Static Pricing and Traffic Management in Networks with Allocation of Resources

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Agenda

- Motivation
- System Model
- Preliminary Results
  - Simulation
  - Analysis
- Conclusions and Further Work

Why Allocation-based Networks ?

- Demand for applications that deliver data, audio, image, video at high speed ⇒ BISDN
- Heterogeneous applications require multiple service classes for adequate QoS
- Future Internet will support multiple classes
- QoS guarantees usually implemented through the allocation of resources (bandwidth, buffers).

Why Pricing ?

- Pricing may affect several traffic management issues
  - congestion control
  - call admission control
  - network performance
- Multi-service networks require incentives for efficient service choices
  - discourage over-allocation
  - maximize statistical multiplexing
- Billing may impact network load and network equipment design

And Why Static Pricing ?

- Dynamic schemes - prices fluctuate as a result of some network condition
  - provide some elegant answers to pricing problems BUT
  - are costly to implement
  - usually require application software redesign
  - may encounter resistance from users
- Static pricing - independent of network utilization
  - generally easier to implement
  - simpler for users to understand
  - used today in the vast majority of commercial networks

Objective

Using a game-theoretic model, we illustrate how a static pricing policy can influence user behavior, with implications to revenue, user satisfaction and the traffic management task.

A Game-theoretic Model

The pricing problem is modeled as a non-cooperative game, consisting of a principal (the network provider) and a finite set of players $\mathcal{N} = \{1, 2, \ldots, N\}$ (the network users).

Each player independently chooses a strategy $\mathbf{s_i}$ seeking to maximize her payoff function C_i.

Users preferences are characterized via utility functions, indicating willingness to pay for a certain amount of resources allocated to the call.

Payoff function is the user surplus, the difference between the utility derived with a service and price paid for it.

Predicting the Outcome of the Game

If a unique Nash Equilibrium exists, it is considered a consistent prediction of the outcome of the game.

- A N.E. is a joint strategy where no individual user can increase her surplus by deviating unilaterally.
- Strategy $\mathbf{s}$ is a Nash equilibrium if $C_i(\mathbf{s}) \geq C_i(\mathbf{s_i^*}, \mathbf{s_{-i}}),\ \forall\ \mathbf{s_i^*} \in \mathcal{S}_i,\ \forall\ i \in \mathcal{N}$.

Allocation-Based Networks

- Users are allowed to allocate bandwidth to their calls
- Excess bandwidth is distributed uniformly among all users
- Strategy space for the game is $\mathcal{S} = \{\mathbf{s} \in [0, L]^N : \sum_{i \in \mathcal{N}} s_i \leq L\}$
  s_i is the amount of bandwidth allocated by user i
- Utility may be expressed as a function of available bandwidth b_i or of some QoS parameter (*e.g.* CLR).
- Price can be a function of allocated bandwidth s_i and/or utilized bandwidth $\hat{b}_i$

Simulation

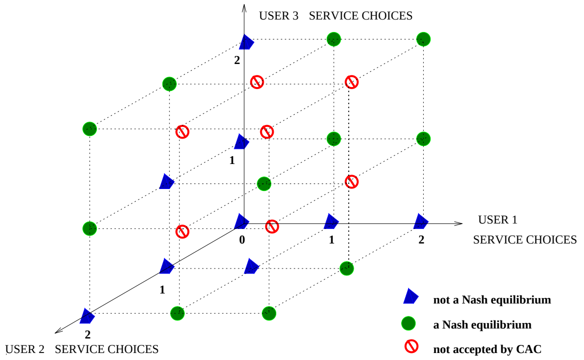
- Three video sources competing for a single link
- Utility is a function of the portion of cells not delivered within a tight delay bound
- Weighted Round Robin (WRR) scheduler
- Each user has a choice of best-effort service, allocation of equivalent bandwidth and peak rate allocation.

Summary of Simulation Results

- If network load is low, under-allocating is always an equilibrium regardless of pricing policy.
- Pricing influences user behavior when network utilization is high
  - If prices do not depend upon allocation, users over-allocate.
  - By adding an allocation-based component to the pricing policy, the provider can induce a unique equilibrium that maximizes aggregate utility.

Results without Allocation-based Pricing

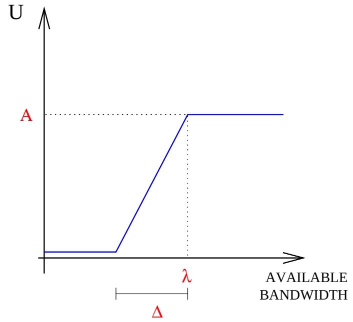
- Users allocate maximum amount of bandwidth allowed by CAC



Analytical Approach

- Users are allowed to allocate a continuum of bandwidth to their calls
- Utility is a function of available bandwidth b_i
- Price is a linear function of allocated bandwidth and utilized bandwidth $\hat{b}_i$
- $C_i(\mathbf{s}) = U_i(b_i) - (c + k_f s_i + k_g \hat{b}_i)$
- Non-linear programming analysis

Generalized approximation to Utility Functions



- Δ indicates user elasticity:

  For inelastic users, $\Delta = 0$; for perfectly elastic users, $\Delta = \lambda$.

Mix of Elastic and Inelastic Users

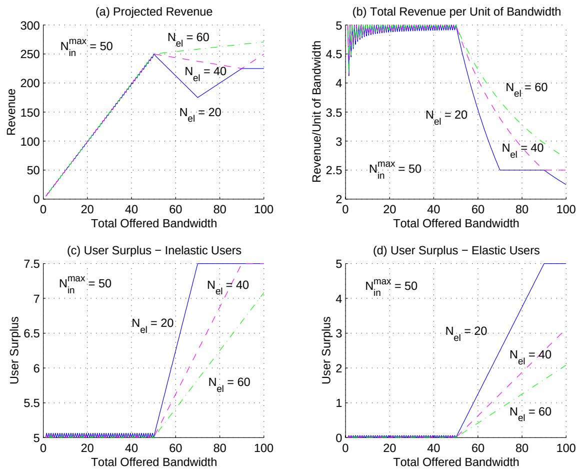
Proposition 1 *If there are N_{in} identical inelastic users and N_{el} identical perfectly elastic users, with utility functions characterized by (λ_{in}, A_{in}) and (λ_{el}, A_{el}), respectively, then as long as service price is a strictly increasing function of allocated bandwidth,*

1. *if $L \leq N\lambda_{in}$, the only Nash equilibrium that is Pareto optimal is $s_i = \frac{N_{in}+N_{el}}{N_{el}}\lambda_{in} - \frac{L}{N_{el}}$ for $i \in \mathcal{N}_{in}$ and $s_i = 0$ for $i \in \mathcal{N}_{el}$;*
2. *if $L > N\lambda_{in}$, the only Nash equilibrium that is Pareto optimal is $s_i = 0 \; \forall \; i \in \mathcal{N}$.*

Dimensioning Problem

- With a fixed prices per unit of utilized and allocated bandwidth, study how revenue and consumer surplus are affected by the total offered bandwidth.
- Results:
  - There are diminishing returns from offering bandwidth in excess of the amount needed by the inelastic users.
  - Increasing total bandwidth may result in a reduction in revenue

Dimensioning Problem



Pricing Problem

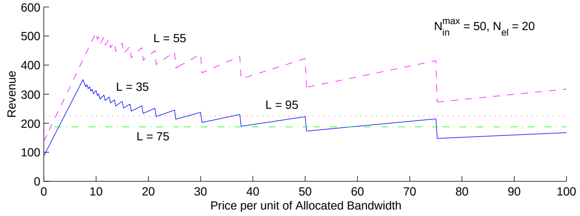
- Fix L and the price per unit of utilized bandwidth k_g. Then the maximum revenue that can be obtained by the provider is:

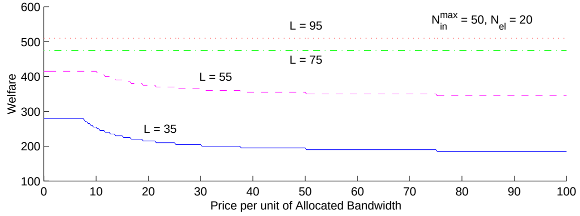
$$R_{max} = \begin{cases} N_{in}A_{in} + N_{el}k_g \min[\lambda_{el}, \frac{L-N_{in}\lambda_{in}}{N_{el}}] & \text{if } L < N\lambda_{in} \\ k_g \min[L, N_{in}\lambda_{in} + N_{el}\lambda_{el}] & \text{otherwise} \end{cases}$$

- Moreover, in the top case revenue is maximized when

$$k_f = \frac{N_{el}}{N\lambda_{in} - L}(A_{in} - k_g\lambda_{in})$$

Pricing Problem


(a) Projected Revenue


(b) Social Welfare

Conclusions and Further Work (I)

- By providing monetary incentives one can tailor the services provided to customers' needs, and users' choices to beneficial network utilization
  - Pricing can be used to ensure the existence of a unique efficient equilibrium and to maximize revenue
- At high network utilization, an appropriate pricing policy will induce users to reveal their bandwidth needs, providing valuable information for network management (and, in particular, CAC)
- Results allow service providers to predict revenue based on some knowledge of customer needs and willingness to pay

Conclusions and Further Work (II)

- In many cases it is an equilibrium for users *not* to allocate any bandwidth (in this static model, even true for inelastic users)

  ⇒ pricing policies must generate revenue through other means besides allocation-based charges

- Model can be employed as a rough approximation to ATM networks
  - in the process of extending results for interpretation in an ATM context