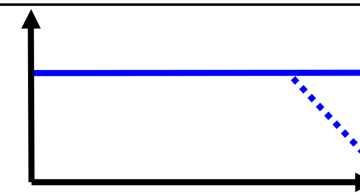
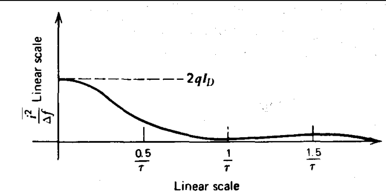
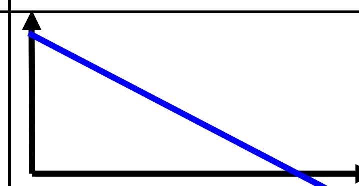
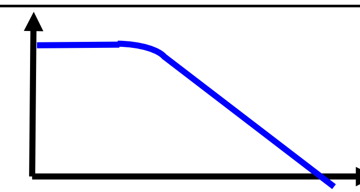
D·I·T

School of Electronic and Communications Engineering

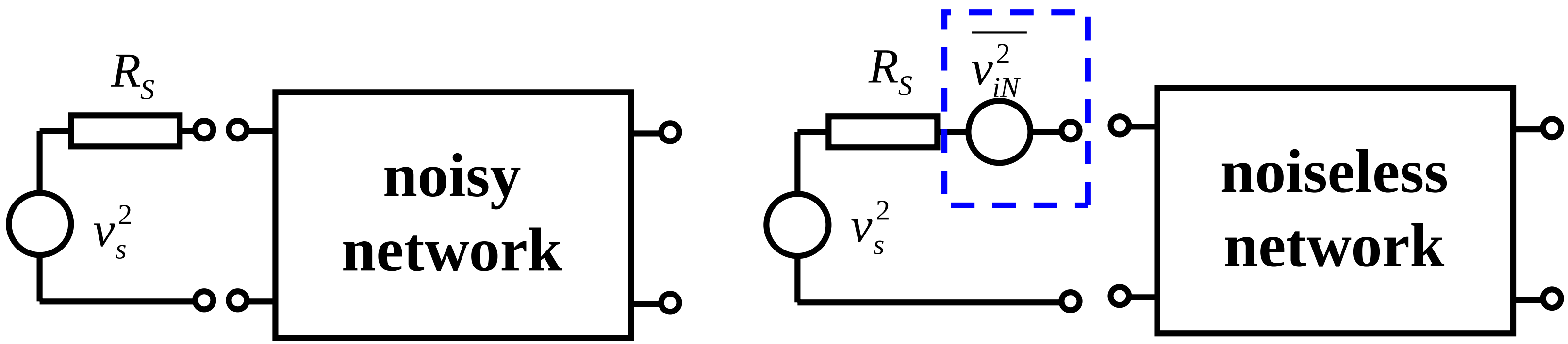
3. Noise Analysis

Noise Sources (resume)

Noise	Origin	Expression	Spectral density
thermal noise (Gaussian)	random fluctuations of velocity	$i_t^2 = 4kT\Delta f / R$	
shot noise (Gaussian)	due to DC current through p-n junction	$\overline{i_s^2} = 2qI_D \cdot \Delta f$	
flicker noise	traps in crystal lattice (semiconductors, carbon , etc.)	$\overline{i_f^2} = K_1 \frac{I_{DC}^{\alpha}}{f^{\beta}} \cdot \Delta f$	
burst noise	heavy-metal ions contamination. (gold , etc.)	$\overline{i_b^2} = K_2 \frac{I^c}{1 + (f / f_c)^2} \cdot \Delta f$	
avalanche noise	due to avalanche breakdown in zener diodes		

Equivalent input noise

- The significance of the noise performance of a circuit is the limitation it places on the smallest input signals the circuit can handle before the noise degrades the quality of the output signal.
- For this reason, the noise performance is usually expressed in terms of an *equivalent input noise signal*, which gives the same output noise as the circuit under consideration.
- The real (noisy) network is replaced by noiseless network and *equivalent input noise signal*, $\overline{v_{iN}^2}$



Equivalent input noise (BJT)

- It was shown that BJT noise can be expressed as:

$$\frac{\overline{v_o^2}}{B} = g_m^2 R_L^2 \frac{|Z|^2}{|Z + r_b + R_S|^2}\left[4kT(R_S + r_b) + (R_S + r_b)^2 2qI_B\right] + R_L^2\left(\frac{4kT}{R_L} + 2qI_C\right)$$

- On the other hand BJT noise can be presented as an amplified equivalent input noise: v_{iN}^2

$$\overline{v_o^2} = g_m^2 R_L^2 \frac{|Z|^2}{|Z + r_b + R_S|^2}\overline{v_{iN}^2}$$

- Combining of these two equations gives the expression for equivalent input noise:

$$\frac{\overline{v_{iN}^2}}{B} = 4kT(R_S + r_b) + (R_S + r_b)^2 2qI_B + \frac{|Z + r_b + R_S|^2}{g_m^2 R_L^2 \cdot |Z|^2} \cdot R_L^2\left(\frac{4kT}{R_L} + 2qI_C\right)$$

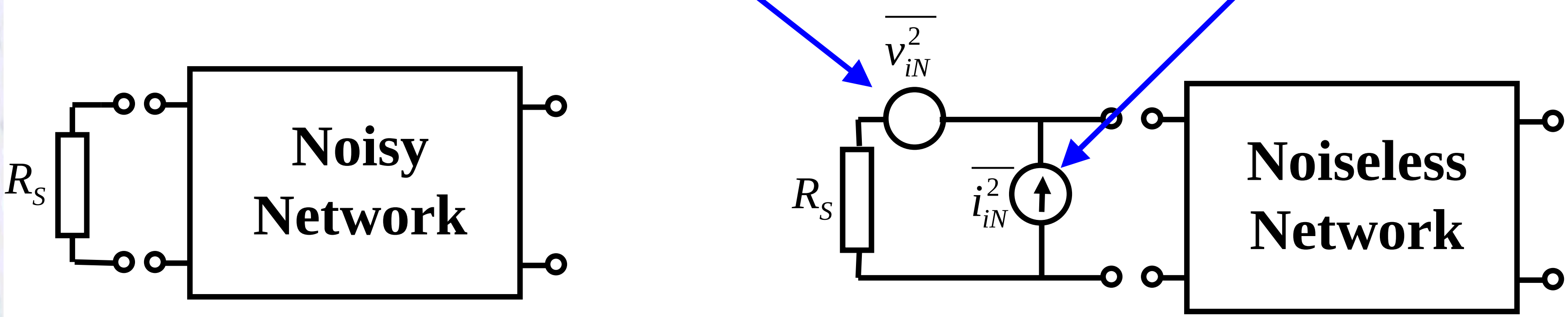
Minimum Detectable Signal

- In this way, the equivalent input noise can be compared directly with incoming signals and the effect of the noise on those signals is easily determined.
- This noise voltage can be used to estimate the smallest signal that the circuit can effectively amplify, sometimes called the *minimum detectable signal* (MDS).
- This depends strongly on the nature of the signal.
- If no special techniques are used, the MDS can be taken as equal to the equivalent input noise voltage in the passband of the amplifier.

$$MDS = \frac{\overline{v_{iN}^2}}{B} \cdot \left(Bandwidth\_of\_the\_Network \right)$$

Equivalent input noise Generators

- The equivalent input noise voltage for a particular configuration is dependent on the source resistance *Rs*, as well as the transistor parameters.
- This method is now extended to a more general and more useful representation by using *two* equivalent input noise generators:
  - equivalent input voltage
  - equivalent input current



BJT Equivalent input noise Generator

- The equivalent input noise for BJT is

$$\frac{\overline{v_{iN}^2}}{B} = 4kT(R_S + r_b) + (R_S + r_b)^2 2qI_B + \frac{|Z + r_b + R_S|^2}{g_m^2 R_L^2 \cdot |Z|^2} \cdot R_L^2 \left(\frac{4kT}{R_L} + 2qI_C \right)$$

- and it can be splitted to
- The equivalent input current for BJT is

$$\frac{\overline{i_{iN}^2}}{B} = 2qI_{eq} = 2q\left(I_B + K_1' \frac{I_B^a}{f} + \frac{I_C}{|\beta(j\omega)|^2} \right)$$

- and it appears as shot noise from $\boldsymbol{I_{eq}}$
- The equivalent input voltage for BJT is

$$\frac{\overline{v_{iN}^2}}{B} = 4kTR_S + 4kT\left(r_b + \frac{1}{2g_m} \right) + R_S^2 2q\left(I_B + K_1' \frac{I_B^a}{f} + \frac{I_C}{|\beta(jf)|^2} \right)$$

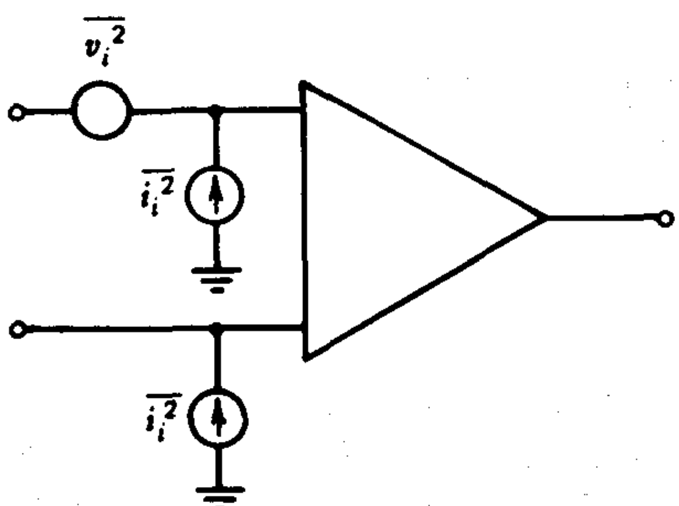
Noise model for linear IC

- IC amplifiers designed for low-noise operation generally use a simple differential-pair input stage with resistive loads.
- The noise of following stages is generally not significant, and the resistive loads make only a small noise contribution.
- However, circuits of this type (the 725 op amp is an example) are inefficient in terms of gain and bandwidth.
- Using active loads (for example in the 741) allows realization of very high gain in relatively few stages,
- However, by their very nature, active loads amplify their own internal noise and cause considerable degradation of circuit noise performance.

Noise model for 741

- Noise model for OpAmp circuits is presented below,
- The concrete form for input noise generators depends on particular circuit design

For 741 OpAmp these expressions are:



$$\frac{\overline{v_i^2}}{B} = 4kT(16000)$$

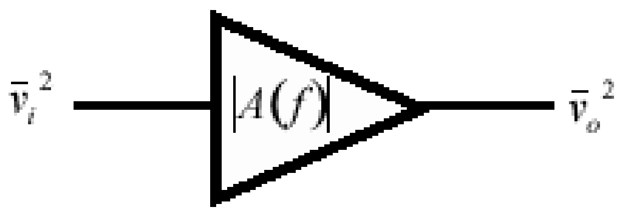
$$\frac{\overline{i_i^2}}{B} = 2qI_B \approx 2q(0.2 \cdot 10^{-6})$$

Noise analysis methods

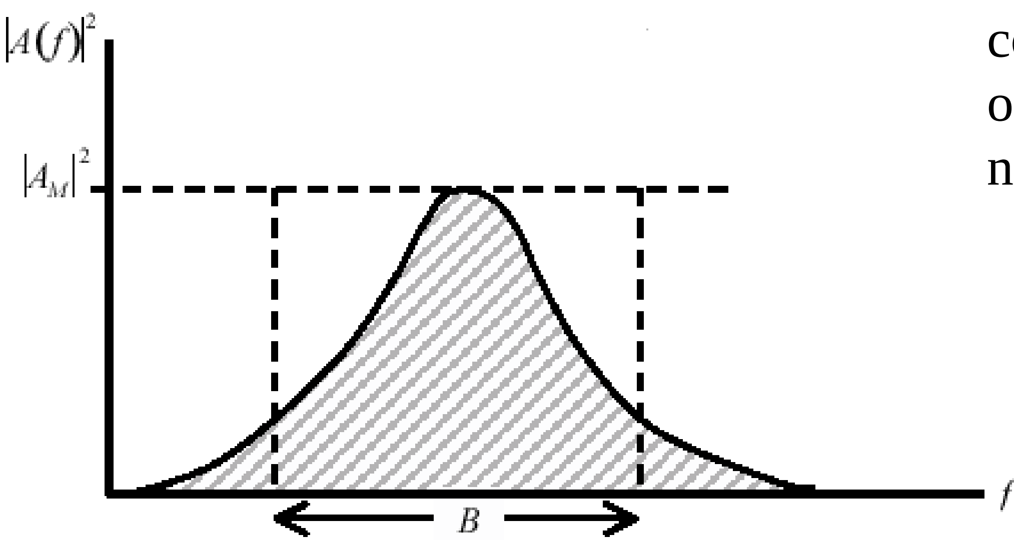
- The most general method of specifying the noise performance of circuits is by specifying input noise generators as described above.
- However, a number of specialized methods of specifying noise performance have been developed that are convenient in particular situations.
- Some of these methods are now described.
  - Noise Bandwidth
  - Signal to Noise Ratio
  - Noise Figure
  - Noise temperature

Noise bandwidth

- The total noise depends on the bandwidth of the system.
- For example, the total noise voltage at the output of a voltage amplifier with the frequency dependent gain *Av (f)* is



$$\overline{v_{on}^2} = \int_0^\infty \frac{\overline{v_n^2}}{\Delta f} A(f)_v^2 \, df$$



Note: Since spectral noise components are non-correlated, one must integrate over the noise power.

Bandwidth of white Noise

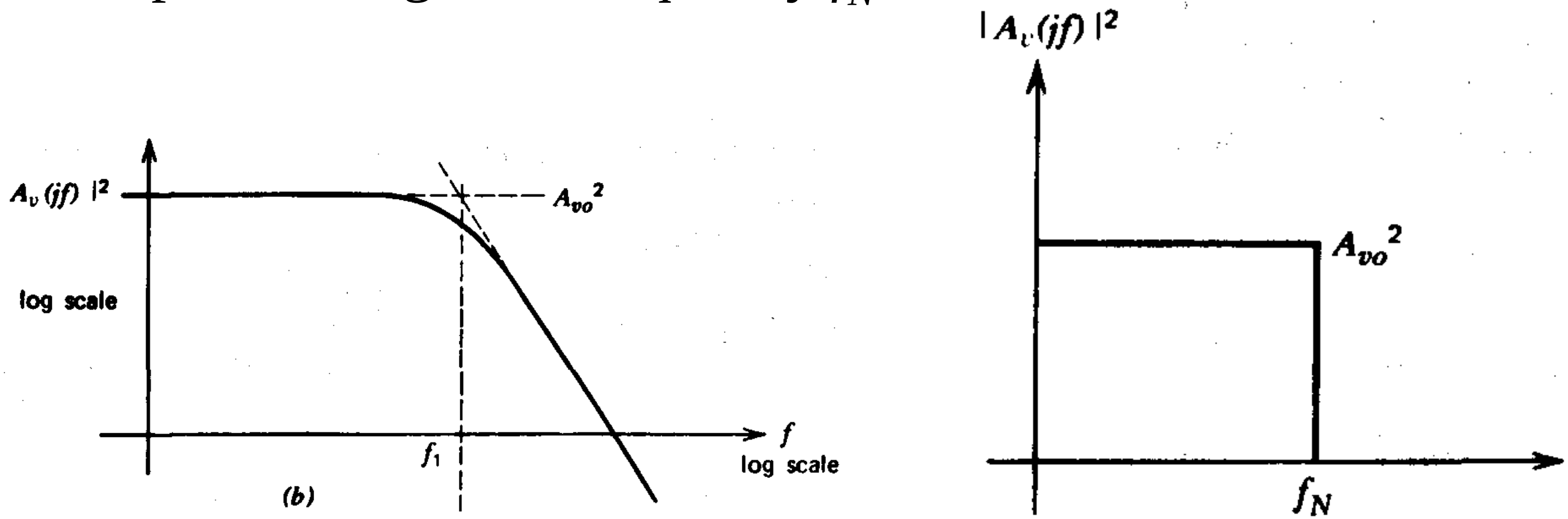
- The evaluation of the integral is a complicated task.
- However, if the equivalent input noise spectral density of a circuit is *constant* (i.e., if the noise is white), we can simplify the calculations using the concept *of noise bandwidth* .

$$\overline{v_{on}^2} = \int_0^\infty \frac{\overline{v_n^2}}{\Delta f} A(f)_v^2 \, df = \frac{\overline{v_n^2}}{\Delta f} \int_0^\infty |A(f)|^2 df = S_{io} \int_0^\infty |A(f)|^2 df$$

- The evaluation of this integral is often difficult except for very simple transfer functions.
- However, if the problem is transformed into a *normalized* form, the integrals of common circuit functions can be evaluated and tabulated for use in noise calculations.

Normalized form

- For this purpose, consider a transfer function as shown with the same low-frequency value A_{vo} as the original circuit but with an abrupt band edge at a frequency f_N.



- Frequency f_N is chosen to give the *same* total output noise voltage as the original circuit when the same input noise voltage is applied. Thus

Normalized Noise Band

- If $\overline{v_{oT}^2}$ and $\overline{v_{oT}^2}$ are evaluated, we obtained the *normalized noise band* , f_N:

$$f_N = \frac{1}{A_{vo}^2}\int_0^\infty |A_v(f)|^2 df$$

- Consider an amplifier with a single-pole frequency response given by

$$A_v(f) = \frac{A_{vo}}{1 + j\frac{f}{f_1}}$$

- substitution gives:

$$f_N = \frac{1}{A_{vo}^2}\int_0^\infty \left|\frac{A_{vo}}{1 + j\frac{f}{f_1}}\right|^2 df = \int_0^\infty \frac{df}{1 + \left(\frac{f}{f_1}\right)^2} = \frac{\pi}{2} f_1 = 1.57 f_1$$

Signal to Noise ratio

- The common parameter to characterize signals is their power.
- In all practical systems the signal always coexists with noise.
- Therefore it can be described by very important parameter Signal-to-Noise Ratio (SNR or S/N):

$$SNR(dB) \equiv 10 \cdot \lg\left(\frac{Signal\ Power}{Noise\ Power}\right)$$

Noise Figure

- The ***noise figure (F)*** specifies the noise performance of a circuit or a device.
- The definition of the noise figure of a circuit is

$$F \equiv \frac{Input\ S/N\ Ratio}{Output\ S/N\ Ratio}$$

- Its disadvantage is that it is limited to situations where the source impedance is resistive
- However, it is widely used as a measure of noise performance in communication systems where the source impedance is often resistive.

Noise Figure (cont.)

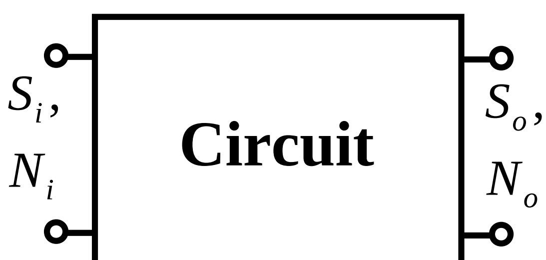
- Consider a circuit as shown below, where S represents signal power and N represents noise power.
- N_O is the total output noise including the circuit contribution and noise transmitted from the source resistance. The noise figure is .

$$F = \frac{S_i}{N_i} \cdot \frac{N_o}{S_o}$$

Noticing that

$$S_o = GS_i \text{ and } N_o = GN_i$$

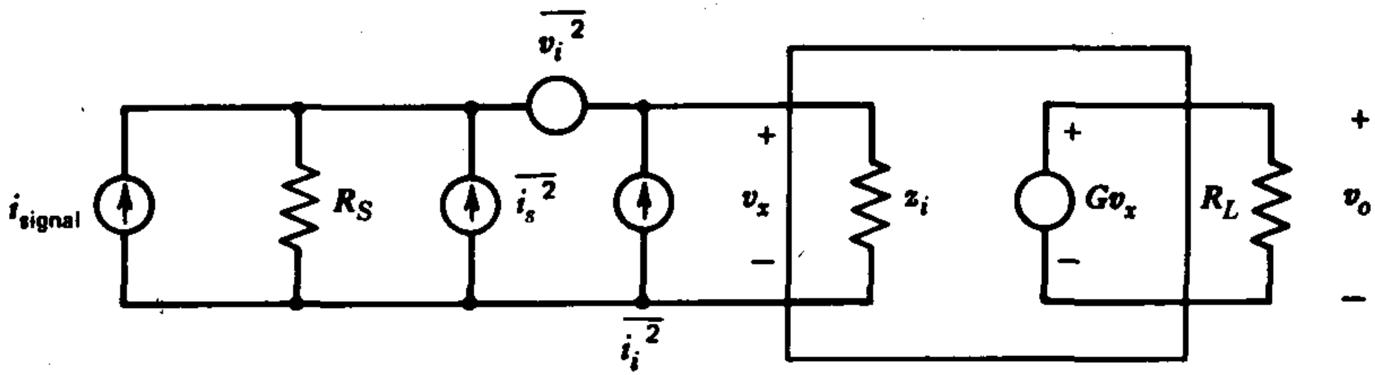
$$F = \frac{N_o}{G \cdot N_i}$$



$$F = \frac{Total\ Noise\ Power}{part\ of\ noise\ due\ to\ source\ \text{resistance}}$$

Noise Figure (cont.)

- Consider the circuit below, consisting of the noise generators and signal source



- The noise at input terminals is

$$v_{xA} = v_i \frac{z_i}{z_i + R_S} + i_i \frac{z_i R_S}{z_i + R_S}$$

and thus

$$\overline{v_{xA}^2} = \overline{v_i^2} \frac{|z_i|^2}{|z_i + R_S|^2} + \overline{i_i^2} \frac{|z_i R_S|^2}{|z_i + R_S|^2}$$

Noise Figure (cont.)

The noise power in R_L from noise generators is

$$N_{oA} = \frac{|G|^2}{R_L}\overline{v_{xA}^2} = \frac{|G|^2}{R_L}\left[\overline{v_i^2}\frac{|z_i|^2}{|z_i + R_S|^2} + \overline{i_i^2}\frac{|z_i R_S|^2}{|z_i + R_S|^2}\right]$$

The noise power in R_L from source resistor is

$$N_{oB} = \frac{|G|^2}{R_L}\frac{|z_i R_S|^2}{|z_i + R_S|^2}\overline{i_s^2} = \frac{|G|^2}{R_L}\frac{|z_i R_S|^2}{|z_i + R_S|^2}\cdot 4kT\frac{B}{R_s}$$

Finally, the Noise Figure is

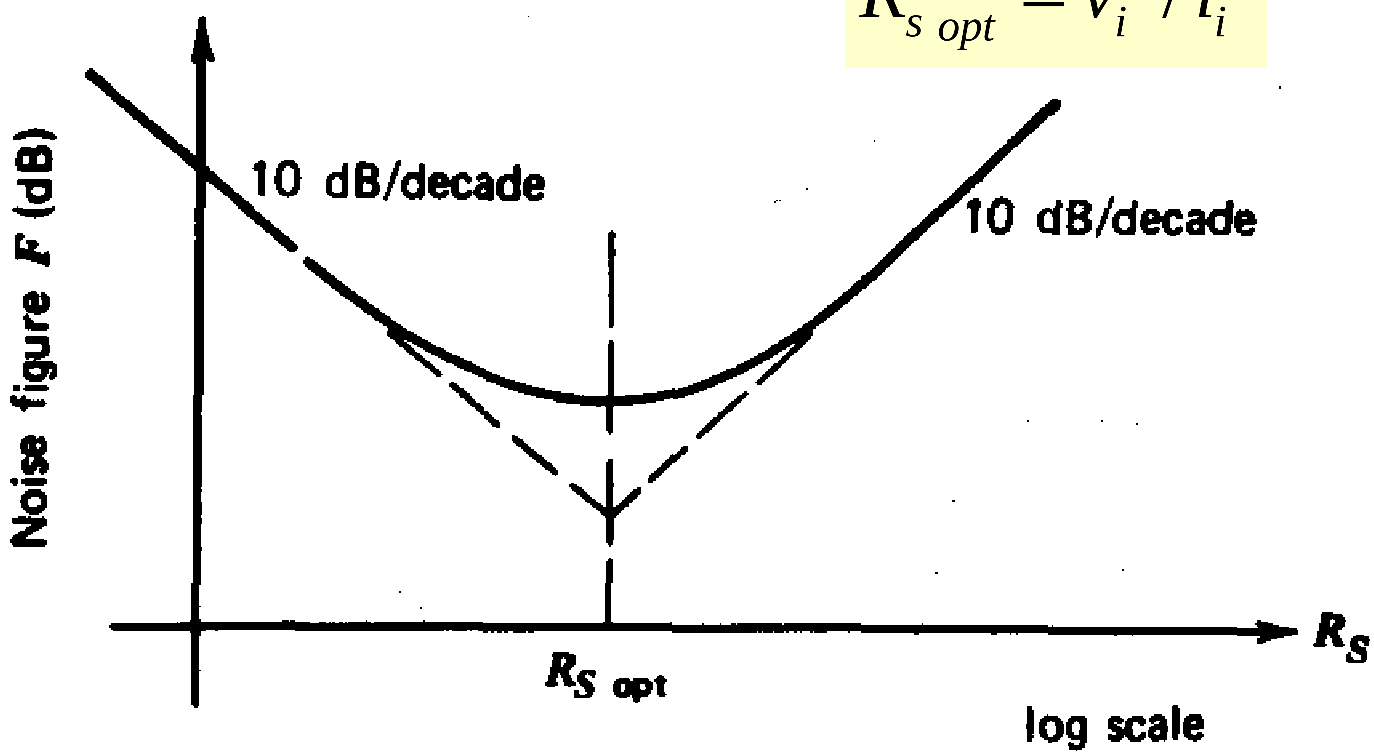
$$F = \frac{N_{oA} + N_{oB}}{N_{oB}} = \frac{|G|^2}{R_L}\frac{\overline{v_i^2}}{4kTBR_s} + \frac{\overline{i_i^2}}{4kTB / R_s}$$

Note that F *is independent of* all circuit parameters except the source resistance *Rs* and the equivalent input noise generators

Noise Figure vs. Rs

- For very low values *of Rs*, the *v*-generator is dominant, whereas for large *Rs* the *i*-generator is the most important.
- It is apparent, that *F* has a minimum as *Rs* varies.
- By differentiating *F* with respect to *Rs*, we can calculate the value of *Rs* giving minimum *F:*

$$R_{s\ opt}^2 = \overline{v_i^2} / \overline{i_i^2}$$



Example for BJT

- Consider the noise figure of a bipolar transistor at low-to-moderate frequencies where both flicker noise and high-frequency effects are neglected. Assume collector current of *Ic* = 1 mA, and with β_f = 100 and r_b = 50.

Solution

$$\frac{\overline{v_{iN}^2}}{B} = 4kTR_S + 4kT\left(r_b + \frac{1}{2g_m}\right) + R_S^2 2q\left(I_B + K_1' \frac{I_B^a}{f} + \frac{I_C}{\left|\beta(jf)\right|^2}\right)$$

$$\frac{\overline{i_{iN}^2}}{B} = 2qI_{eq} = 2q\left(I_B + K_1' \frac{I_B^a}{f} + \frac{I_C}{\left|\beta(j\omega)\right|^2}\right)$$

$$\overline{v_{iN}^2} \approx 4kT\left(r_b + \frac{1}{2g_m}\right)B$$

$$\overline{i_{iN}^2} \approx 2q\frac{I_C}{\beta_F}B$$

$$R_{s\,opt} = \frac{\overline{v_i^2}}{\overline{i_i^2}} = \frac{\sqrt{\beta_f}}{g_m}\sqrt{1+2g_m r_b}$$

$$F_{opt} \approx 1 + \frac{1}{\sqrt{\beta_f}}\sqrt{1+2g_m r_b} \approx 1.22\,(0.9dB)$$

Noise Temperature

- The noise temperature T_n of a circuit is defined as the temperature at which the source resistance Rs must be held so that the noise output from the circuit due to Rs equals the noise output due to the circuit itself.
- For the previous circuit, the output noise N_{oA} due to the circuit itself is unchanged but the output noise due to the source resistance becomes

$$N'_{oB} = \frac{|G|^2}{R_L} \frac{|z_i R_S|^2}{|z_i + R_S|^2} \cdot 4kT_n \frac{B}{R_s}$$

$$N'_{oB} = N_{oB} \frac{T}{T_n}$$

- From definition:

$$F = 1 + \frac{N_{oA}}{N_{oB}}$$

- Substitution gives:

$$(F - 1) = \frac{T_n}{T}$$

T_n vs. F

- Thus noise temperature and noise figure are directly related.
- The main application of noise temperature provides a convenient expanded measure of noise performance near $F = 1$ for very-low-noise amplifiers.
- A noise figure of $F = 2$(3 dB) corresponds to $\boldsymbol{T_n}$ = 290°K and
- $F = 1.1$ (0.4 dB) corresponds to $\boldsymbol{T_n}$ = 29°K

Low-Noise Design

- The main objective of the noise analysis is to design a low-noise circuit.
- One of the efficient way to do that is a matching of the source resistance to the noise parameters of the circuits as discussed before (to find R_{Sopt})
- Usually requirements low-noise design contradict to the other important circuit parameters such as: bandwidth, gain, input/output characteristics, etc.
- The low-noise design is different for each specific case, but there are some general recommendations.

Multistage Circuits

- In case of multistage circuits the particular effort should be applied for the very first stage.
- The noise of following stages is generally not significant.
- In particular multistage circuit - OpAmp designed for low-noise operation - generally use a simple differential-pair input stage with resistive loads instead of active loads
- JFET transistors are preferable.
- Zener Diodes should not be used in OpAmp Design to avoid very high avalanche noise.

- The following stages can use BJTs and active loads to allow very high gain in relatively few stages.
- Avoid to use carbon resistors due to flicker noise.
- The total Bandwidth of the circuit should be limited to the bandwidth of the signal spectrum.
- So that, acoustic amplifiers bandwidth should be 20 Hz to 20kHz or little wider.