Survivability Function – A Measure of Disaster-Based Routing Performance

Journal Club Presentation on


July 6th 2007

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Outline

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• Survivability Assessment Models
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  – Heuristic Procedures
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Introduction – Internet Traffic

• Explosive growth in traffic carried by telcom networks

• Potential impact of network failure
  – significant traffic carried by a single fiber
  – WDM adds to this capacity (up to Tbits per sec)
  – network survivability is crucial
  – higher impact in terms of traffic loss, scope & users affected

• Network survivability assessment
  – survivability function & survivability attributes
  – measure of disaster-based performance
  – illustrated example on polish backbone network
Introduction – Objectives

- **Disaster type**
  - one or more node and/or arc failures
- **Survivability function**
  - “the probability function of the percentage of total data flow delivered after failure”
- **Survivability attributes**
  - “measures of disaster–based routing performance”
  - $p$-percentile values, worst case survivability
- **Factors considered**
  - varying traffic environments
  - affect of routing protocols, traffic priorities, network load
Related Work

Related works in network survivability

- **Fiber network survivability (Tsong-Ho Wu)**
  - optical network survivability

- **ANSI T1A1 subcommittee**
  - concept of disaster-based survivability
  - performance following the occurrence of a failure

- **Survivability function (Liew & Lu)**
  - probabilistic measure of network survivability
  - number of users/nodes still connected, fraction of working links

- **Shortcomings**
  - no formal definitions
  - no formal models to express survivability as a whole
Network Survivability Model

- Network model
  - network modeled as a directed graph $\Gamma(N, A)$ where
    $N$ is a set of nodes $|N| = N$;
    $A$ is set of directed arcs $|A| = M$
  - topology described with node-to-node incidence matrix
    $$ a_{ij} = \begin{cases} 
    1, & \text{if arc } e_m \equiv (i, j) \in A \\
    0, & \text{otherwise}
    \end{cases} $$
  - each arc has a finite capacity $C_m; m = 1, 2, \ldots, M.$
  - flows between all pair of nodes given by demand matrix $R$ where
    $r_{pq}; p = 1, 2, \ldots, N; q = 1, 2, \ldots, N; p \neq q$ is a commodity
  - failures
    + randomness and statistical independence is assumed
    + network nodes and arcs fail with known distribution
Network Survivability Model

- Survivability function
  - failure scenario (denoted by $\zeta$) is set of component failures
    subset $N_D$ out of $N$ nodes or subset $M_D$ out of $M$ arcs
  - note: different scenarios ($\zeta$) may result in similar surviv. values
  - survivability function:
    "probability function of total data flow delivered after failure"
    \[
    S(x) = \sum_{\zeta: X(\zeta) = x} P(\zeta)
    \]
- $X(\zeta)$: random variable
  percentage of flow delivered after failure $\zeta$
- $P(\zeta)$: probability of scenario $\zeta$ characterized by the percentage $x$ of total data flow still delivered
- $S(x)$: always less than 1
Network Survivability Model

- Survivability attributes based on $S(x)$
  - expected percentage of total data flow delivered after a failure

  $$E(x) = \sum_{x} x \cdot S(x)$$

  - $p$ – percentile survivability

  $$P_{px} = P(X = p)$$

  - worst case survivability

  $$x^* = \min_{S(x) > 0} x$$
Survivability Assessment Models

• Complexity of calculations
  – assume arc failures only in the given $M$ arcs
  – number of scenarios to be investigated to calculate $S(x)$ is $2^M$
  – intractable even for small networks

• Simplified calculations
  – assume probability distribution of arc failures to be uniform
  – each arc $M$ fails with a probability of $M^{-1}$
  – Probability $P(\zeta)$ of scenario $\zeta$ with $M_D$ failed arcs

\[
P(\zeta) = \prod_{l=1}^{M_D} \frac{1}{M} \prod_{k=1}^{M-M_D} \frac{M-1}{M}
\]

  – $P(\zeta)$ decreases rapidly with increasing $M_D$
Survivability Assessment Models

- Simplified calculations
  - the number of such scenarios with $M_D$ failed arcs
    \[
    L_{M_D} = \binom{M}{M_D} = \frac{M!}{M_D!(M - M_D)!}
    \]
  - increase in the cumulative probability mass (c.p.m) due to all scenarios with $M_D$ failed arcs
    \[
    \nu_{M_D} = L_{M_D}(\zeta) \times P(\zeta)
    \]
  - actual value of c.p.m achieved due to all scenarios investigated from no arc failures to $M_D$ failed arcs
    \[
    \nu_0 + \nu_1 + \ldots + \nu_{M_D}
    \]
  - calculate $S(x)$ until c.p.m of all examined scenarios reaches a given threshold (e.g. 99%)
Survivability Assessment Models

- Simplified calculations

### TABLE I

**Rate of Increase of the c.p.m. for a Network**

With $M = 500$ Arcs and $0 \leq M_D \leq 8$

<table>
<thead>
<tr>
<th>$M_D$</th>
<th>$P(\zeta)$</th>
<th>No. of scenar.</th>
<th>No. of steps</th>
<th>Comp. of mass</th>
<th>c.p.m.f.</th>
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Survivability Assessment Models

• Finding survivability functions
  – depends on the routing protocol, user priorities and network load
  – consider only the routing protocol – DV and flooding
  – in case of DV
    + find optimal commodity for each failure scenario $\zeta$
    + multi-flow commodity route optimization can be set as linear programming problem
    + can be solved using a linear programming package (e.g. PPRN)
    + constraint matrix too large with few non-zero elements
  – heuristic procedure based on Seidler’s result
    + optimal multi-commodity flows $x^*$ are a superposition of the nonbifurcated flows on all shortest paths where the length of the path is the cost of sending unit data on all the component links
Survivability Assessment Models

- Finding survivability functions

Find optimum multicommodity flows, minimizing cost function \( \varphi(x) \) subject to:

- flow conservation constraint

\[
\sum_{m \in \{m \in \{\{i, j\} \in A; i = 1, 2, ..., N; j \neq n\} \}} x_{k,m} - \sum_{m \in \{m \in \{\{n, j\} \in A; j = 1, 2, ..., N; j \neq n\} \}} x_{k,m} = \begin{cases} -r_k, & \text{if } n = s_k \\ +r_k, & \text{if } n = t_k \\ 0, & \text{otherwise} \end{cases}
\]

- finite arc capacities

\[
\sum_{k=1}^{K} x_{k,m} \leq c_m; \quad m = 1, 2, \ldots, M
\]

- nonnegativity constraints

\[
x_{k,m} > 0; \quad k = 1, 2, \ldots, K; \quad m = 1, 2, \ldots, M
\]

The linear cost function is defined as

\[
\varphi(x) = \sum_{k=1}^{K} \sum_{m=1}^{M} \text{cost}_{k,m} x_{k,m}
\]
Survivability Assessment - Heuristic procedure (DV)

0. preparatory step:
   - define disaster type (node/arc), c.p.m threshold,
   - calculate prob. of no failures and let $S(x = 0) = 0$

1. generate the next most probable failure scenario $\zeta$
2. set to zero all elements of A corresponding to failures in $\zeta$
   1. check for net. partitioning, if so add $P(\zeta)$ to value of $S(x = 0)$ else go to step 1
Survivability Assessment - Heuristic procedure (DV)

0. preparatory step:
   define disaster type (node/arc), c.p.m threshold,
   calculate prob. of no failures and let \( S(x = 0) = 0 \)

1. generate the next most probable failure scenario \( \zeta \)

2. set to zero all elements of \( A \) corresponding to failures in \( \zeta \)
   1. check for net. partitioning, if so add \( P(\zeta) \) to value of \( S(x = 0) \) else go to step 1

3. for each commodity, find the shortest path and its capacity
   1. if demand < capacity, set flow = demand; calculate residual capacity on the path and go to set 3.3.
   2. if demand > capacity, set flow = capacity; find the next shortest path and allocate the remaining portion of the demand to it
   3. calculate which percentage of total flow \( x_s \) has been found and save pair \( (S_x, P(\zeta)) \) to the database

4. if c.p.m < threshold, go to step 1, otherwise go to step 5

5. group, order results and calculate the survivability function for whole range (0 – 100%) of \( x \) with specified granularity (say 1%)
Survivability Assessment - Heuristic procedure (FL)

1. For flooding algorithm
   - same as DV except for step 3
   - 3.1 allocates multi commodity flows on shortest paths
   - 3.2 the possibility of sending additional flows is examined.
     + at a selected node the procedure finds any other non-zero capacity paths
     + if found, either the unsatisfied demand or the capacity of the new path is assigned for this commodity flow (as supplementary fraction).
     + stop if no path with non-zero capacity available.
Survivability Assessment

- Demand matrix
  - how are the demands in the previous algorithms determined?
  - find maximum feasible and fair flows in fully operational network
  - denote this flow matrix as the optimal flows matrix $F^*$
  - for a given load, the matrix of required flows (demand matrix) is
    \[ R = \text{load} \times F^* \]
  - in case of priority traffic, demand matrices for each priority are specified
    \[ R_1, R_2, \ldots, R_p \]
Backbone Network Example

- Polish backbone network
  - \( N = 12, \quad M = 34 \); \( C_m = 2.5 \text{ Gb/s} \)
  - survivability function in case of routing protocols
    + based on minimum number of hops and on flooding

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</table>

Source: Molisz-2004
Fig 1: Survivability function $S(x)$ for node (or arc) failures
load=60%, routing based on minimum number of hops

$E[x] = 80.41\%$

$E[x] = 95.69\%$

[Source: Molisz-2004]
Backbone Network Example - Analysis

Fig 1: Survivability function $S(x)$ for node (or arc) failures
load=90%, routing based on minimum number of hops

$E[x] = 78.03\%$

$E[x] = 89.60\%$

[Source: Molisz-2004]
Backbone Network Example - Analysis

Fig 1: Survivability function $S(x)$ for node failures only. Load=60%, routing based on minimum number of hops; traffic has five priorities.

$E[x] = 80.47\%$

[Source: Molisz-2004]
Backbone Network Example - Analysis

Fig 1: Survivability function $S(x)$ for node failures only. Load=90%, routing based on minimum number of hops. Traffic has five priorities.
Backbone Network Example - Analysis

Fig 1: Survivability function $S(x)$ for node failures only, load=200%, routing based on minimum number of hops, traffic has five priorities.

[Source: Molisz-2004]
Backbone Network Example - Analysis

Fig 1: Survivability function $S(x)$ for node failures only; three loads, routing based on minimum number of hops

[Source: Molisz-2004]
Backbone Network Example - Analysis

Fig 1: Survivability function $S(x)$ for node failures only; load=60%, routing based on flooding; traffic has five priorities.

[Source: Molisz-2004]
Conclusions

• lowest survivability occurs due to node failures
  • routing protocol has significant impact
    – min. num of hops gives relatively good results
    – flooding diminishes survivability dramatically
• congestion further reduces survivability
• greater degradation is observed at higher loads
  – more visible when dealing with priority traffic
• Quantitative evaluation of flow degradation due to failure scenarios is achieved
THANK YOU!