Mutual Coupling Between Antennas—Optimization of Transistor Parameters in Active Antenna Design

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Abstract—**It is demonstrated theoretically that mutual coupling effects on the radiation pattern of an element in the presence of another one can be reduced with appropriate loads. Constant coupling circles are calculated and plotted on a Smith chart. The choice of transistors for active antenna design then becomes clearer either for reception or for emission.**

I. Introduction

In a previous publication [1], the term "coupling" was defined in order to describe interactions between antennas for either emission or reception. According to the chosen function, two different definitions were given and used in the case of an array of two active dipoles.

This problem was reexamined by showing the mutual coupling effects on the radiation pattern of one element in the presence of the other one. The application of the reciprocity theorem makes it possible to demonstrate that one and the same definition is sufficient to describe the coupling for receiving and transmitting antennas—to be named "radiation coupling."

Then, the existence of optimal load impedances, which permit a zero coupling to be obtained, will be shown; the locus of load impedances for a given coupling is a circle on the Smith chart plane. Hence, the knowledge of the constant coupling circles makes it possible to choose the most appropriate microwave transistors for the design of active antenna arrays for either emission or reception.

II. Problem of Mutual Coupling Between Passive Antennas

For simplicity, only a very simple array of two identical linear antennas will be considered here.

The two-port equivalent network (Fig. 1) is characterized by two matrices $[S]$ and $[B]$ as follows [1]:

$$b = [S]a + [B]U^c \tag{1}$$

where

- b column vector of reflected waves,
- a column vector of incident waves,
- $U^c = E/\beta$,
- E column vector $\begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$, where E_1 and E_2 are the incident fields strength on the dipoles 1 and 2 (each field is parallel to the dipoles (Fig. 2)),
- β free space propagation constant.

Let the base currents of the dipoles be I_1 and I_2, the base voltages be V_1 and V_2, and the normalization impedance be Z_0. Then

$$I_i = \frac{a_i - b_i}{\sqrt{Z_0}} \quad \text{and} \quad V_i = \sqrt{Z_0}\,(a_i + b_i), \qquad i = 1,2.$$

Here, the directivity pattern of one element (for instance, number 1), in the presence of the other, is calculated within the H-plane defined by $\theta = 90°$ (Fig. 2). The two antennas are dipoles parallel at axis Oz of which the coordinates are:

$$\text{dipole 1:} \quad X_1 = 0,\ Y_1 = 0$$

$$\text{dipole 2:} \quad X_2 = 0,\ Y_2 = d.$$

Manuscript received June 25, 1974; revised January 8, 1975.
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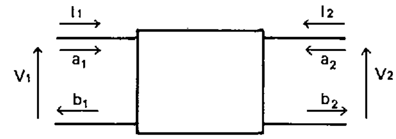

Fig. 1. Equivalent network of two-antenna array.

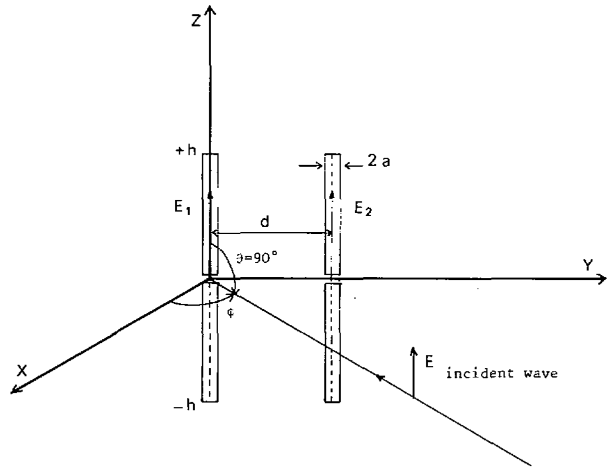

Fig. 2. Relative positions of two identical dipoles.

Directivity Pattern of Dipole 1

Emission: Let dipole 1 be fed by means of a voltage generator of impedance Z_0, and dipole 2 be terminated in impedance Z_0. The radiated electrical field strength E_i, from a single dipole i, which delivers the power P_i, is given by [3]:

$$E_i = \frac{e^{-j\beta r}}{r}\sqrt{\frac{2\xi_0 P_i}{4\pi}}\, g_i(\theta,\phi)$$

where $\xi_0 = 120\pi$ is the intrinsic wave impedance;

$$g_i(\theta,\phi) = j\beta \frac{\xi_0}{4\pi}\sqrt{\frac{4\pi}{2\xi_0 P_i}} \sin\theta \int_{-h_i}^{h_i} I_i(Z_i) \cdot \exp\{j\beta[\sin\theta(X_i\cos\phi + Y_i\sin\phi) + Z_i\cos\theta]\}\, dZ_i$$

$I_i(Z_i)$ is the total current,

$$P_i = Z_0 \frac{1 - |S_{11}|^2}{|1 - S_{11}|^2} |I_i(0)|^2.$$

Hence, the total far-zone field radiated from the array in the XOY plane is

$$E = \sum_i E_i = \frac{e^{-j\beta r}}{r}\sqrt{\frac{2\xi_0}{4\pi} Z_0 \frac{1 - |S_{11}|^2}{|1 - S_{11}|^2}}\, |I_1(0)|\, g_1 \cdot \left\{1 + \left|\frac{I_2(0)}{I_1(0)}\right| \frac{g_2}{g_1} e^{j\beta d \sin\phi}\right\}.$$

Introducing $g_2/g_1 = |g_2/g_1|e^{j\alpha}$ and $\Phi = \beta d \sin\phi$ the directivity pattern $d_1(\phi)$ of dipole 1 in presence of dipole 2 is obtained:

$$d_1(\phi) = \frac{|E|^2/2\xi_0}{P_1/4\pi r^2} = |g_1|^2\left[1 + \left|\frac{I_2(0)}{I_1(0)}\right|^2 \left|\frac{g_2}{g_1}\right|^2 + 2\left|\frac{I_1(0)}{I_2(0)}\right| \left|\frac{g_2}{g_1}\right| \cos(\Phi + \alpha)\right].$$

Accordingly for the expression $|I_2(0)/I_1(0)|$ we find:

$$d_1(\phi) = |g_1|^2 \left[1 + \frac{|S_{21}|^2}{|1 - S_{11}|^2} \left|\frac{g_2}{g_1}\right|^2 + 2 \left|\frac{S_{21}}{1 - S_{11}}\right| \left|\frac{g_2}{g_1}\right| \cos(\Phi + \alpha)\right]. \quad (2)$$

Reception: Let the two dipoles be terminated in impedance Z_0, and be illuminated by a plane wave in direction ϕ. Since the loads are chosen equal to normalization impedance, we have $a_1 = a_2 = 0$, hence from (1) the power P_1' received by the dipole 1 is

$$P_1' = |b_1|^2 = |B_{11} + B_{12}\, e^{j\Phi}|^2 \frac{|E|^2}{\beta^2}.$$

The power received, if the load is matched, would be

$$P_1 = \frac{P_1'}{1 - |S_{11}|^2} = \frac{|B_{11}|^2}{1 - |S_{11}|^2} \frac{|E|^2}{\beta^2} \cdot \left[1 + \left|\frac{B_{12}}{B_{11}}\right|^2 + 2\left|\frac{B_{12}}{B_{11}}\right| \cos(\eta + \Phi)\right]$$

where

$$\frac{B_{12}}{B_{11}} = \left|\frac{B_{12}}{B_{11}}\right| e^{j\eta}.$$

The directivity pattern is obtained by comparing P_1 with the power received by the isotropic radiator in the same field

$$P_{\text{iso}} = \frac{\lambda^2}{4\pi} \frac{|E|^2}{\xi_0} = \frac{1}{120} \frac{|E|^2}{\beta^2}.$$

Consequently,

$$d_1(\phi) = 120 \frac{|B_{11}|^2}{1 - |S_{11}|^2} \left[1 + \left|\frac{B_{12}}{B_{11}}\right|^2 + 2\left|\frac{B_{12}}{B_{11}}\right| \cos(\eta + \Phi)\right]. \quad (3)$$

If the array is reciprocal the directivity pattern is the same for emission and for reception. By identifying the expressions (2) and (3) and their derivatives in functions of ϕ, it can easily be shown that

$$\begin{cases} \left|\dfrac{B_{12}}{B_{11}}\right| = \left|\dfrac{S_{21}}{1 - S_{11}}\right| \left|\dfrac{g_2}{g_1}\right| \quad \text{and} \quad \alpha = \eta \\ |g_1|^2 = 120 \dfrac{|B_{11}|^2}{1 - |S_{11}|^2}. \end{cases}$$

It can be seen that, the weaker $|B_{12}/B_{11}|$ is, all the more weaker is the coupling effect on the omnidirectionality of the dipole 1. From a radiation pattern point of view it seems logical to define the coupling C_r by

$$C_r = \left|\frac{B_{12}}{B_{11}}\right|^2.$$

Furthermore it is easy to show that C_r is also equal to P_2/P_1, where P_2 represents the power received in the load of antenna 2, and P_1 that of antenna 1 when only the latter is illuminated. This last definition was used in a previous publication [1] to characterize the receiving antennas coupling. Thus, from now on, we shall adopt the following definition for mutual coupling:

$$C_r = \frac{P_2}{P_1} \quad (4)$$

where P_2 and P_1 are as defined in the preceding.

It should be observed that the "coupling" defined in this way, is relatively useful, since it has the same value for emission and for reception. Moreover, we shall see that this definition of radiation coupling is also available in the case of active antennas.

Choice of Optimal Loads

The value C_{r_0} of C_r which has been previously calculated is dependent on geometrical dimensions of the array, on frequency, and on the normalization impedance Z_0 (chosen as load until now). Using a load Z_T of receiver or an internal impedance Z_T of generator, allows a reduction of coupling to be obtained and even a null. It is possible to repeat the preceding calculations for emission and for reception. The directivity pattern $d_1'(\phi)$ of dipole 1 in the presence of dipole 2 is now calculated.

Emission:

$$d_1'(\phi) = |g_1'|^2 \left[1 + \left|\frac{I_2'}{I_1'}\right|^2 \left|\frac{g_2'}{g_1'}\right|^2 + 2\left|\frac{I_2'}{I_1'}\right| \left|\frac{g_2'}{g_1'}\right| \cos(\Phi + \alpha')\right]$$

with

$$\frac{g_2'}{g_1'} = \left|\frac{g_2'}{g_1'}\right| e^{j\alpha'}. \quad (5)$$

Reception:

$$d_1'(\phi) = \frac{1 - |\Gamma_T|^2}{1 - |\Gamma|^2} |B_{11}'|^2 .120 \cdot \left[1 + \left|\frac{B_{12}'}{B_{11}'}\right|^2 + 2\left|\frac{B_{12}'}{B_{11}'}\right| \cos(\Phi + \eta')\right]$$

with

$$\frac{B_{12}'}{B_{11}'} = \left|\frac{B_{12}'}{B_{11}'}\right| e^{j\eta'} \quad (6)$$

and

$$B_{11}' = \frac{1}{1 - \Gamma_s \Gamma_T} \left[B_{11} + \frac{S_{12}\Gamma_T B_{21}}{1 - S_{22}\Gamma_T}\right]$$

$$B_{12}' = \frac{1}{1 - \Gamma_s \Gamma_T} \left[B_{12} + \frac{S_{12}\Gamma_T B_{22}}{1 - S_{22}\Gamma_T}\right]$$

$$\Gamma_s = S_{11} + \frac{S_{12}S_{21}\Gamma_T}{1 - S_{22}\Gamma_T} \qquad \Gamma_T = \frac{Z_T - Z_0}{Z_T + Z_0}$$

$$|\Gamma| = \left|\frac{\Gamma_T - \Gamma_s^*}{1 - \Gamma_T \Gamma_s}\right|.$$

Then using the reciprocity theorem the following is obtained:

$$\left|\frac{B_{12}'}{B_{11}'}\right|^2 = \left|\frac{I_2'}{I_1'}\right|^2 \left|\frac{g_2'}{g_1'}\right|^2, \quad \text{with } \alpha' = \eta'$$

$$|g_1'|^2 = 120 \frac{1 - |\Gamma_T|^2}{1 - |\Gamma|^2} |B_{11}'|^2.$$

Lack of omnidirectionality depends on term $|B_{12}'/B_{11}'|^2$; it is easily found that $C_r = P_2'/P_1' = |B_{12}'/B_{11}'|^2$ (with P_2' being the power received by antenna 2, P_1' being the power received by antenna 1 when only the latter is illuminated).

The expression of C_r is therefore given by

$$C_r = C_{r_0} \left|\frac{1 - \dfrac{\Gamma_T}{\Gamma_0}}{1 - \dfrac{\Gamma_T}{\Gamma_M}}\right|^2 \quad (7)$$

TABLE I
COMPUTED VALUES OF Γ_0 AND Γ_M BETWEEN 0.8 AND 1.2 GHz

Fréquence (GHz)	Γ_o Module $\lvert\Gamma_o\rvert$	Γ_o Phase (degrees) Γ_o	Γ_M Module $\lvert\Gamma_M\rvert$	Γ_M Phase (degrees) Γ_M
0.80	0.974	7.68	1.48	51.99
0.85	0.978	8.28	2.19	51.40
0.90	0.982	8.84	4.54	44.54
0.95	0.986	9.38	4.22	-15.19
1.00	0.989	9.90	2.44	-26.20
1.05	0.992	10.42	1.84	-23.68
1.10	0.994	10.95	1.57	-19.64
1.15	0.997	11.50	1.42	-15.89
1.20	1.000	12.05	1.34	-12.66

with

$$C_{r_0} = \left|\frac{B_{12}}{B_{11}}\right|^2$$

$$\Gamma_0 = \frac{B_{12}}{S_{22}B_{12} - S_{12}B_{11}}$$

$$\Gamma_M = \frac{B_{11}}{S_{22}B_{11} - S_{12}B_{21}}.$$

It is noticed that $\Gamma_T = \Gamma_0$ involves $C_r = 0$, while $\Gamma_T = \Gamma_M$ involves $C_r = \infty$. Now the problem is to know if Γ_0 and Γ_M are physically feasible and moreover to know variations of C_r when Γ_T differs from Γ_0. Table I gives the complex value of Γ_0 and Γ_M of two passive dipoles, the dimensions of which are $h = 7.5$ cm, $a = 0.15$ cm, and $d = 3.75$ cm.

B_{ij} and S_{ij} have been calculated using methods previously developed [1], [2]. $|\Gamma_0|$ appears to be lower than 1 (or equal to 1) while $|\Gamma_M|$ is always greater than 1. So it is possible to choose a load impedance leading to a null coupling.

Now values of Z_T which maintain $C = C_r/C_{r_0}$ constant are found. Equation (7) shows that locus of Γ_T is a circle on the Smith chart (for a given C). Different circles are plotted in Fig. 3 with C as the parameter (given in dB), at 0.8, 0.9, 1 GHz. The circles are dependent on geometrical dimensions of the array, frequency, and normalization impedance. They enable values of load impedance for a given radiation coupling to be chosen.

III. MUTUAL COUPLING BETWEEN ACTIVE ANTENNAS

Coupling Formulas

The equivalent networks of two active dipoles are given in Fig. 4, either for emission or for reception (the arrow indicates the direction of net power flow). S_T parameters define transistors, which are identical.

Emission:

$$\begin{cases} b_i' = S_{T_{11}}a_i' + S_{T_{12}}b_i \\ a_i = S_{T_{21}}a_i' + S_{T_{22}}b_i, \end{cases} \quad \text{with } i = 1,2.$$

Reception:

$$\begin{cases} a_i = S_{T_{11}}b_i + S_{T_{12}}a_i' \\ b_i' = S_{T_{21}}b_i + S_{T_{22}}a_i', \end{cases} \quad \text{with } i = 1,2.$$

Suppose that transistors are loaded with impedances Z_C (internal impedance of generator or of receiver). Then the

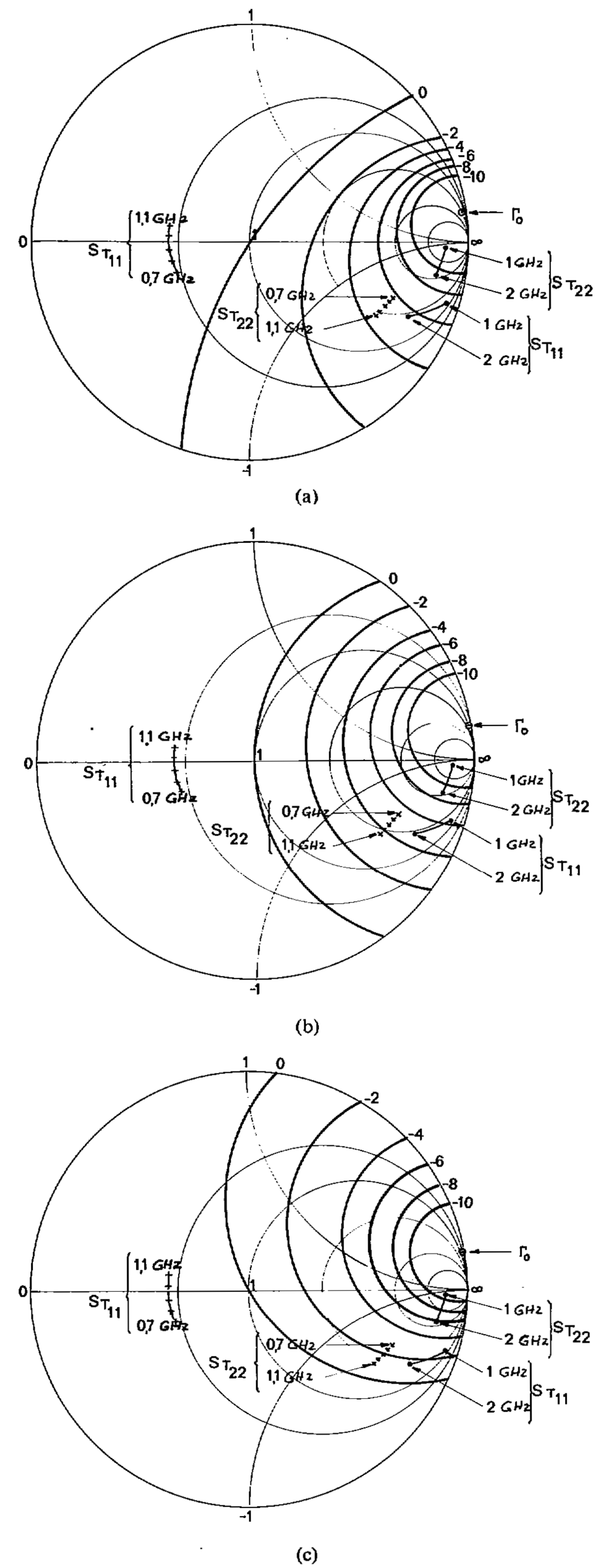

Fig. 3. Constant mutual coupling circles for two dipoles at three frequencies (and typical values of $S_{T_{11}}$ and $S_{T_{22}}$ parameters: ✱✱ bipolar transistor, ●● FET). (a) 0.8 GHz ($C_{r_0} = -8.92$ dB). (b) 0.9 GHz ($C_{r_0} = -5.81$ dB). (c) 1.0 GHz ($C_{r_0} = -5.82$ dB).

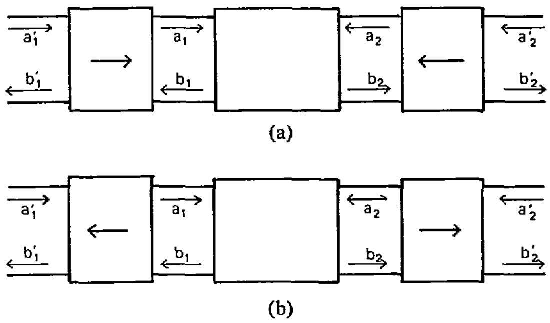

Fig. 4. Equivalent network of two active antennas. (a) Emission. (b) Reception.

reflection coefficient Γ_T at any port of the passive array is the following.

Emission:

$$\Gamma_T = S_{T_{22}} + \frac{S_{T_{12}} S_{T_{21}} \Gamma_C}{1 - S_{T_{11}} \Gamma_C} \quad \text{and} \quad \Gamma_C = \frac{Z_C - Z_0}{Z_C + Z_0}.$$

Reception:

$$\Gamma_T = S_{T_{11}} + \frac{S_{T_{12}} S_{T_{21}} \Gamma_C}{1 - S_{T_{22}} \Gamma_C}.$$

Since the product $|S_{T_{12}} S_{T_{21}}|$ is generally small, Γ_T is reduced to $S_{T_{22}}$ (emission) and $S_{T_{11}}$ (reception). Then mutual couplings for radiation are given by

$$C_r^e = C_{r_0} \left| \frac{1 - \dfrac{S_{T_{22}}}{\Gamma_0}}{1 - \dfrac{S_{T_{22}}}{\Gamma_M}} \right|^2 \quad \text{and} \quad C_r^r = C_{r_0} \left| \frac{1 - \dfrac{S_{T_{11}}}{\Gamma_0}}{1 - \dfrac{S_{T_{11}}}{\Gamma_M}} \right|^2.$$

The choice of transistors will be made, using essentially $S_{T_{22}}$ for emission or $S_{T_{11}}$ for reception, and Γ_0 and Γ_M.

Example—Design of the Active Antenna

Typical values of parameters $S_{T_{11}}$ and $S_{T_{22}}$ are given in Fig. 3(a)–3(c) for two types of transistors: a bipolar HP 35823 E ($V_{CE} = 15$ V, $I_C = 6$ mA) and an FET Plessey GAT2 ($V_{DS} = 5$ V, $V_{GS} = 0$). Consider now the emission. Looking at Fig. 3(a)–3(c), the constant mutual coupling circles, and $S_{T_{22}}$, it appears that a bipolar transistor yields a reduction of coupling of about 5 or 6 dB at 0.8 GHz, and about 3 dB at 1.0 GHz. With an FET an improvement (with respect to C_{r0}) of better than 10 dB between 0.8 and 1 GHz is obtained. Both these active devices can be used but the reduction with the FET is better than with the bipolar device.

In the reception case, $S_{T_{11}}$ must be considered. The bipolar parameter has a bad position relative to constant coupling circles, and it does not offer any improvement for coupling. On the contrary the FET produces a reduction of about 7 or 8 dB around 0.8 GHz and 2 or 3 dB around 1 GHz. So an improvement in mutual coupling is apparent only with the FET.

The theoretical results support the experimental work of Anderson and Dawoud [4], [5]. However, the idea that suppression of mutual coupling effects is obtained when the radiating elements operate under substantially open-circuit conditions remains fairly indeterminate. The knowledge of constant coupling circles for each frequency enables us to choose the most appropriate transistor and the possible correction in order to lower mutual coupling. For instance, $S_{T_{11}}$ of the FET moves toward the inductive part of the Smith chart if a shunt inductance L is placed on the input part of this transistor. If L is chosen such

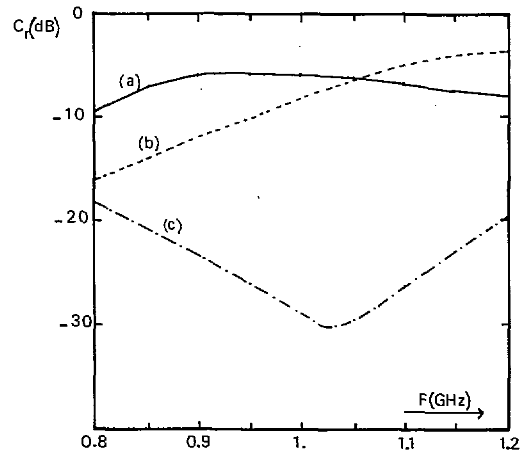

Fig. 5. Curves of coupling versus frequency (in GHz). (a) Passive dipoles. (b) Active receiving dipoles (using FET). (c) Active receiving dipoles (using FET plus shunt inductances).

that $S'_{T_{11}}$ (the new parameter) is into the circle—10 dB between 0.8 and 1 GHz—an important reduction of coupling is obtained. Fig. 5 shows mutual coupling between 0.8 and 1 GHz, for a passive dipole, an active dipole (using an FET), and an active dipole (using an FET and a shunt inductance).

IV. Conclusion

The definition of radiation coupling between antennas allows the characterization, with a numerical value, of the influence of interactions relative to the pattern of an aerial in the presence of another aerial. We prove theoretically that an optimal load exists for a null coupling; moreover, we define constant coupling circles which allow us to choose the load for a given coupling. The design of the active antenna array leads to the selection of transistors whether emission or reception is considered. The FET appears to be useful either for emission or reception, whereas the bipolar transistor is efficient only for emission.

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Optical Technique for Broadbanding Phased Arrays

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***Abstract*—An optical method for broadbanding a phased array is considered. A narrow band feed-through aperture lens comprised of pick-up elements, radiating elements, and 360° type phase shifters is fed by a small feed array with an intervening passive lens. The lens has fixed frequency-sensitive properties. The feed array has variable time delay compensators which are selected in accordance with the desired scan angle in the far field. Using geometrical optics, design formulas are**

Manuscript received March 22, 1974; revised January 19, 1975.
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