D·I·T
School of Electronic and Communications Engineering

1. Noise sources:

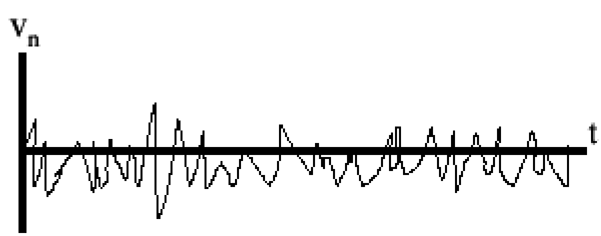
classification, representation and dependence

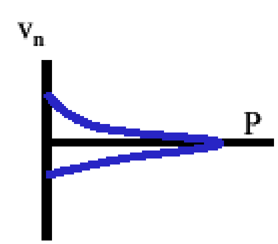
Topic Structure

- This course first reviews classification, representation and dependence of noise sources.
- Next, noise models (resistor, diode, BJT, FET, OpAmp) are introduced and
- calculations are applied to different amplifiers and analog signal processors.
- And finally the basic methods for noise analysis and calculations

What is a Noise?

- Any unwanted random disturbance
- Random carrier motion produces a current. Frequency and phase are not predictable at any instant in time
- The noise amplitude is often represented by a Gaussian probability density function.
- The cumulative area under the curve represents the probability of the event. Total area is normalized to 1.





Introduction

- Noise is a random signal and therefore cannot be analyzed by **common** methods of circuit theory.
- Noise is inherent to any electronic device and arises from different sources.
- Noise limits our ability to perceive small changes in the amplifier input signal, hence its resolution.
- A goal in analog signal processing is that amplifiers should not limit the overall resolution.
- Rather, the resolution should be limited by the sensor, the signal source or the ADC

Signals vs. Noise

- Signals are usually described by an explicit mathematical equation with small number of parameters.
- A sine wave, for example, is described by its amplitude, frequency, and phase relative to a reference.

$$v(t) = V_0 \sin(\omega t + \varphi)$$

- Noise, instead, is a random signal: its precise value at any future moment cannot be predicted
- This in no way implies we cannot know anything about noise; it only means that the knowledge we gather from noise is of a ***different*** nature: we can only predict average values.
- For stationary noise, as considered here, these ***average*** values remain constant over the time.

Noise description

- Noise, like any random signal, can be described in different domains: amplitude, time, and frequency. Noise power or intensity is also of interest.
- The *mean-square value*, or intensity, of a signal *x(t)* is the average of the squares of the instantaneous values of the signal,

$$\psi_x^2 \equiv \lim_{T\to\infty} \frac{1}{T} \int_0^T x^2(t)dt$$

- If only a small number of values of a random signal *x(f)* are considered, that is, if *T* is not very long, then different calculations of ψ_x^2 yield different results.

Invariants

- The mean-square value can be separated into a time-invariant part and a time-varying part.
- The time-invariant or static part is the square of the *signal average* or *mean value,*

$$\mu_x \equiv \lim_{T\to\infty}\frac{1}{T}\int_0^T x(t)dt$$

- The time-varying or dynamic part of the mean-square value is the *signal variance*, which is denned as the mean-square value of *x(t)* about its mean value,

$$\sigma_x^2 \equiv \lim_{T\to\infty}\frac{1}{T}\int_0^T [x(t)-\mu_x]^2 dt$$

It follows that

$$\psi_x^2 = \mu_x^2 + \sigma_x^2$$

The positive square root of the variance is the *standard deviation*.

Power dissipation

- The power dissipated by a random voltage on a resistor is proportional to the mean-square voltage. In most cases the mean value for electronic noise is zero.
- Therefore, the noise variance equals the noise power.
- The standard deviation then equals the root-mean-square voltage.
- Rather different signals can convey the same power. A large amplitude during a short time, for example, can yield the same power as a smaller amplitude during a longer time.

Probability Density Function

- The amplitude distribution of a random signal is described by the *probability density function* (PDF), $p(x)$, defined as

$$p(x) \equiv \lim_{\Delta x \to 0} \frac{\text{Prob}\left[x < x(t) < x + \Delta x\right]}{\Delta x} = \lim_{\Delta x \to 0} \frac{1}{\Delta x}\left[\lim_{T \to \infty} \frac{T_x}{T}\right]$$

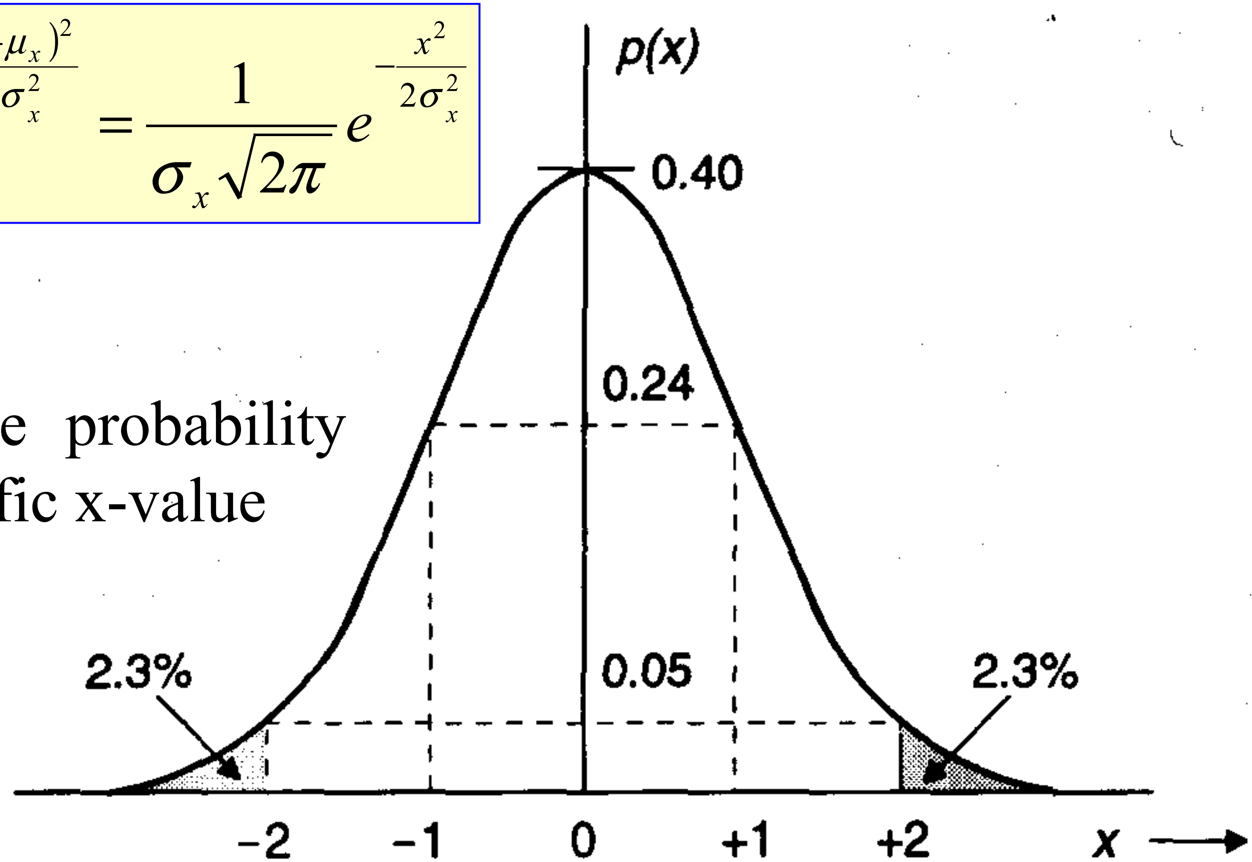
- where Tx is the amount of time in which $x(t)$ falls inside the amplitude interval from x to $x + \Delta x$.
- Therefore, the PDF gives the probability that the signal amplitude at any arbitrary moment lies inside a given amplitude range.

Gaussian (normal) PDF

- Electronic noise has a Gaussian PDF because it results from a large number of random, independent events.
- This means that its PDF is bell-shaped and follows the equation:

$$p(x)=\frac{1}{\sigma_x\sqrt{2\pi}}e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}}=\frac{1}{\sigma_x\sqrt{2\pi}}e^{-\frac{x^2}{2\sigma_x^2}}$$

Ordinates are the probability density of a specific x-value (in s–units).



Crest Factor (CF)

- The peak value of a signal divided by its root-mean-square (rms) value is termed *crest factor* (CF).
- Statistical tables for the normal distribution provide the CF values shown in Table.

Probability (%)	Crest Factor
4.6	2
1	2.6
0.37	3
0.1	3.3
0.01	3.9
0.006	4
0.001	4.4
0.0001	4.9

- For example, CF = 3.3 for a 0.1% probability means that the peak value will exceed 3.3 times the rms value only 0.1% of the time.

Power Spectral Density (PSD)

- The PDF does not take into account the time when the different amplitudes of a random signal appear.
- Therefore, very different amplitude sequences (or waveforms) can lead to the same Gaussian distribution.
- In order to better characterize a random signal, one must consider the distribution of its power in different frequency bands. The *power spectral density* (PSD) of a random signal *x(t)* is

$$G_{xx}(f) = \lim_{\Delta f \to 0} \frac{\psi_x^2(f, \Delta f)}{\Delta f} = \lim_{\Delta f \to 0} \frac{1}{\Delta f}\left[\lim_{T \to \infty} \frac{1}{T} x^2(t, f, \Delta f) dt\right]$$

- where $\psi_x^2(f, \Delta f)$ is the signal power in the frequency band from f to $f+\Delta f$ and $x(t, f, \Delta f)$) is that part of $x(t)$ contributing to power in the frequency band from f to $f+\Delta f$.

PDF vs. PSD

- The power of a signal can be obtained by integrating its PSD over the entire frequency range.
- The PSD does not completely specify a signal either because signals with different phase can have the same spectrum. However, signal phase is not considered in noise analysis.
- Gaussian noise is completely described by its variance and power spectral density.

Basic Noise Mechanisms

- Consider ***n*** carriers of charge ***e*** moving with a velocity ***v*** through a sample of length ***l***. The induced current *i* at the ends of the sample is

$$i = \frac{n \cdot e \cdot v}{l}$$

- The fluctuation of this current is given by the total differential

$$\langle di \rangle^2 = \left(\frac{n \cdot e}{l} \cdot \langle dv \rangle \right)^2 + \left(\frac{v \cdot e}{l} \cdot \langle dn \rangle \right)^2$$

- where the two terms are added in quadrature since they are statistically uncorrelated.

Two contributions to noise

- Two mechanisms contribute to the total noise:

1. velocity fluctuations, v
    1. thermal noise or Johnson or Nyquist noise
2. number fluctuations, n
    1. shot noise
    2. “flicker” or “1/ *f* “ or “low-frequency” or excess noise
    3. avalanche noise
3. unknown mechanism,
    1. burst noise or “popcorn” noise

Thermal Noise (origin)

- The most common noise sources are the random fluctuations at the atomic and molecular level because of the thermal energy in the medium.
- Random charge movements yield instantaneous differences in voltage between any two points in every conductor.
- The available noise power from a conductor at temperature T is

$$P_{noise} = 4kT \cdot B \equiv 4kT \cdot \Delta f$$

- where k = Boltzmann constant, T = absolute temperature, B (or Δf)= Noise bandwidth

Thermal Noise (spectrum)

Thermal noise and shot noise are both "white" noise sources, i.e. power per unit bandwidth is constant:

$$\frac{dP_{noise}}{df} = const$$ or $$\frac{dv^2_{noise}}{df} = const \equiv V_n^2 \equiv E_n^2$$

whereas for "$1/f$" , Burst noises:

(α= 0.5 – 2)

$$\frac{dP_{noise}}{df} = \frac{1}{f^{\alpha}}$$

Thermal Noise in Resistors

- The most common example of noise due to velocity fluctuations is the thermal noise of resistors.
- Spectral noise power density vs. frequency *f*

$$\frac{dP_{noise}}{df} = 4kT \quad \text{or} \quad P_{noise} \equiv P_n = 4kT \cdot B \equiv 4kT \cdot \Delta f$$

since $P = V^2 / R = I^2 R$

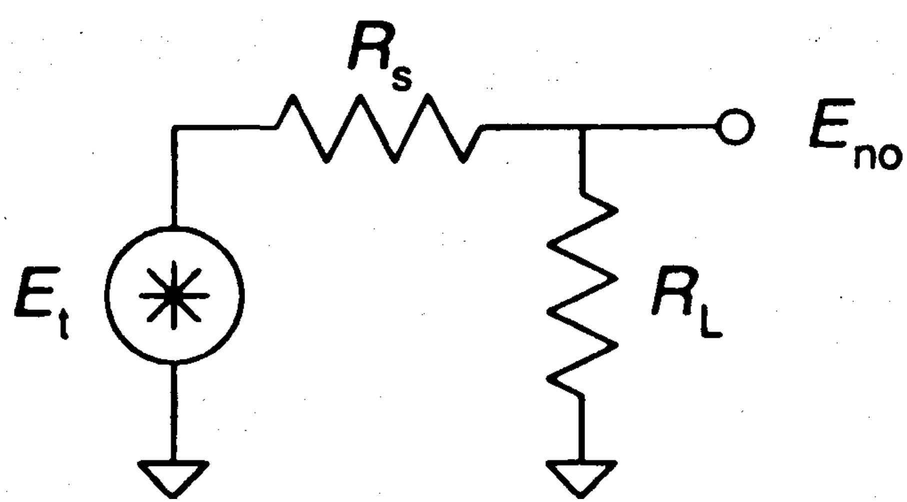
- Therefore the noise *voltage* and *current* are

$$\overline{v_n^2} \equiv \overline{v_t^2} = 4kT \cdot R \cdot B \qquad \overline{i_n^2} \equiv \overline{i_t^2} = \frac{4kT \cdot B}{R}$$

Thermal Noise Voltage

- The noise power from a resistor is the power that can be delivered to a resistive load equal in value to the source resistance.
- Therefore, if in Figure the load R_L is noiseless and $\boldsymbol{R_S} = \boldsymbol{R_L} = \boldsymbol{R}$, then

$$P_{no} = \frac{E_{no}^2}{R_L} \cdot B = \frac{(E_t/2)^2}{R_S} \cdot B = kT \cdot B$$



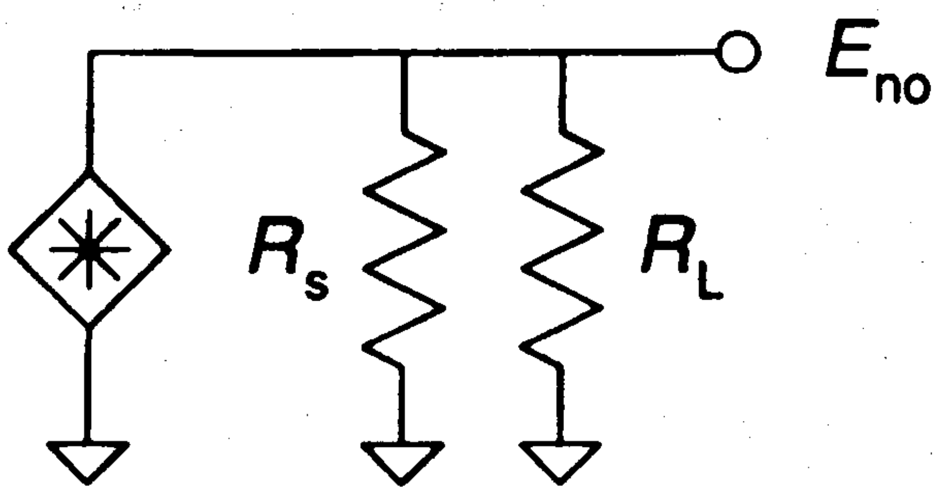
thus, $\overline{v_t^2} = 4kT \cdot B \cdot R$

and spectral density

$$\overline{V_n^2} \equiv \frac{d\overline{v_n^2}}{df} = 4kT \cdot R$$

Thermal Noise Current

- Analogously, the thermal noise from a resistive source can be modeled as a current source



$$\overline{i_n^2} = \frac{4kT \cdot B}{R}$$

and spectral density

$$I_n^2 \equiv \frac{d\overline{i_n^2}}{df} = \frac{4kT}{R}$$

Example

- Calculate the thermal noise voltage for a 1 kΩ resistor when *B (Δf)*= 1 Hz and the temperature is 25° C,
- 77 K (liquid nitrogen) or 4.2 K (liquid helium).

Solution

- At 25 °C, *T* = (273.16 + 25) K ~ 298° K.

$$\overline{v_t^2} = 4 \cdot 1.38 \text{ x } 10^{-23} \text{ J/K} \cdot 298 \text{ K} \cdot 1 \text{ Hz} \cdot 1000\,\Omega =$$

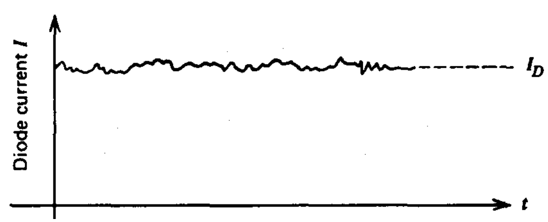
$$= 1.65 \text{ x } 10^{-17} \text{ W}\Omega = 1.65 \text{ x } 10^{-17} \text{ V}^2$$

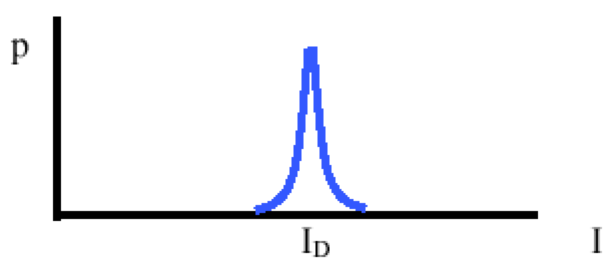
$$v_t\,(\text{rms}) \equiv \sqrt{\overline{v_t^2}} = 4\text{nV}$$

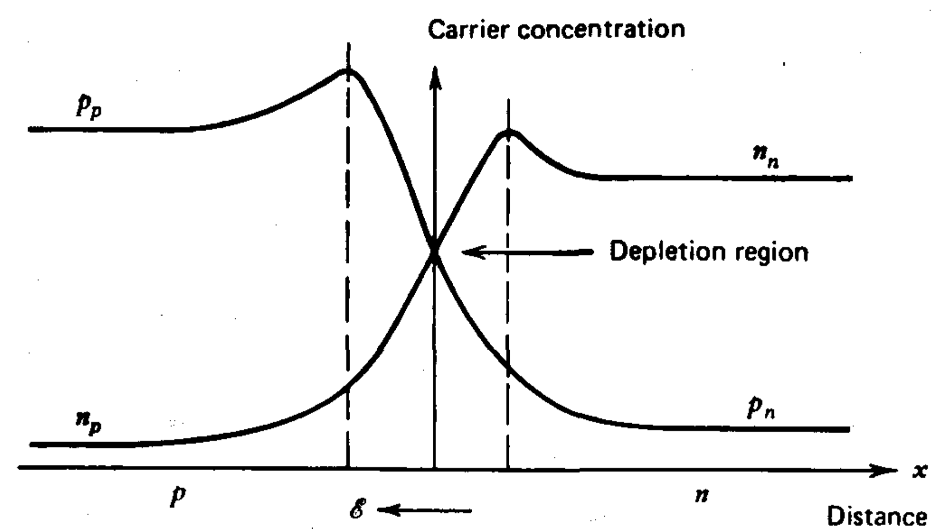
- At 77° K, $\overline{v_t^2} = 4.25 \text{ x } 10^{-18} \text{ V}^2 \text{ and } v_t\,(\text{rms}) = 2\text{nV}$
- At 4.2° K, $\overline{v_t^2} = 2.32 \text{ x } 10^{-19} \text{ V}^2 \text{ and } v_t\,(\text{rms}) = 0.5\text{nV}$

Shot Noise origin

- Shot noise is *always* associated with a direct-current flow and is present in diodes and bipolar transistors.
- The passage of each carrier across the junction is a purely *random event* and is dependent on the carrier having sufficient energy and a velocity.
- Thus external current ***I***, which appears to be a steady current, is, in fact, composed of a large number of random independent current pulses.







Shot noise

The fluctuation in ***I*** is termed *shot noise* and is generally specified in terms of its mean-square variation about the average value.

$$\overline{i_n^2} \equiv \overline{(i - I_D)^2} = \lim_{T\to\infty} \frac{1}{T} \int_0^T (i - I_D)^2 \, dt$$

, *where* q_e = electron charge , I_{DC} = DC current

It can be shown that if a current *I is* composed of a series of random independent pulses with average value I_D, then the resulting noise current has a mean-square value

$$\overline{i_n^2} = 2q_e I_{DC} \cdot \Delta f \equiv 2qI_D \cdot B$$

$$I_n^2 \equiv \overline{i_n^2} / B \equiv 2qI_D$$

The shot noise current (in rms): $$i_{sh} = \sqrt{2qI_D \cdot B}$$

The total current will therefore be: $$i(t) = i_{sh}(t) + I_D$$

Shot noise in p-n junction

- Conductors do not have shot noise because there are no potential barriers in them and electron movements are correlated.
- In a *p-n* junction, however, there is a potential barrier and the current through it obeys the equation

$$i_d = I_S\left(\exp(qv_d / kT) - 1\right)$$

- where I_S is the reverse saturation current, v_d is the voltage across the junction
- The current $\boldsymbol{i_d}$ consist of two currents $I_S \exp(qv_d / kT)$
- and I_S each one with its own shot noise.
- The mean-square noise current will be the sum of mean-square noise currents

biased p-n junction

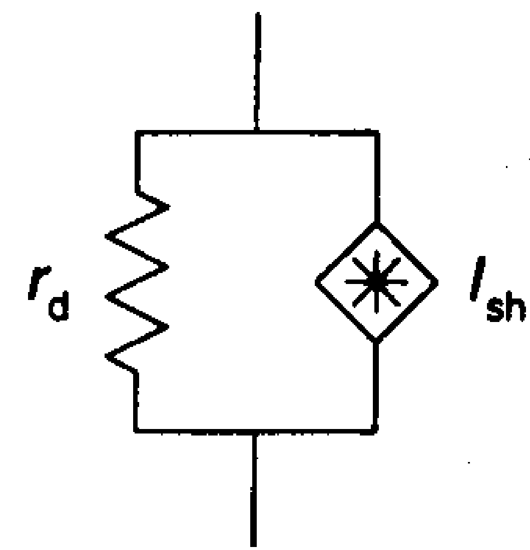
- For zero Bias (v_d=0) i_d=0 and

$$\overline{i_{sh}^2} = 2 \times 2qI_S B = 4qI_S B$$

- For forward biased p-n junction , the exponential term of i_d is much larger that I_S, and the shot current is given by and the equivalent circuit is shown, where r_d is

$$r_d = dv_d / di_d = kT / qi_d$$

- at room temperature r_d=0.025 V/i_d and



$$i_{sh}(rms) = \sqrt{2q_e I_{DC} \cdot B}$$

$$v_{sh}(rms) = i_{sh} \cdot r_d = \sqrt{2qI_{DC} \cdot B}\,\frac{kT}{qi_d} = kT\sqrt{\frac{2B}{qi_d}}$$

Thermal vs Shot

- Across the base-emitter junction of a forward-biased transistor with I_E = 10 mA at 25 °C, calculate the voltage noise produced by shot noise in a 10 kHz bandwidth.
- Compare that noise with the thermal noise of a conductor having the same resistance.

Solution

- $I_{dc} = I_E$ and $B = 10^4$ Hz, Therefore

$$i_{sh} = \sqrt{2q_e I_{DC} \cdot B} = \sqrt{2 \cdot 1.6 \cdot 10^{-19} C \cdot 10^{-2} A \cdot 10^4 Hz} = 5.7nA$$

$$r_E = kT / qI_d = (1.38 \cdot 10^{-23} J/K \cdot 298K)/(1.6 \cdot 10^{-19} C \cdot 10^{-2} A) = 2.6\Omega$$

$$v_{sh} = i_{sh} r_d = 5.7A \cdot 2.6\Omega = 15nV$$

Thermal vs Shot

- the thermal noise voltage is

$$v_t = \sqrt{4kT \cdot B \cdot R} = 21nV$$

- The power of the shot noise is independent of I_d

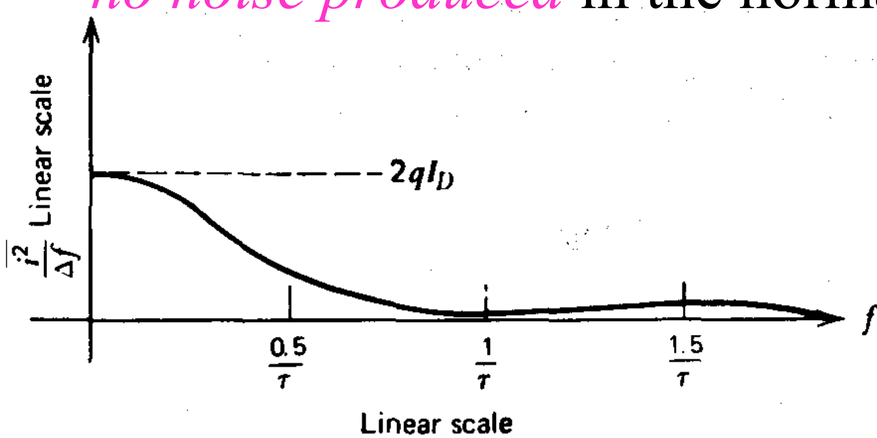
$$P_{sh} = i_{sh}^2 r_d \cdot B = 2q_e I_d \cdot kT / q_e I_d \cdot B = 2kT \cdot B = 0.83 \times 10^{-16} W$$

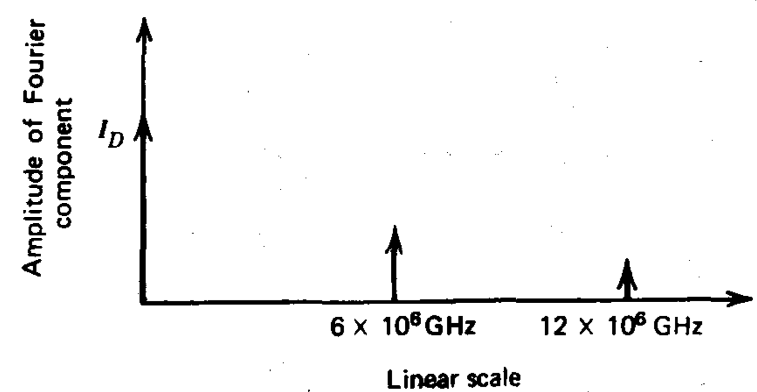
- and twice smaller that thermal noise.

$$P_T = 4kT \cdot B = 1.65 \times 10^{-16} W$$

Spectral density of Shot Noise

- Equation $\overline{I_{sh}^2} = 2q_e I_{DC} \cdot \Delta f$ is valid until the frequency becomes comparable to *1/τ*, where t is the carrier transit time through the depletion region. A sketch of noise-current spectral density versus frequency for a diode is shown below (left)
- Assuming that all the carriers made transitions with uniform time separation, the Fourier analysis of such a waveform would give the spectrum. Thus the first harmonic is at 6 x 10^6 GHz, which is far beyond the frequency of the device. There would be *no noise produced* in the normal frequency range of operation.





Flicker ("1/f") Noise origin

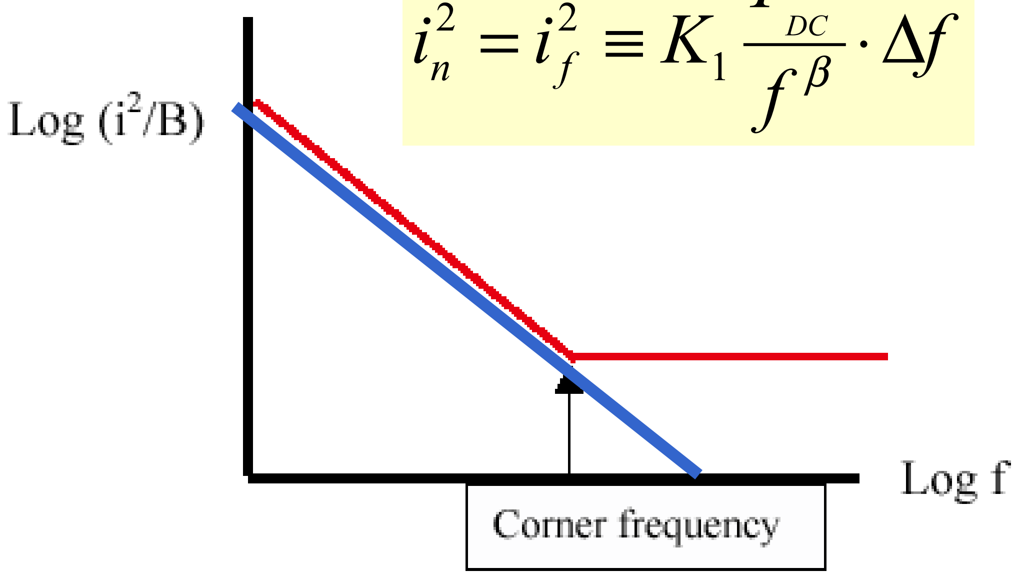
- This is a type of noise found in all active devices, as well as some discrete passive elements such as carbon resistors.
- The origins of flicker noise are varied, but in bipolar transistors it is caused mainly by traps associated with contamination and crystal defects in the emitter-base depletion layer.
- These traps capture and release carriers in a random fashion and the time constants associated with the process give rise to a noise signal with energy concentrated at low frequencies.

Flicker Noise

- Flicker noise is always associated with a flow of direct current and displays a spectral density of the form :

$$\overline{i_n^2} = \overline{i_f^2} \equiv K_1 \frac{I_{DC}^{\alpha}}{f^{\beta}} \cdot \Delta f$$

$$I_f^2 = K_1 \frac{I_{DC}^{\alpha}}{f^{\beta}}$$



- I_{DC} is a direct current
- K_1 is a constant for a particular device
- α is a constant in the range 0.5 to 2
- β is a constant of about unity

❖ The final characteristic of flicker noise its amplitude distribution, and it is often non-Gaussian.

Burst noise origin

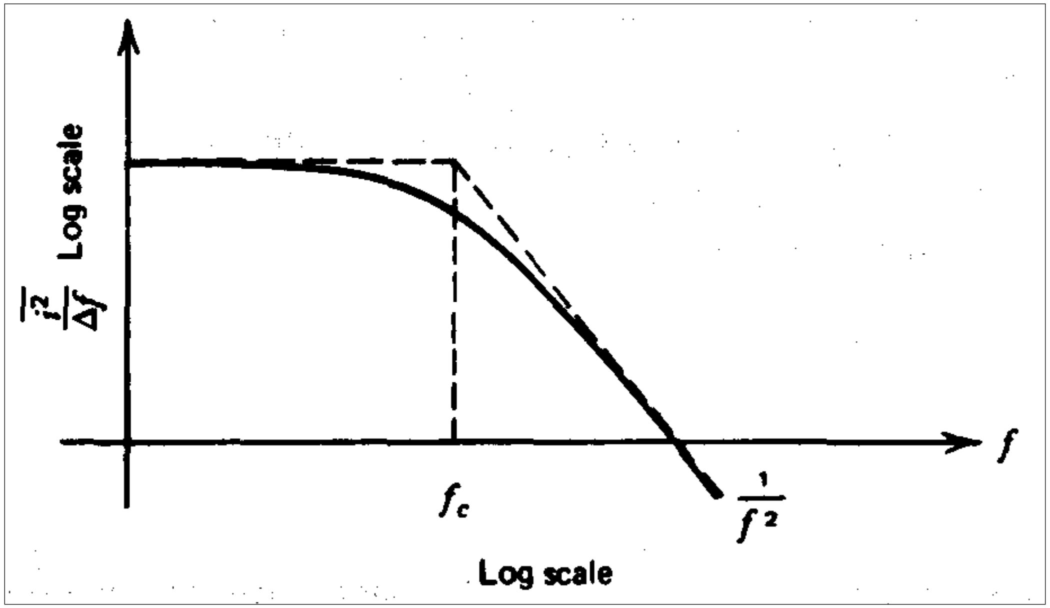
- This is another type of low-frequency noise found in some integrated circuits and discrete transistors.
- The source of this noise is related to the presence of heavy-metal ion contamination.
- Gold-doped devices show very high levels of burst noise.
- The amplitude distribution (PDF) is non-Gaussian
- The spectral density of burst noise can be shown to be of the form :

$$\overline{i_n^2} = \overline{i_b^2} \equiv K_2 \frac{I^c}{1 + \left(f / f_c \right)^2} \cdot \Delta f$$

Burst noise (spectral density)

$$I_b^2 = K_2 \frac{I^c}{1+(f/f_c)^2}$$

- I is a direct current
- K_2 is a constant for a particular device
- c is a constant in the range 0.5 to 2
- f_c is a particular frequency for given noise process



Avalanche Noise origin

- This is a form of noise produced by Zener or avalanche breakdown in a *p-n* junction.
- In avalanche breakdown, holes and electrons in the depletion region of a reverse-biased *p-n* junction acquire sufficient energy to create hole-electron pairs by colliding with silicon atoms.
- This process is cumulative, resulting in the production of a random series of large noise spikes.
- The noise is always associated with a direct-current flow, and the noise produced is much greater than shot noise in the same current, as given by $i^2 = 2qI_D \cdot B$

- The most common situation where avalanche noise is a problem occurs when are used in the circuit.
- The spectral density of the noise is approximately flat, but the amplitude distribution is generally non-Gaussian

Terminology (resume)

- The terminology you may find in different books varies from author to author.
- In our course we use for mean square values:
  - for noise voltage $\overline{v_n^2}$ or $\overline{v^2}$ and for current $\overline{i_n^2}$ or $\overline{i^2}$
- For Voltage/Current Root Mean Square (rms) values:

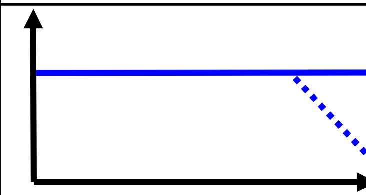
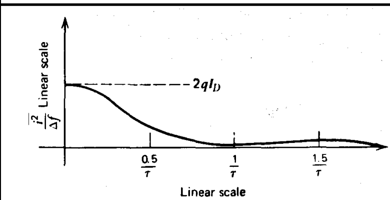
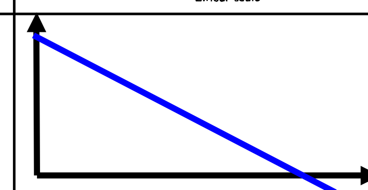
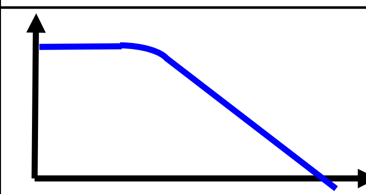
$$v_n = \sqrt{\overline{v_n^2}} \ [\mathrm{V}] \qquad i_n = \sqrt{\overline{i_n^2}} \ [\mathrm{A}]$$

- For Voltage/Current Spectral densities:

$$V_n^2 \equiv E_n^2 = \overline{v_n^2} / \Delta f \ [V^2 / Hz] \qquad I_n^2 = \overline{i_n^2} / \Delta f \ [\mathrm{A}^2/\mathrm{Hz}]$$

- For noise bandwidth both B or Δf.
- Subscripts: t, sh, b, f - for different noise mechanisms

Noise Sources (resume)

Noise	Origin	Expression	Spectral density
thermal noise (Gaussian)	random fluctuations of velocity	$I_t^2 = 4kT / R$	
shot noise (Gaussian)	due to DC current through p-n junction	$I_{sh}^2 = 2qI_D$	
flicker noise	traps in crystal lattice (semiconductors, carbon , etc.)	$I_f^2 = K_1 \frac{I_{DC}^{\alpha}}{f^{\beta}}$	
burst noise	heavy-metal ions contamination. (gold , etc.)	$I_b^2 = K_2 \frac{I^c}{1+(f/f_c)^2}$	
avalanche noise	due to avalanche breakdown in zener diodes		