Thus, we can write:

$$Y = \frac{T_1 + T_e}{T_2 + T_e} \tag{14.14}$$

or

$$T_e = \frac{T_1 - YT_2}{Y - 1} \tag{14.15}$$

From a measurement of T_1, T_2 and Y, the unknown amplifier's noise temperature (T_e) can be found.

POINT OF CAUTION: *To obtain an accurate value for Y, the two temperatures ideally must be far apart; otherwise, $Y \approx 1$ and the denominator of Equation 14.15 will create relatively inaccurate results.*

NOTE: *A noise source "hotter" than room temperature, as used in the Y-factor measurement, would be a solid-state noise source (such as an IMPATT diode) or a noise tube. Such active sources, providing a calibrated and specific noise power output in a particular frequency range, are most commonly characterized by their "excess noise ratio" values versus frequency. The term excess noise ratio or ENR is defined as:*

$$ENR(\text{dB}) = 10\log_{10}\left(\frac{P_N - P_o}{P_o}\right) = 10\log_{10}\left(\frac{T_N - T_o}{T_o}\right) \tag{14.16}$$

where P_N and T_N are the noise power and equivalent noise temperature of the active noise generator, and P_o and T_o are the noise power and temperature of a room-temperature passive source (e.g., a matched load), respectively.

14.8 DEFINITIONS OF NOISE FIGURE

As discussed earlier, a noisy amplifier can be characterized by an equivalent noise temperature (T_e). An alternate method to characterize a noisy amplifier is through the concept of noise figure, which we need to define first.

DEFINITION-NOISE FIGURE: *The ratio of the total available noise power at the output, $(P_o)_{tot}$, to the output available noise power $(P_o)_i$ due to thermal noise coming only from the input resistor at the standard room temperature ($T_o = 290°$ K).*

To formulate an equation for noise figure (F), let us transfer the noise generated inside the amplifier (P_n) to its input terminals and model it as a "noiseless" amplifier that is connected to a noisy resistor (R) at noise temperature (T_e) in series to another resistor (R) at $T = T_o$, both connected at the input terminals of the "noiseless" amplifier, as shown in Figure 14.9.

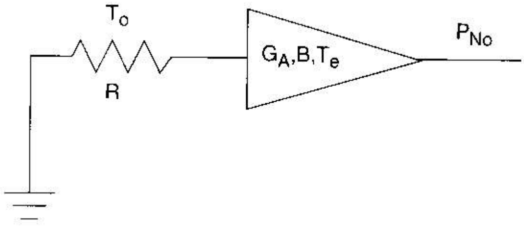
From this configuration, we can write:

$$P_n = G_A k T_e B \tag{14.17a}$$

$$(P_o)_i = G_A P_{Ni} = G_A k B T_o \tag{14.17b}$$

$$(P_o)_{tot} = P_{No} = P_n + (P_o)_i \tag{14.18}$$

FIGURE 14.9 A noisy amplifer.



$$F = \frac{(P_o)_{tot}}{(P_o)_i} = \frac{(P_o)_i + P_n}{(P_o)_i} = 1 + \frac{P_n}{G_A P_{Ni}} \tag{14.19a}$$

or

$$F = 1 + \frac{T_e}{T_o} \tag{14.19b}$$

Or, in dB, we can write:

$$F = 10 \log_{10}\left(1 + \frac{T_e}{T_o}\right) \tag{14.20}$$

From Equations 14.19, we can see that F is bounded by:

$$1 \leq F \leq \infty \tag{14.21}$$

The lower boundary ($F = 1$) is the best-case scenario and is the noise figure of an ideal noiseless amplifier where $T_e = 0$.

From Equation 14.19b, we can write:

$$T_e = (F - 1)T_o \tag{14.22}$$

NOTE 1: *Temperature (T_e) is the equivalent noise temperature of the amplifier referred to the input.*

NOTE 2: *Either F or T_e can interchangeably be used to describe the noise properties of a two-port network. For small noise figure values (i.e., when $F \approx 1$), use of T_e becomes preferable.*

POINT OF CAUTION: *It is interesting to note that the noise figure is defined with reference to a matched input termination at room temperature (T_o= 290° K). Therefore, if the physical temperature of the amplifier changes to some value other than T_o, we still use the room temperature (T_o = 290° K) to find the noise figure value.*

14.8.1 Alternate Definition of Noise Figure

From Equations 14.17 and 14.18, we can write:

$$P_{No} = G_A P_{Ni} + P_n \tag{14.23}$$

$$(P_o)_i = G_A P_{Ni} \tag{14.24}$$

where $P_n = G_A k T_e B$ is the generated noise power inside the amplifier. The noise figure can now be written as:

$$F = \frac{P_{No}}{(P_o)_i} = \frac{P_{No}}{G_A P_{Ni}} \tag{14.25}$$

The available power gain (G_A) by definition is given by:

$$G_A = \frac{P_{So}}{P_{Si}}$$

where P_{So} and P_{Si} are the available signal power at the output and the input, respectively. Thus, Equation 14.25 can now be written as:

$$F = \frac{P_{Si}/P_{Ni}}{P_{So}/P_{No}} = \frac{(SNR)_i}{(SNR)_o} \tag{14.26}$$

where $(SNR)_i$ and $(SNR)_o$ are the available signal-to-noise ratio at the input and output ports, respectively.

Equation 14.26 indicates that the noise figure can also be defined in terms of the ratio of the available signal-to-noise power ratio at the input to the available signal-to-noise power ratio at the output.

14.8.2 Noise Figure of a Lossy Two-Port Network

This is an important case, where the two-port network considered earlier is a lossy passive component, such as an attenuator or a lossy transmission line, as shown in Figure 14.10.

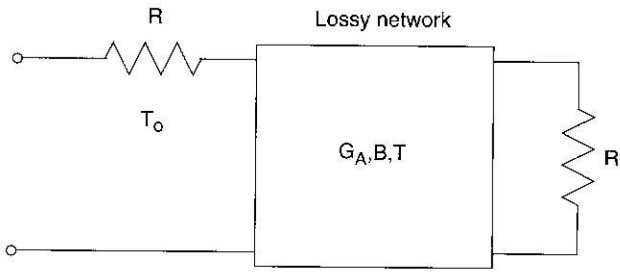


FIGURE 14.10 A lossy two-port network.

A lossy network has a gain $\left(G_A = \frac{P_o}{P_i}\right)$ less than unity, which can be expressed in terms of the loss factor or attenuation (L) as:

$$G_A = \frac{1}{L} \quad (G_A < 1) \tag{14.27}$$

Because the gain of a lossy network is less than unity it follows that the loss or attenuation factor (L) is more than unity (i.e., $L = P_i/P_o > 1$) for any lossy network or component.

Expressing the attenuation factor (L) in dB gives the following:

$$L(\text{dB}) = 10\log_{10}\left(\frac{P_i}{P_o}\right) \tag{14.28}$$

For example, if the lossy component attenuates the input power by ten times, then we can write: