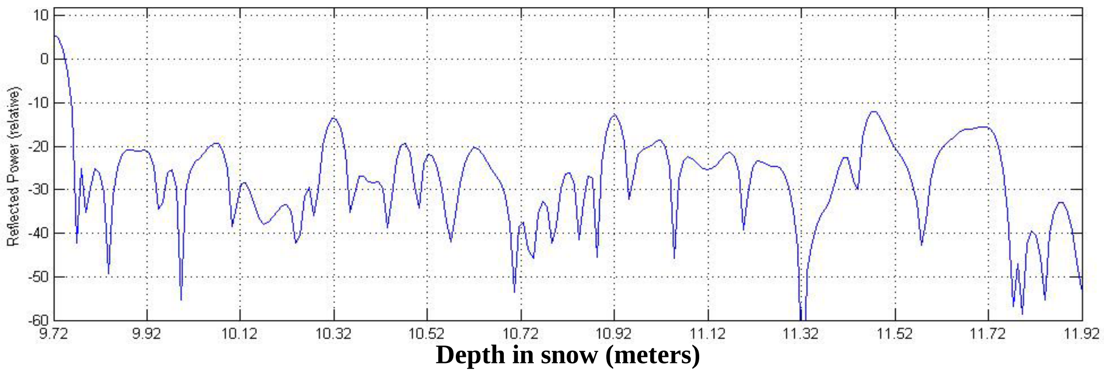
Measured data



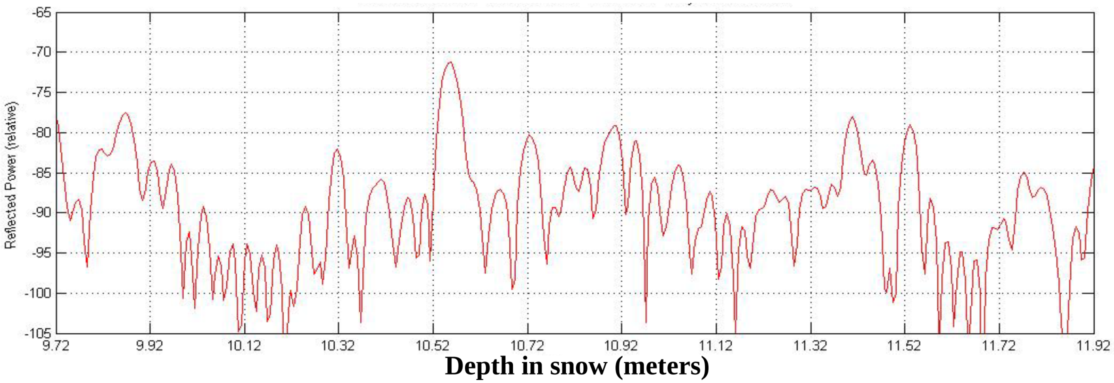
PLANE WAVE DATA INVERSION

Analysis

Simulated range profile of Pit using ADS



Actual radar return



- Inconsistencies in measured data
- Internal reflections have higher amplitudes than surface reflection
- Inversion yielded very high permittivity estimates

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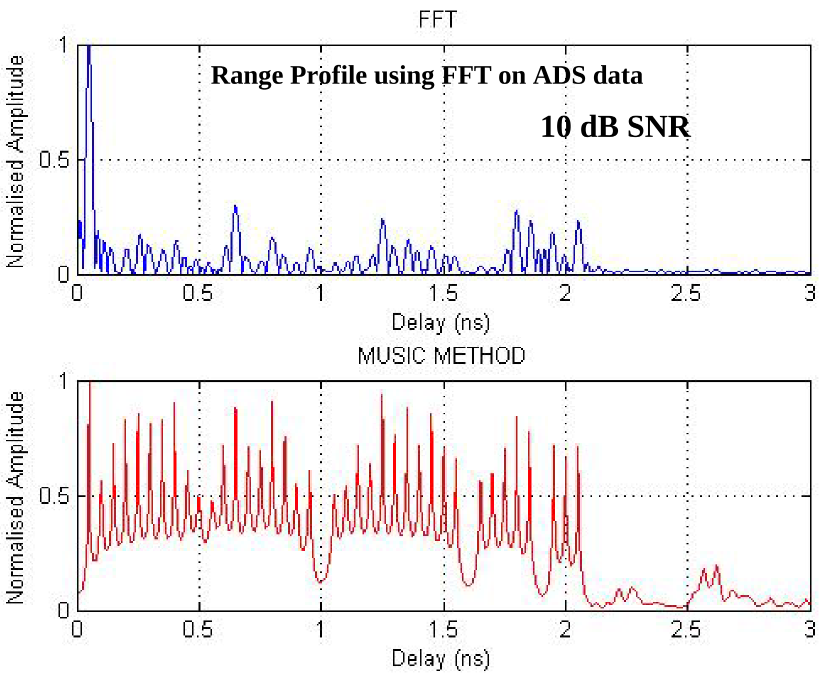
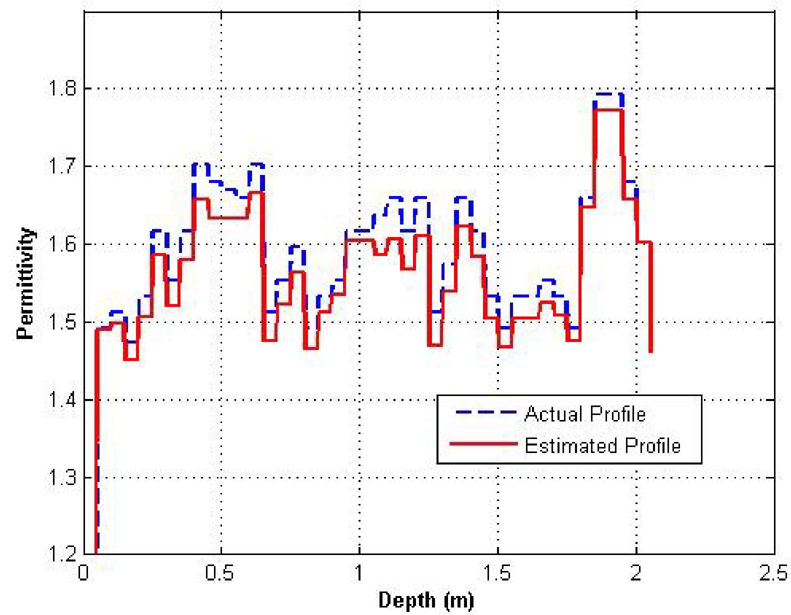
PLANE WAVE DATA INVERSION

Inversion test on ADS simulated data



Range Profile using MUSIC



MUSIC works well in the case of multiple reflections

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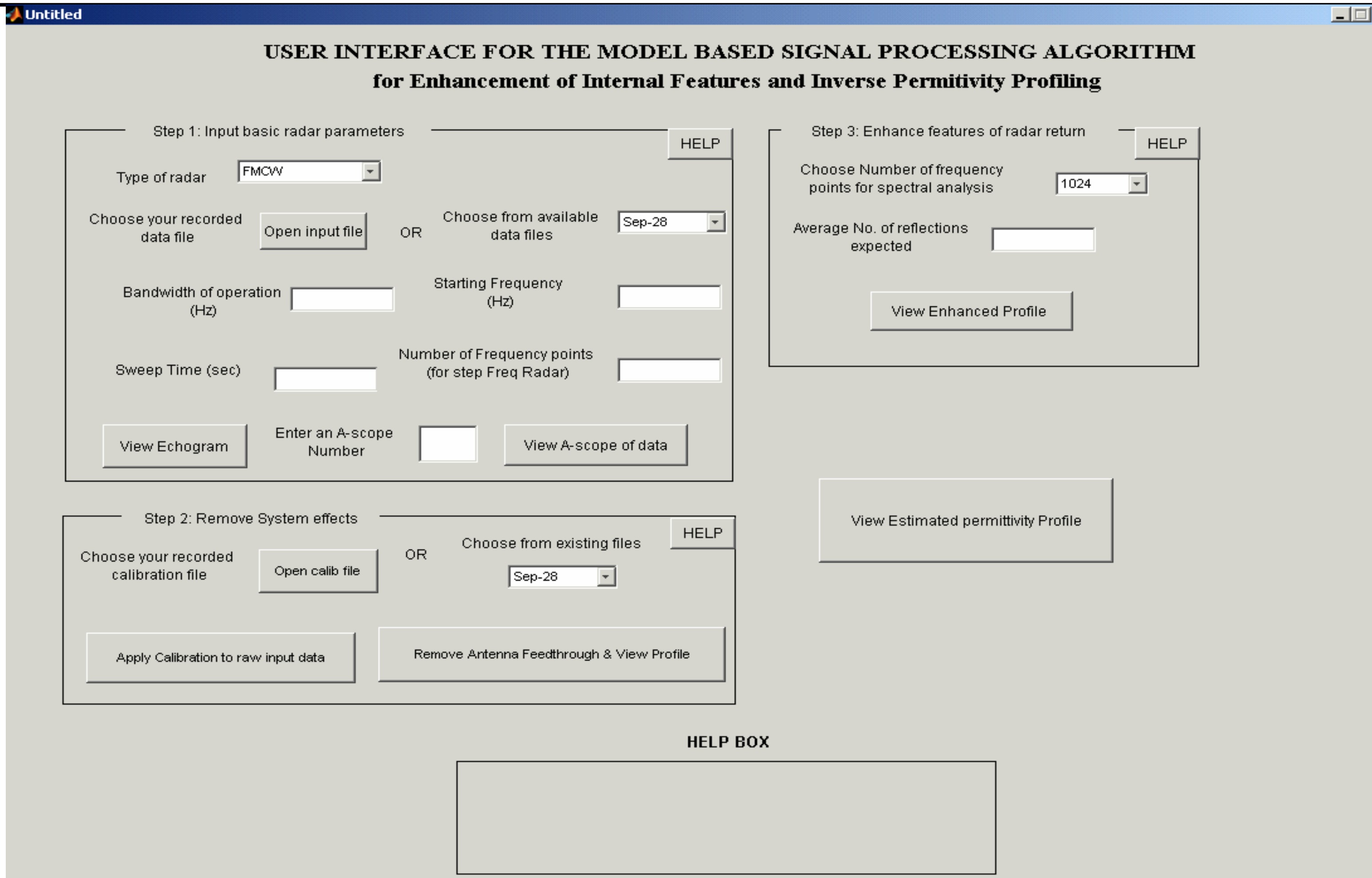
G.U.I FOR DATA INVERSION



OUTLINE

- **Introduction**
  - GPR Applications
  - Thesis Objectives
- **The Inverse Problem**
  - Forward Modeling – FMCW Radar
  - Layer Stripping Approach
  - The Model Based Approach
- **Model Based Parameter Estimation**
  - MMSE based (Gauss-Newton)
  - Spectral Estimation based (MUSIC)
- **Inversion on actual radar data**
  - Tests on Antarctic snow radar data
  - Tests at the Sandbox lab
  - Tests on Greenland Plane wave data
  - GUI for data inversion algorithm
- **Conclusions and Future Work**

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SUMMARY

- Studied, simulated and analyzed inversion schemes
  - Layer Stripping
  - Gauss Newton
  - MUSIC → yields acceptable results in simulation
- Implemented the MUSIC algorithm to enhance and invert GPR data
  - Tested on actual radar data
  - Successful in Snow radar data inversion
  - Partly successful in Sandbox test (Enhanced Profile)
- Developed a GUI for the algorithm

FUTURE WORK

- Incorporate effects of scattering due to rough surface and losses due to attenuation into the forward model
- Pre-whitening filter may be used to obtain Gaussian Noise statistics (or look at techniques for amplitude estimation in colored noise)
- 3 - Dimensional FDTD, MOM can be used to represent forward model for better inversion results

THANK YOU!

QUESTIONS/COMMENTS?