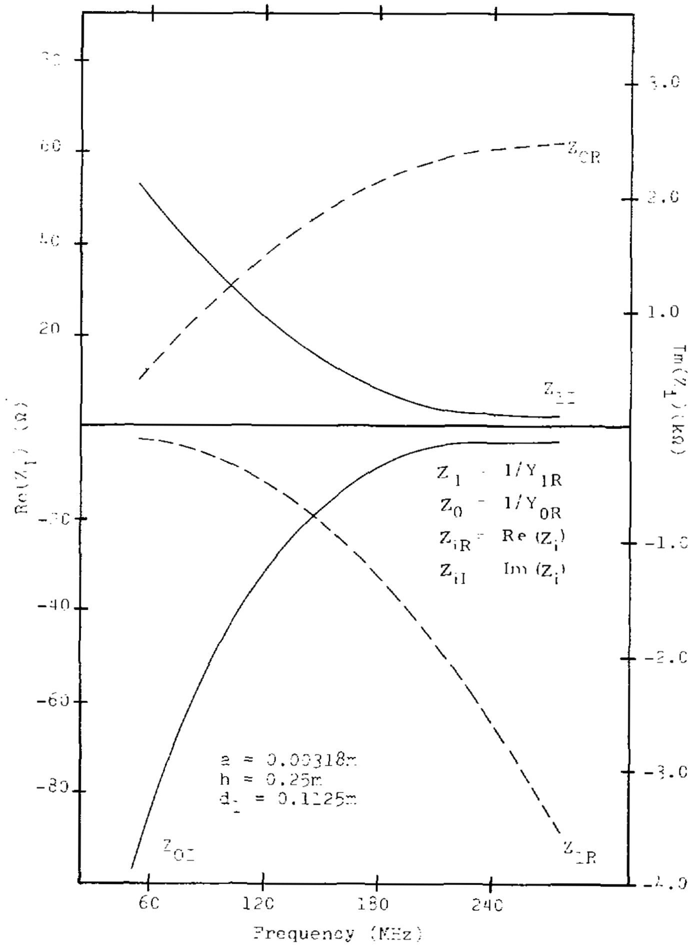

Fig. 4. Load impedance versus frequency for receiving antenna to maintain resonance instability.

to the antenna axis, the resonance of a receiving antenna should be quite narrow-banded.

Fig. 3 shows the values of loading impedences as a function of frequency for a radiating antenna with $h = 0.25$ m, $a = 0.00318$ m, $d_1 = 0.1125$ m, and $d_2 = 0.1675$ m to reach two types of instabilities. Curves 1 and 2 represent the real and imaginary parts of $1/Y_{1R}$ and curves 3 and 4 that of $1/Y_{2R}$. In this particular example, Y_{2R} is an active element and Y_{1R} is passive. With these values of Y_{1R} and Y_{2R} loaded at d_1 and d_2, this antenna will encounter the first kind of instability. In the same figure, curves 5 and 6 represent $1/Y_{1c}$ and curves 7 and 8, $1/Y_{2c}$. If Y_{1c} and Y_{2c} are loaded at d_1 and d_2, they will cause the second kind of instability in this antenna. For this instability, Y_{2c} is passive and Y_{1c} is active.

Fig. 4 gives the loading impedance, $Z_1 = 1/Y_{1R}$, and the center-loading impedance, $Z_0 = 1/Y_{0R}$, which cause the instability to a receiving antenna with $h = 0.25$ m, $a = 0.00318$ m, and $d_1 = 0.1125$ m, as a function of frequency. The results of Fig. 4 are very interesting. It is seen that Z_0 is a passive impedance with a real part near 50 Ω, a typical active load impedance for a receiving antenna. The value of Z_1 indicates a typical active loading impedance. The results of Fig. 4 provide important information for the designers of actively loaded receiving antennas. That is, for a conventional receiving antenna with a load impedance of 50 Ω, single loading with active elements wlll most likely cause the unstable problem at some frequencies. To avoid the problem of instability, an actively loaded receiving antenna may choose a load impedance which is drastically different from an impedance near 50 Ω.

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Mutual Coupling Between Antennas for Emission or Reception—Application to Passive and Active Dipoles

JEAN-PIERRE DANIEL

Abstract—**Two different definitions of coupling between antennas are given, for emission or reception. Using these definitions, computations have been made for two passive dipoles and two active dipoles (passive dipoles fed with a transistor). It appears that reduction of coupling is obtained only with an emissive active dipole array.**

Introduction

Our purpose is to discuss the notion of coupling between antennas either on emission or on reception. Ordinarily we define interaction between two radiating elements by a matrix such as an impedance matrix Z (or admittance matrix). Interaction between these antennas decreases with the term Z_{12} (or Z_{21}) when the spacing increases. For emission, the passive network allows us to define coupling with four complex terms. Reception is another problem because the network becomes an active one, in the sense that there are some internal sources imposed by the incident wave. It seems quite evident that interaction between the different ports of the network is dependent on terms relating to the passive quadripole (Z matrix, for instance), and terms relating to the internal sources.

Here we consider two identical dipoles working either on emission or on reception, and for each function we give a definition of coupling. Then, using the definition just given, we theoretically calculate the coupling of passive or active dipoles for the emission and reception cases.

Definitions—Example

We consider, here, two coupled antennas fed by generators of internal impedance Z_0 or by impedances Z_0.

In the emission case, we define coupling by the following expression:

$$C_e = \frac{P_2}{P_1} \tag{1}$$

where P_2 is the power received by antenna 2 and P_1 is the power delivered by antenna 1.

If we consider reception, we define coupling by the following expression:

$$C_r = \frac{P_2}{P_1} \tag{2}$$

where P_2 is the power received by antenna 2 (where the external field = 0) and P_1 is the power received by antenna 1 (where the external field ≠ 0).

Now we calculate the terms C_e and C_r in the particular case of an array of two parrallel and identical dipoles. We have already shown [1] that for an array of N parallel dipoles (identical or not), there

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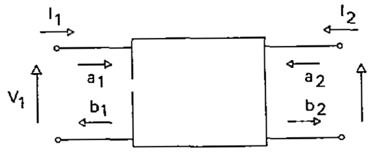

Fig. 1. Equivalent network of two-antenna array.

is a relation between currents, voltages at the base of dipoles, and the external applied electromagnetic field

$$I = [Y_A]V_0 + [T_A]U^c \tag{3}$$

where I is the column vector of base currents of dipoles; V_0 is the column vector of voltages at the base of dipoles; and $U^C = E^C/\beta$ which is the columm vector of the applied field divided by β (β is the free space propagation constant). $[Y_A]$ and $[T_A]$ are two matrices which depend upon geometrical systems and frequency. With only two dipoles, we obtain the quadripole of Fig. 1 where I_1 and I_2 are input currents at ports 1 and 2 (elements of I); V_1 and V_2 are applied voltages at ports 1 and 2 (elements of V_0); a_1 and a_2 are incident waves at ports 1 and 2; b_1 and b_2 are reflected waves at ports 1 and 2; and Z_0 is the normalization impedance, with

$$a_1 = \frac{1}{2}\left(\frac{V_1}{(Z_0)^{1/2}} + (Z_0)^{1/2}I_1\right) \qquad a_2 = \frac{1}{2}\left(\frac{V_2}{(Z_0)^{1/2}} + (Z_0)^{1/2}I_2\right)$$

$$b_1 = \frac{1}{2}\left(\frac{V_1}{(Z_0)^{1/2}} - (Z_0)^{1/2}I_1\right) \qquad b_2 = \frac{1}{2}\left(\frac{V_2}{(Z_0)^{1/2}} - (Z_0)^{1/2}I_2\right).$$

Introducing the incident and reflected waves in (3) we obtain

$$b = [S]a - \frac{(Z_0)^{1/2}}{2}[[S] + [W]][T_A]U^C \tag{4}$$

where $[W]$ is the unity matrix and $[S]$ a matrix which is a function of $[Y_A]$ and Z_0.

Coupling for emission

Here we have $U_C = 0$, port 1 fed with a generator of impedance Z_0, and port 2 fed with an impedance Z_0. Therefore, $a_2 = 0$, $b_1 = S_{11}a_1$, $b_2 = S_{21}a_1$, $P_1 = |a_1|^2 - |b_1|^2$, $P_2 = |b_2|^2$, and

$$C_2 = \frac{P_2}{P_1} = \frac{|S_{21}|^2}{1 - |S_{11}|^2}. \tag{5}$$

Coupling for reception

Here U^C is such that $U_1{}^C = E_1/\beta \neq 0$, $U_2{}^C = E_2/\beta = 0$. Ports 1 and 2 are fed by an impedance Z_0. Therefore, $a_1 = a_2 = 0$ and

$$b_1 = -\frac{(Z_0)^{1/2}}{2}[(1 + S_{11})T_{11} + S_{21}T_{21}]U_1{}^C$$

$$b_2 = -\frac{(Z_0)^{1/2}}{2}[S_{21}T_{11} + (1 + S_{22})T_{21}]U_1{}^C$$

$$P_2 = |b_2|^2$$

$$P_1 = |b_1|^2.$$

Thus,

$$C_r = \frac{P_2}{P_1} = \frac{|S_{21}T_{11} + (1 + S_{22})T_{21}|^2}{|(1 + S_{11})T_{11} + S_{21}T_{21}|^2}. \tag{6}$$

Comments

Coupling of antenna 2 relative to antenna 1 is different for emission and reception. Even with a large distance between the two antennas we obtain

$$C_e = \frac{|S_{21}|^2}{1 - |S_{11}|^2} \quad \text{and} \quad C_r = \frac{|S_{21}|^2}{|1 + S_{11}|^2}.$$

Thus, $C_e \neq C_r$ except when $S_{11} = 0$.

If the two antennas are different, coupling of antenna 2 relative to antenna 1 is different than the coupling of antenna 1 to antenna 2, because the reciprocity theorem only imposes $S_{21} = S_{12}$, and generally $S_{11} \neq S_{22}$, $T_{21} \neq T_{12}$, and $T_{11} \neq T_{22}$.

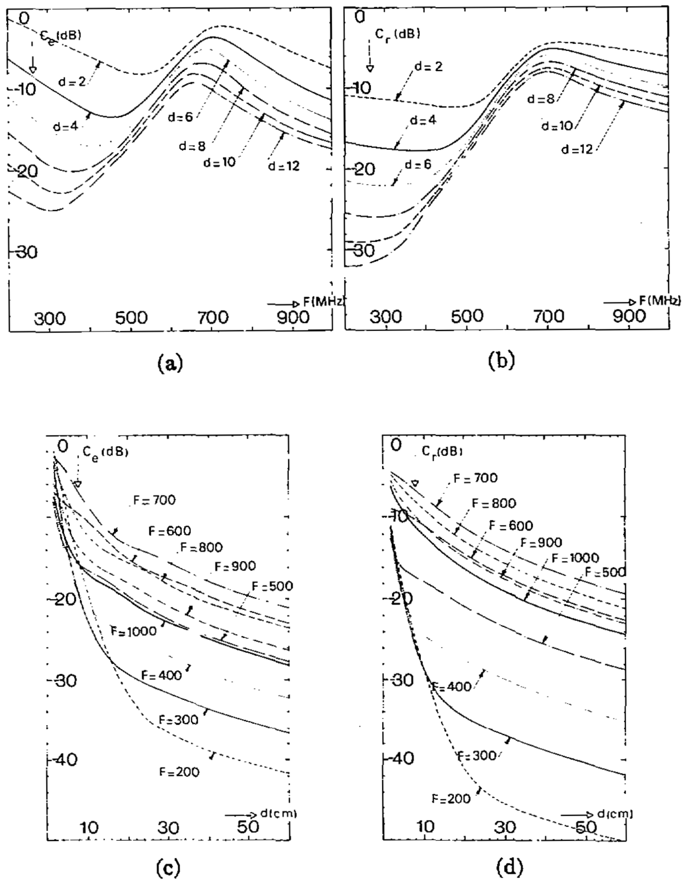

Fig. 2. Curves of C_e and C_r for passive dipoles. (a) and (b) Versus frequency in MHz. (c) and (d) Versus distance in cm.

Results

We have calculated the parameters C_e and C_r for two dipoles, the characteristics of which are $h = 10$ cm, $a = 0{,}15$ cm, d is 2–60 cm, and the frequency band is 200–1000 MHz.

In Figs. 2(a) and 2(b), C_e and C_r are plotted versus frequency for some distance d. It appears that the shape of the curves remains similar, with a maximum between 700–800 MHz. However, coupling is lower for reception than for emission below 600 MHz. Between 600 and 900 MHz, values are of the same order, and over 900 MHz C_e becomes lower than C_r.

In Figs. 2(c) and 2(d), C_e and C_r are plotted versus distance d for different frequencies. When the distances are small (lower than 20 cm), $C_e(d)$ and $C_r(d)$ are more or less rapidly decreasing functions of d, according to the frequency considered. On the other hand, when d is quite large ($d > 40$ cm), $C_e(d)$ and $C_r(d)$ become practically linear functions of d. We can write $C = C_0(\nu) - kd$ where C_0 is a function of frequency, and ν and k are constant. The slope k remains the same for emission and reception. Between $d = 45$ cm and $d = 60$ cm we obtain $0.15 < k < 0.16$, with k in dB/cm. The previous definition of coupling enables us to give a numerical value to the interaction between antennas used as emission elements or reception elements. We shall see that definitions appear more interesting when considering active dipoles either for reception or emission.

Application to Active Dipoles

Passive dipoles fed by active devices such as microwave transistors, for instance, are called active dipoles. Use of transistors, which are not reciprocal devices, allows us to think that the effects of mutual coupling in the feed network of an array will be reduced.

Some authors [1], [2] have effectively shown this possibility. With a two active dipole array, for instance, it can be proved that $|Y_{12}/Y_{11}|_{\text{active}}$ is lower than $|Y_{12}/Y_{11}|_{\text{passive}}$; ($Y_{ij}$ are terms of the admittance matrix) [1].

If we look only upon the terms of the admittance matrix (or impedance matrix), it seems that coupling remains the same at either emission or reception. We will see here that there is an im-

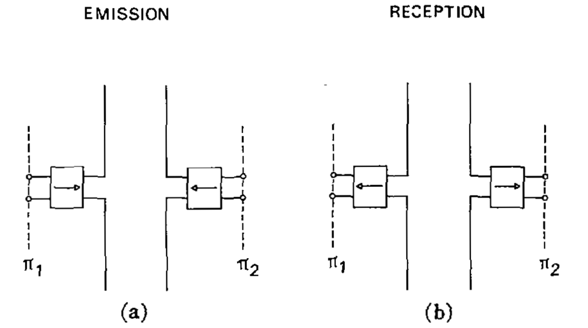

Fig. 3. Array of active dipoles. (a) Emission. (b) Reception.

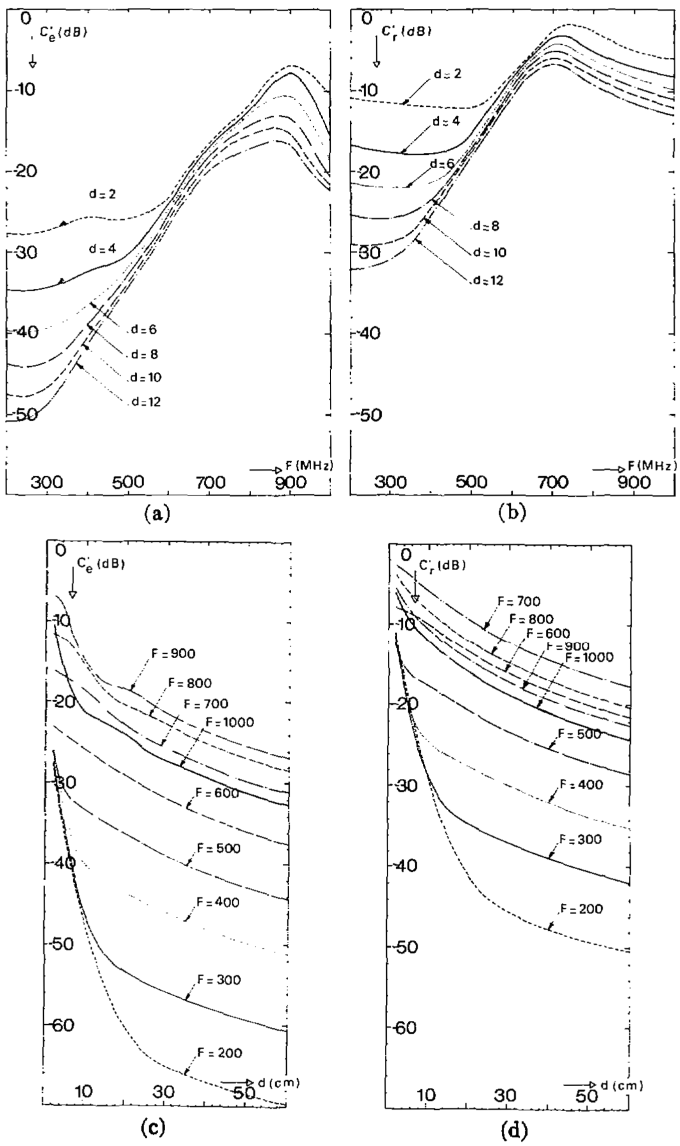

Fig. 4. Curves of C_e' and C_r' for active dipoles. (a) and (b) Versus frequency in MHz. (c) and (d) Versus distance in cm.

portant difference in coupling between active dipoles for emission and active dipoles for reception.

Physical arrangement

Transistors amplify the energy that is given by a generator or the energy that is received by the antenna. The direction of power flow is shown with an arrow on the quadripole (Fig. 3).

The properties of the quadripole included between the two ports π_1 and π_2 are defined by the following relation:

$$I^S = [Y_T]V^S + [T_T]U^C \tag{7}$$

where I^S is the vector of input currents in ports π_1 and π_2 and V^S is the vector of voltages at ports π_1 and π_2. $[Y_T]$ and $[T_T]$ are two matrices calculated from $[Y_A]$ and $[T_A]$, and the transistor parameters [1].

Since (7) is similar to (3), we obtain the same formulas

$$C_e' = \frac{|S_{T21}|^2}{1 - |S_{T11}|^2} \tag{8}$$

$$C_r' = \frac{|S_{T21}T_{T11} + (1 + S_{T22})T_{T21}|^2}{|(1 + S_{T11})T_{T11} + S_{T21}T_{T21}|^2}. \tag{9}$$

$S_{T_{ij}}$ are "S parameters," which can be deduced from $[Y_T]$ and the normalization impedance Z_0.

Results

The geometrical parameters of dipoles are those that have been defined for passive dipoles. H P 35823 E microwave transistors (with the bias $V_{CB} = 15$ V, $I_c = 15$ mA) have been used.

For emission, active dipole coupling is lower than coupling of passive dipoles. For instance, with $d = 2$ cm at 400 MHz, we obtain $C_e = -6$ dB and $C_e' = -26$ dB. Moreover, the maximum of coupling moves to upper frequency (900 MHz).

Coupling for reception is the same (Figs. 2 and 4) for active or passive antennas. These results prove that incorporation of nonreciprocal elements (such as bipolar transistors) at the base of dipoles does not change coupling between the two ports of the network when reception is considered. For upper frequencies (above 750 MHz), coupling of passive dipoles is even better than coupling of active dipoles.

We have also plotted on Fig. 4 curves C_e' and C_r' versus distances for different frequencies. When d is small, variations of C_e' and C_r' look like those of C_e and C_r (with better values of C_e'). When d becomes sufficiently large ($d > 40$ cm), we obtain a linear function C' (at emission or reception), where $C' = C_0'(\nu) - kd$. The slope k has the same value as the previous one.

Conclusion

We propose two definitions of mutual coupling of antennas, either for reception or for emission. Computations prove that coupling of passive dipoles has the same order of magnitude for emission and reception. On the contrary, coupling between the two ports of the active dipole network is very different according to its function. Reduction of coupling seems to be useful only with an emissive active array of dipoles.

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A Modified Backfire Antenna

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Abstract—**The three-term theory is applied to study the circuit and field properties of linear and crossed-dipole arrays of the backfire type. The arrays are composed of both driven and parasitic elements. It is found that with proper spacings and lengths of the elements, both broadside and endfire patterns can be obtained by changes in the relative phases of only a fraction of the total number of elements.**

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