Printed Plane-Filling Fractal Antennas for UHF Band

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I. INTRODUCTION

Printed Hilbert antenna has been used for UHF band since it is small, easy to fabricate and impedance-match to common types of planar transmission lines [1]. Vinoy [2] studied the resonant frequency of wire Hilbert antenna in free space by using an approximate method. Anguera [3] studied a vertical monopole Hilbert antenna. Chen [1] studied the characteristics of printed Hilbert Antenna used as a dipole, including the dependence of its resonant frequency and input impedance on different parameters, current distribution at resonant frequency, miniaturization capability and radiation pattern.

There are many different plane-filling fractal curves other than Hilbert curve, for example, Moore, Peano, Wunderlich 1, 2 and 3 curves [5-7], as shown in Fig. 1. However, previous researches have focused on Hilbert curve, and how do other types of space-filling curves affect the current distribution, resonant frequency and input impedance of antennas shaped in them, is less explored.

In this research we studied printed plane-filling antennas of different shapes, including Hilbert, Moore, Peano, Wunderlich 1, 2 and 3 curves. The input impedance and current distribution are calculated through method of moments (MoM) with mixed-potential integral equations (MPIE) formulation. The resonant frequencies are obtained by searching for the roots of the imaginary part of the input impedance. The dependence of the resonant frequency on the shape and size of the antennas are studied, and the current distributions along the antennas at resonant frequency are compared.

II. SIMULATION METHOD

The plane-filling fractal curves are generated through a recursive procedure based on descriptions given by [7]. The printed antenna is defined as a trace with specified width running along the curve. The trace is then meshed into triangles as shown in Fig. 1.

The surface current density on the trace is unknown quantity to be solved, which is expanded by a set of RWG [8] basis functions $\vec{J}_n$ defined on the triangles,

$$\vec{J}(\vec{r}) = \sum_{n=1}^{N} \vec{J}_n(\vec{r})$$

The MPIE for PEC surface is [9]:

$$\vec{n} \times \vec{E} = -\mu_0 \mu_0 \iiint_{\Omega} \left[ \vec{G}_s(\vec{r},\vec{r}') \cdot \vec{J}(\vec{r}') d\vec{r}' \right] + \iiint_{\Omega} \left[ \vec{G}_s(\vec{r},\vec{r}') \cdot \vec{J}(\vec{r}') d\vec{r}' \right]$$

where $S$ is the PEC surface, $\vec{n}$ is the unit normal vector of $S$, $\vec{E}$ is incident field, $\vec{S}$ and $\vec{G}_s$ are Green’s functions for $\vec{A}$ and $\vec{S}$, respectively. The formulas of $\vec{S}$ and $\vec{G}_s$ is given by [9], and they are calculated by using complex image method (CIM) [10].

The feed structure is modeled by a delta-gap voltage source. After applying the Galerkin’s procedure [8], (2) is converted to a matrix equation:

$$[Z]\{I\} = \{V\}$$

which is then solved and the current distribution is obtained. The input impedance is calculated based on the current distribution.
TABLE I

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Fig. 1 Plane-filling fractal antennas. (a) 3rd order Hilbert, (b) 3rd order Moore, (c) 2nd order Peano, (d) 2nd order Wunderlich 1, (e) 2nd order Wunderlich 2, (f) 2nd order Wunderlich 3.

III. NUMERICAL RESULTS

Printed plane-filling fractal antennas of different shapes and outer dimensions have been simulated. The substrate is a common 1/16" FR4 PCB board, with dielectric constant around 4.3. The ground plane is removed to increase the radiation.

Fig. 2 shows the input impedance of a 3rd order Moore antenna around its first resonant frequency. The antenna is fed at half of the total length by a delta-gap voltage source. From this figure we can see that the behavior of input impedance of Moore antenna is equivalent to a series resonant circuit, which is quite similar to a Hilbert antenna [1]. Peano, Wunderlich 1, 2 and 3 antennas showed similar behavior around their first resonant frequency. It is noted that the input resistance is relatively small at the resonant frequency, but it can be increased by shifting the feeding position toward the end of the antenna, as demonstrated as follows.

Fig. 2 Input impedance of 3rd order Moore antenna.
Width=1mm. Outer dimension=30mm.

Fig. 3 shows a 3rd order Moore antenna fed at position A and its input impedance is shown in Fig. 4. The dimension of the antenna is the same as the one used for Fig. 2. The resonant frequency is 646.6MHz and the input resistance at the resonant frequency is 45.7 ohm. The same antenna fed at center has resonant frequency of 646.5MHz and input resistance of 0.69 ohm.

Table I and Fig. 5 show the resonant frequency of printed plane-filling fractal antennas of different shapes and sizes. The total length of the antennas is listed in Table II. From Fig. 5 we can see that as dimension increases, resonant frequency decreases. The resonant frequencies of 3rd order Hilbert and Moore antennas and 2nd order Peano.
TABLE II
THE TOTAL LENGTH OF PRINTED PLANE-FILLING FRACTAL ANTENNAS (THE NUMBER IN THE BRACKET DESIGNATES THE ORDER OF THE CURVE.)

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Wunderlich 1 and 2 antennas have similar resonant frequency, while 2\textsuperscript{nd} order Wunderlich 3 antenna has slightly lower resonant frequency than others.

Fig. 6 shows the current distribution along different antennas. The outer dimensions of these antennas are all 3cm. Current distribution along antennas of the same shape but different outer dimensions are also obtained. It is found the current distribution depends mainly on the shape but not the outer dimension, i.e., antennas with the same shape but different outer dimensions have similar current distribution. Of the six curves Wunderlich 3 curve is special because it is not symmetric, which may be the reason why its current distribution is also not symmetric. All other curves have symmetric current distribution. Peano and Wunderlich 2 antennas have similar current distribution because their main shapes are similar (the curve at four corners and center are the same). Other than these observations, it is difficult to explain the association between a curve and its current distribution.

Fig. 3. A 3rd order Moore antenna fed at position A.

Fig. 4 Input impedance of 3\textsuperscript{rd} order Moore antenna fed at position A.

Fig. 5 Resonant frequency versus outer dimension for different printed plane-filling fractal antennas.
IV. CONCLUSIONS

In this paper printed plane-filling fractal antennas of different shapes are studied. It is found all printed plane-filling antennas behave like series resonant circuits near their first resonant frequency. The resonant frequency depends mainly on the total length but not the shape except Wunderlich 3 antenna.

ACKNOWLEDGEMENTS

The authors would like to acknowledge Digital Security Controls Ltd. (Canada) for supporting this research.

REFERENCES


