Noise Characteristics of Gallium Arsenide Field-Effect Transistors

HERMANN STATZ, HERMANN A. HAUS, FELLOW, IEEE, AND ROBERT A. PUCEL, SENIOR MEMBER, IEEE

Abstract—Small signal and noise characteristics for GaAs field-effect transistors are derived with the saturated drift velocity of the carriers underneath the gate taken into account. The noise contributed by the saturated carriers is nonnegligible and in most cases, exceeds the noise generated by the unsaturated region. Parasitic elements contribute importantly by preventing the full cancellation of the correlated noise of the intrinsic transistor and by adding their own Johnson noise. The theory predicts the experimentally observed trend of noise figure dependence on drain current and on source-to-drain voltage. The present theory does not take into account the effects of a possible short negative resistance region underneath the gate.

I. INTRODUCTION

High-frequency gallium arsenide field-effect transistors have shown astonishingly low noise figures of approximately 3–4 dB at 10 GHz [1], [2]. Even though considerable work in understanding this noise performance has been done, most explanations omit important features of the microwave FET. Van der Ziel, in pioneering work, first analyzed noise in field-effect transistors [3], [4]. Van der Ziel, however, restricted himself to source–drain voltage differences below the pinch-off value. All microwave transistors operate above the so-called pinch-off point.

Van der Ziel omitted, in addition, the nonohmic conductivity of gallium arsenide. The nonohmic behavior is of importance in virtually all GaAs microwave transistors. For example, a voltage drop of 1 V across a typical gate length of 10^{-4} μm corresponds to an average field strength of 10^4 V/cm. The velocity of carriers as a function of field in GaAs reaches a maximum at 3×10^3 V/cm followed by a negative resistance region and then a nearly constant saturated drift velocity. Baechtold [5], [6] corrected for the nonconstant mobility by assuming a constant mobility up to a critical field, and then for fields larger than the critical field, he assumed a constant velocity. This approach neglects the effects of the negative resistance region. Since the negative resistance region occupies in general only a small space under the gate, the approximation may be justified.

Baechtold also allows for a field dependent electron temperature. For gallium arsenide, he assumes

$$\frac{T_e}{T_o} = 1 + \delta\left(\frac{E}{E_{\rm sat}}\right)^3 \qquad (1)$$

where T_e is the effective noise temperature, T_o is the reference temperature, i.e., 300 K, δ is an empirical constant, E is the electric field, and $E_{\rm sat}$ is the saturation field. Baechtold confirmed (1) by experiments. However, Baechtold neglects the saturated velocity region of the transistor and its contribution to the noise behavior. As stated before, the average field in the transistor under the gate is generally much larger than the saturation field $E_{\rm sat}$. The region within which the carriers drift at saturated velocity can occupy a major fraction of the total channel length. Therefore, the characteristics of the transistor in the current saturation regime should strongly depend on this satuated velocity region.

This paper includes the effect upon the signal and noise parameters of the saturated drift region. We derive the small signal parameters of the transistor such as transconductance, drain resistance, etc., including the effects of the drift region. The noise analysis in the drift region in some ways follows a suggestion by Shockley *et al.* [7] according to which the high-field diffusion constant describes the formation of dipole layers which drift through the high-field region to the drain contact. Ruch and Kino [8] have measured the diffusion constant in GaAs at high fields, and Fawcett and Rees [9] have calculated it. These measurements and calculations show a peak in the parallel diffusion constant near the saturation field followed by a rapid drop to rather low values. The measurement of the diffusion constant is complicated by trapping effects. We obtain rather good agreement with experimental noise figure measurements if we use in our expressions values which are consistent with the calculations of Fawcett and Rees [9]. We neglect variations of the diffusion constant with electric field.

It turns out that the noise in the saturated velocity region is rather important and may be larger than the one contributed by the unsaturated region. On the other hand, the correlation between drain noise and gate noise current is rather large, so that the overall noise figure is still attractively low. Parasitic resistances consisting of contact resistance at the source and the series resistance between the source and the region under the gate as well as gate metallization resistance are very important. They prevent the full cancellation of the correlated noise of the intrinsic field-effect transistor and contribute their own Johnson noise. It appears that a reduction of these parasitic resistances would substantially lower the noise figure.

II. THE DC CONDITIONS

In this paper, we consider both the symmetric FET as well as the FET constructed on an insulating substrate.

Manuscript received January 17, 1974.
H. Statz and R. A. Pucel are with the Research Division, Raytheon Company, Waltham, Mass.
H. A. Haus is with the Electrical Engineering Department and Research Laboratory of Electronics, M.I.T., Cambridge, Mass.

The latter has a field configuration which approximately corresponds to the field configuration of the symmetric FET on one side of the symmetry plane; the nonconducting substrate surface plays the role of the symmetry plane. Fig. 1 illustrates the situation. Instead of placing the source and drain contacts on the side of the gate, as would normally be done, the contacts are placed below the gate in the positions corresponding to the idealized symmetric FET.

We follow van der Ziel's [3], [4] convention of considering positive charge carriers in order to avoid unnecessary confusion between his and our paper. Nevertheless, all GaAs transistors are made of n-type material, and we, therefore, use in our numerical examples saturated velocities, noise temperatures, mobilities, etc., which are characteristic of electrons. In contrast to van der Ziel's transistor biased below pinchoff, we consider a transistor biased beyond pinchoff. The onset of pinchoff is here defined to occur when the maximum longitudinal electric field first reaches the saturation value E_{sat}. The velocity-E field characteristic of the medium (GaAs) is assumed to follow the idealized plot of Fig. 2, which has already been used by Baechtold [5], [6]. The analysis of the FET is carried out separately in two regions: Region I of ohmic conductivity and Region II of saturated velocity.

We use in our paper some results which have previously been derived by Shockley [10] and van der Ziel [3], [4]. For clarity and continuity of the analysis, however, we repeat these results where needed. It is convenient to introduce the gate potential with respect to the source V_g and the channel potential V_p at the end of Region I, also measured with respect to the source. The effective potential difference W between the gate and the channel is then $W = W_s = V_g + V_{dif}$ at the source side and $W = W_p = V_g + V_{dif} - V_p$ at the pinch-off point. V_{dif} is the built-in diffusion potential due to the doping difference between channel and gate. Most of the expressions to be used in the paper become simplified if we introduce the reduced potentials

$$s = \left(\frac{W_s}{W_{oo}}\right)^{1/2} \tag{2}$$

$$p = \left(\frac{W_p}{W_{oo}}\right)^{1/2} \tag{3}$$

where W_{oo} is the value of the gate-to-channel potential required to deplete the channel of carriers, as seen by

$$W_{oo} = \frac{\rho}{2\epsilon} a^2. \tag{4}$$

In (4), ρ is the space-charge density in the channel region due to the built-in acceptor centers, ϵ is the dielectric constant of the material, and $2a$ is the undepleted channel thickness of the symmetrical transistor. As mentioned, we divide the region under the gate of length L into Region I of length L_1 which is ohmic, and a region of length L_2 which is saturated. In Region I, we have [3]

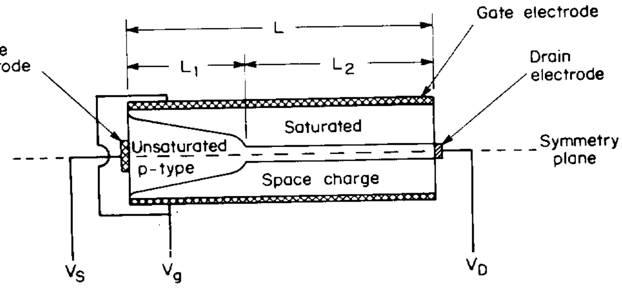

Fig. 1. Idealized geometry of a field-effect transistor.

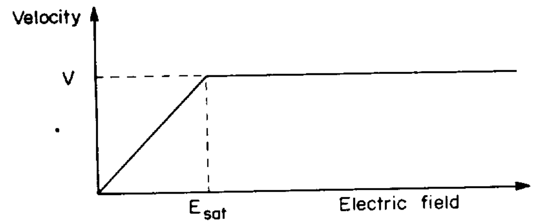

Fig. 2. Simplified velocity versus electric field relationship used in text.

$$I_d = \frac{l g_o W_{oo}}{L_1} \{p^2 - s^2 - (2/3)(p^3 - s^3)\}. \tag{5}$$

In (5), I_d is the source-to-drain current, l is the width of the transistor; g_o is defined for the symmetric transistor by

$$g_o = 2\sigma a \tag{6}$$

where σ is the conductivity of the channel material. For the asymmetric FET, $g_o = \sigma a$.

The undepleted channel width $2b_p$ at the pinch-off point and beyond is given by

$$2b_p = 2a(1 - p). \tag{7}$$

For applied voltages exceeding the pinch-off value, the longitudinal channel field exceeds the saturation field E_{sat}. By definition, the end of Region I has a longitudinal channel field E_{sat}. The total current that can be carried at this point is equal to

$$I_d = l g_o E_{sat}(1 - p). \tag{8}$$

By equating (5) and (8), we can determine the position of L_1 as a function of gate and pinch-off potentials, as seen by

$$L_1 = \frac{W_{oo}}{E_{sat}} \frac{p^2 - s^2 - (2/3)(p^3 - s^3)}{1 - p}. \tag{9}$$

Up to the pinch-off point, ohmic conduction occurs. Beyond the pinch-off point, the carriers drift at constant velocity, occupying the width $2b_p$, as seen in (7). The longitudinal electric field is equal to the saturation field E_{sat} at the pinch-off point. The electric field continues to increase beyond the pinch-off point reaching its maximum value at the drain contact, thus assuring saturated drift throughout Region II.

III. EVALUATION OF EQUIVALENT CIRCUIT PARAMETERS

We are concerned mainly with the low-frequency limit of the signal analysis. The noise analysis takes intermediate frequencies into account by considering the induced gate noise. Fig. 3 shows the equivalent circuit elements of the field-effect transistor. Van der Ziel evaluated the transconductance g_m and the drain conductance for the transistor below pinchoff and found

$$g_m = -\frac{\partial I_d}{\partial V_g} = \frac{g_o l}{L}(d - s) \qquad (10)$$

$$g_d = -\frac{\partial I_d}{\partial V_d} = \frac{g_o l}{L}(1 - d) \qquad (11)$$

where $d = (W_d/W_{oo})^{1/2}$, and W_d is the channel potential at the drain with respect to the gate. V_d is the drain voltage. Here we want to go through an analysis that includes the effect of the saturated velocity pinch-off region. The drain resistance r_d is defined by

$$r_d = -\frac{\partial V_{sd}}{\partial I_d}. \qquad (12)$$

In (12), V_{sd} is the total voltage difference between source and drain including the voltage contribution V_p for Region I, plus the voltage between L_1 and L. From the definition of p and s, we have

$$V_p = -W_{\infty}(p^2 - s^2). \qquad (13)$$

In Region II, we assume the stream of carriers to continue with constant density and uniform cross-section $2b_p$. We have to solve Poisson's equation assuming a uniform acceptor density and including the space charge of the flowing carriers. We have to satisfy the boundary conditions at both ends of Region II. A particular solution of Poisson's equation is obviously that of a constant potential $-W_{\infty}(p^2 - s^2)$ everywhere in the carrier stream, and outside of the stream, a potential increasing parabolically toward the gate electrode. To satisfy the boundary condition at the drain end of Section II, we have to add a homogeneous solution of Poisson's equation. Obviously, a series expansion of the form

$$V = \sum_{n=-\infty}^{n=\infty} A_n \cos (2n + 1)\frac{\pi y}{2a} \exp (2n + 1)\frac{\pi}{2a}x, \qquad n \text{ integer} \qquad (14)$$

is the most general solution [11]. This form satisfies the boundary conditions along the gate. We shall approximate (14) by the lowest space harmonics, with $n = 0, -1$. For reasons of continuity of potential and field ($E_x = E_{sat}$ at L_1) in the middle of the structure ($y = 0$), the homogeneous solution in Region II is of the form

$$V = -\frac{2a}{\pi}E_{sat} \sinh \frac{\pi x}{2a} \cos \frac{\pi y}{2a} \qquad (15)$$

where the origin of x is at the pinch-off point. For a drain contact placed at the end of the channel at $y = 0$, the source-to-drain voltage therefore becomes

$$V_{sd} = -W_{oo}(p^2 - s^2) - \frac{2a}{\pi}E_{sat} \sinh \frac{\pi}{2a}L_2. \qquad (16)$$

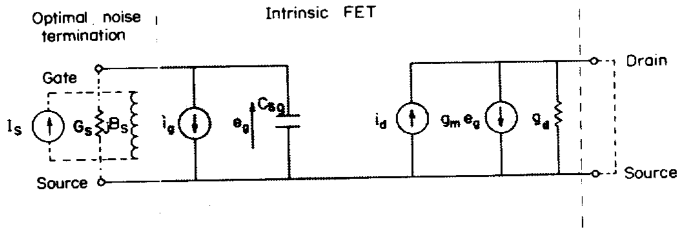

Fig. 3. Equivalent circuit for minimum noise figure calculation.

Differentiation of V_{sd} with respect to current I_d gives the drain resistance. We have to keep in mind that p and L_2 depend upon I_d, therefore,

$$r_d = -\frac{dV_{sd}}{dI_d} = 2pW_{oo}\frac{dp}{dI_d} + E_{sat}\left(\cosh \frac{\pi}{2a}L_2\right)\frac{dL_2}{dI_d}. \qquad (17)$$

The quantity dp/dI_d is evaluated by differentiating (8), seen by

$$\frac{dp}{dI_d} = -\frac{1}{g_o E_{sat} l}. \qquad (18)$$

The quantity dL_2/dI_d is also easily obtained, as seen by the equation

$$\frac{dL_2}{dI_d} = \frac{-dL_1}{dI_d} = \frac{-dL_1}{dp}\frac{dp}{dI_d} = \frac{1}{g_o E_{sat} l}\left\{\frac{2pW_{\infty}}{E_{sat}} + \frac{L_1}{1 - p}\right\} \qquad (19)$$

where dL_1/dp follows from differentiation of (9). Inserting (18) and (19) into (17) finally gives

$$r_d = \frac{2pW_{00}}{g_0 E_{sat} l}\left(\cosh \frac{\pi}{2a}L_2 - 1\right) + \frac{L_1}{g_o l(1 - p)} \cosh \frac{\pi}{2a}L_2. \qquad (20)$$

For $L_2 = 0$, (20) reduces to the well-known formula for the drain resistance [3] of a nonsaturated FET, with $p \to d$, $L_1 \to L$, and is seen as

$$r_d = \frac{L}{lg_o}\frac{1}{1 - d}. \qquad (21)$$

We next evaluate the transconductance under velocity saturation conditions. The transconductance g_m is defined as the change in drain current dI_d divided by the change in gate voltage dV_g holding the potential difference between source and drain V_{sd} constant. Using (8) for the drain current in the pinch-off region and remembering the definition of s in (2), we obtain

$$g_m = -\frac{dI_d}{dV_g} = g_o E_{sat} l \frac{dp}{dV_g} = \frac{g_o E_{sat} l}{2sW_{oo}}\frac{dp}{ds}. \qquad (22)$$

In our composite transistor consisting of Regions I and II, the value of the potential at the pinch-off point, and thus p, must change in order to accommodate a varying current in the region of saturated velocity. The saturated stream of carriers has to change in width, and according to (8), this leads to a variation of p. Likewise, L_1 and L_2 also change. To evaluate dp/ds, we first use the fact that dV_{sd} is zero. From (16), with $dL_1 = -dL_2$,

$$dV_{sd} = 0 = W_{\infty}(2pdp - 2sds) - E_{sat}dL_1 \cosh \frac{\pi}{2a} L_2. \tag{23}$$

To eliminate dL_1 from (23), we need another relationship between dL_1, dp, and ds. We obtain this additional relationship by differentiating (9), seen by

$$\frac{E_{sat}}{W_{\infty}} dL_1 = dp \left\{2p + \frac{E_{sat}L_1}{W_{\infty}} \frac{1}{1-p}\right\} - ds \frac{2s(1-s)}{1-p}. \tag{24}$$

From (23) and (24), one may eliminate dL_1, and an expression for dp/ds is obtained. Inserting this expression for dp/ds into (22), we obtain the desired expression for the transconductance

$$g_m = \frac{lg_o E_{sat}}{W_{oo}} \frac{(1-s) \cosh (\pi/2a)L_2 - (1-p)}{\{2p(1-p) + E_{sat}L_1/W_{oo}\} \cosh (\pi/2a)L_2 - 2p(1-p)}. \tag{25}$$

In the limit of large L_2, (25) reduces to

$$g_m = \frac{lg_o E_{sat}}{W_{oo}} \frac{1-s}{\{2p(1-p) + E_{sat}L_1/W_{\infty}\}}. \tag{26}$$

The latter expression is identical, for $p = d$, to the one used in [6]. In the limit of $L_2 = 0$, and thus $L_1 = L$ and $p = d$, we arrive at

$$g_m = \frac{lg_o}{L}(d - s).$$

This is the expression derived by Shockley [10] and van der Ziel [3]. It has already been presented in (10).

IV. THE NOISE EQUIVALENT CIRCUIT AND THE MINIMUM NOISE FIGURE

In this section, we consider the noise of the ideal transistor. Later, we shall include the noise contributions due to parasitic resistances. The equivalent circuit is shown by the solid lines in Fig. 3. The basic transistor equivalent circuit is identical to the one commonly used. In the input circuit, we have the gate–source capacity, C_{sg}. In the output circuit, we have a current generator $g_m e_g$, where e_g is the signal voltage across the gate–source capacitor. Parallel with the output generator is the output conductance g_d of the transistor. As noise sources we use two generators. In the input we use a noise current generator $\overline{i_g^2}$. This type of noise generator has first been introduced and analyzed by van der Ziel [4]. Basically, noise current flows to the gate because of Johnson-type noise voltage fluctuations in the channel region. As Baechtold has shown, this noise is enhanced by hot electrons in the channel. In addition, we introduce the diffusion noise of the pinch-off region and its contributions to the gate current noise. In the output circuit, we use a drain noise voltage generator $\overline{v_d^2}$. This noise voltage results from hot carrier enhanced Johnson noise in Region I, and, as we shall show, from spontaneously generated dipole layers drifting to the drain contact in Region II.

To calculate the noise figure of the circuit in Fig. 3 as a function of the noise generators, we must include an input and output circuit. By dashed lines we show an input admittance circuit $Y_s = G_s + jB_s$. Since the noise figure F will be independent of the particular output circuit, we simply use a short circuit. The quantity $F - 1$ is defined as the mean-square noise current in the output due to the two noise generators, divided by the mean-square noise current in the output circuit produced by the Johnson noise of the input admittance G_s. This is seen as

$$F - 1 = \frac{\overline{| i_g g_m/(Y_s + j\omega C_{gs}) + v_d g_d |^2}}{4kTG_s\Delta f g_m^2 \, | 1/(Y_s + j\omega C_{gs}) |^2}. \tag{27}$$

In general, there will be some correlation between i_g and v_d. This is the case because the Johnson noise in the channel contributes both to the drain voltage and to the gate current. We define a correlation coefficient C through the equation

$$\overline{v_d i_g^*} = jC(\overline{v_d^2} \cdot \overline{i_g^2})^{1/2}. \tag{28}$$

Thus $F - 1$ may be written as

$$F - 1 = \frac{1}{4kTG_s\Delta f g_m^2} \{\overline{i_g^2} g_m^2 + g_d^2 \overline{v_d^2} \, | Y_s + j\omega C_{gs} |^2 + jC(\overline{v_d^2})^{1/2}(\overline{i_g^2})^{1/2} g_d g_m (Y_s + j\omega C_{gs}) - jC(\overline{v_d^2})^{1/2}(\overline{i_g^2})^{1/2} g_d g_m (Y_s^* - j\omega C_{gs})\}. \tag{29}$$

Since we are interested in the minimum possible noise figure, we first find the optimum input susceptance B_s. Taking the derivative of $F - 1$ with respect to B_s, and setting the resulting expression equal to zero, we get

$$B_{s\,\mathrm{opt}} = -\omega C_{gs} + \frac{g_m}{g_d} C \frac{(\overline{i_g^2})^{1/2}}{\overline{v_d^2}} \tag{30}$$

and $F - 1$ becomes

$$F - 1 = \frac{1}{4kTG_s\Delta f g_m^2} \{\overline{i_g^2} g_m^2 + g_d^2 \overline{v_d^2} G_s^2 - C^2 g_m^2 \overline{i_g^2}\}. \tag{31}$$

Next, we minimize with respect to G_s and obtain the optimum source conductance

$$G_{s\,\mathrm{opt}} = \frac{g_m (\overline{i_g^2})^{1/2}}{g_d (\overline{v_d^2})^{1/2}} (1 - C^2)^{1/2} \tag{32}$$

and minimum excess noise figure

$$(F-1)_{\min} = \frac{1}{2kTg_m\Delta f}\,(\overline{i_g^2})^{1/2}(\overline{i_d^2})^{1/2}(1-C^2)^{1/2}. \quad (33)$$

In (33), we introduced the drain noise current i_d through

$$\overline{i_d^2} = g_d^2\overline{v_d^2}. \quad (34)$$

In the following sections of the paper, we evaluate the quantities $\overline{i_g^2}$, $\overline{i_d^2}$, and C as a function of the device parameters.

V. OPEN-CIRCUIT VOLTAGE FLUCTUATION DUE TO ENHANCED JOHNSON NOISE

In this section, we calculate voltage fluctuations at the drain due to enhanced Johnson noise in the nonsaturated Region I. In order to save space, we make use of earlier work by van der Ziel [3], [4]. Essentially, we have to add two features to van der Ziel's work. First, the noise temperature in the present treatment is field dependent as shown in (1). The implications of a field dependent noise temperature in Region I have been considered briefly by Klaassen [12] and van der Ziel [13], [14], and in more detail by Baechtold [6]. Second, the noise voltage as calculated at the end of Region I propagates and becomes enhanced in Region II.

In Appendix I, we give the derivation of the noise voltage at the end of Region I, Δv_1, due to the Johnson noise of an infinitesimal section of channel Δx located at a position x where the channel potential with respect to the gate is $W_o(x)$. The result is

$$\Delta\overline{v_1^2} = \frac{4kT_e\Delta f}{I_d}\left[\frac{1-w(x)}{1-p}\right]^2 2W_{oo}w(x)\,\Delta w. \quad (35)$$

We introduced in (35) the reduced potential

$$w(x) = \left(\frac{W_o(x)}{W_{oo}}\right)^{1/2} \quad (36)$$

and the voltage drop due to the current I_d flowing from source to gate in the channel section Δx is equal to $\Delta W_o(x) = 2W_{oo}w(x)\Delta w$.

We now modify (35) by introducing the field dependent noise temperature. The longitudinal electric field E in the channel due to the current I_d may be written as

$$E = \frac{I_d}{lg_o}\,\frac{1}{1-w}. \quad (37)$$

If we use (5) for I_d and if we introduce

$$f_1 = p^2 - s^2 - \tfrac{2}{3}(p^3 - s^3) \quad (38)$$

then we may reexpress (1) in the form

$$T_e = T_o\left[1 + \delta\left(\frac{E}{E_{\text{sat}}}\right)^3\right] = T_o\left[1 + \delta f_1^3\left(\frac{W_{oo}}{E_{\text{sat}}L_1}\right)^3\frac{1}{(1-w)^3}\right]. \quad (39)$$

Inserting (39) into (35) and using once more (5) for I_d then gives

$$\Delta\overline{v_1^2} = \frac{4kT_o\Delta f}{l(g_o/L_1)}\,\frac{1}{f_1(1-p)^2}\,2w(1-w)^2 \cdot\left[1 + \delta f_1^3\left(\frac{W_{oo}}{E_{\text{sat}}L_1}\right)^3\frac{1}{(1-w)^3}\right]\Delta w. \quad (40)$$

The total voltage fluctuations at the end of Region I follow from a straightforward integration over w between the limits of $w = s$ and $w = p$. The final result may be written

$$\overline{v_1^2} = \frac{4kT_o\Delta f}{l(g_o/L_1)}\,\frac{P_o + P_\delta}{(1-p)^2}. \quad (41)$$

The quantities P_o and P_δ are defined as

$$P_o = (f_1)^{-1}[(p^2 - s^2) - \tfrac{4}{3}(p^3 - s^3) + \tfrac{1}{2}(p^4 - s^4)] \quad (42)$$

and

$$P_\delta = 2\delta\left(\frac{W_{oo}}{E_{\text{sat}}L_1}\right)^3 (f_1)^2\left[(s-p) + \ln\frac{1-s}{1-p}\right]. \quad (43)$$

Equation (41) with $P_\delta = 0$ is identical to van der Ziel's equation (13) in [3]. P_δ contains the new noise contributions. Note that our expressions for P_o and P_δ are different from the expressions P_1 and P_2 defined by Baechtold [6].

Next, we evaluate the open-circuit voltage fluctuations at $x = L_2$. Because the current is fixed, the width of the pinch-off channel or p remains constant also (8). Since p is fixed at the pinch-off point, so is the voltage (as seen in 3). In other words, noise fluctuations occurring at a fixed point in Region I have as a consequence that L_1 and L_2 fluctuate. Near the pinch-off point, the electrical field is equal to E_{sat}, thus a noise voltage of amplitude Δv_1 will cause a change of L_1 by ΔL_1 where

$$\Delta L_1 E_{\text{sat}} = \Delta v_1. \quad (44)$$

A variation of (16) gives, with channel width constant and hence, p = constant,

$$\Delta V_{sd} = -E_{\text{sat}}\Delta L_2 \cosh\frac{\pi}{2a}L_2. \quad (45)$$

Inserting (44) into (45) with $\Delta L_1 = -\Delta L_2$ gives

$$\Delta V_{sd} = \Delta v_1 \cosh\frac{\pi}{2a}L_2. \quad (46)$$

The noise voltage at $x = L_1$ given by (41) thus becomes transformed into

$$\overline{v_{d1}^2} = \frac{4kT_o\Delta f}{l(g_o/L_1)}\,\frac{P_o + P_\delta}{(1-p)^2}\cosh^2\frac{\pi}{2a}L_2 \quad (47)$$

when referred to the true drain $x = L_2$. Our notation v_{d1} refers to noise at the true drain caused by fluctuations in Region I. Equation (47) also holds for the asymmetric transistor, provided the value of g_o appropriate for a one-sided device is used. Numerically, (47) gives twice as large a value for the asymmetric transistor.

VI. INDUCED GATE NOISE CURRENTS WITH SHORT-CIRCUITED DRAIN

We have shown in the previous section that there are noise voltage fluctuations along the channel. Since the gate is capacitatively coupled to the channel, there will be induced noise charges on the gate, and likewise, since these charges are time dependent, there will be noise currents flowing into the gate. In the present section, we calculate induced gate currents due to elementary noise fluctuations in Region I only.

Van der Ziel has evaluated these noise currents for a nonsaturated transistor. Because of the additional saturated transistor Region II, the boundary conditions at the end of Region I are different in the present case. Since the drain is assumed short circuited, the noise voltage is zero at the end of Region II, while in van der Ziel's single region transistor, the noise voltage was assumed to be zero at the end of Region I. When we evaluate the induced gate current, we have to add effects due to the "breathing" of the channel in Region II. Finally, we have to allow for a field dependent noise temperature.

Van der Ziel has given a relationship (as derived in Appendix I, (I.6) between the voltage jump ΔW_p at the end of Region I and the noise voltage fluctuation $-\Delta W_x$ at position x in Region I, as seen by the equation

$$\Delta W_p = -\Delta W_x \frac{1 - w(x)}{1 - p}. \tag{48}$$

In (48), $w(x)$ is the reduced channel potential defined in (36). Similarly, p represents the value of w at the end of Region I, i.e., at $x = L_1$. Equation (48) is valid under open circuit conditions. We may obtain the noise voltage at the end of Region II by multiplying ΔW_p in (48) by $\cosh (\pi/2a) L_2$ in exact analogy to (46). Thus

$$\Delta v_d = -\Delta W_x \frac{1 - w(x)}{1 - p} \cosh \frac{\pi}{2a} L_2. \tag{49}$$

Under short-circuit conditions, the voltage change of (49) transforms itself into a drain current fluctuation Δi_d, where

$$\Delta i_d = -\frac{\Delta v_d}{r_d} = \Delta W_x \frac{1}{r_d} \frac{1 - w(x)}{1 - p} \cosh \frac{\pi}{2a} L_2. \tag{50}$$

Equation (50) replaces (11) of [4]. We may obtain (50) from (11) of [4] by multiplying the latter by the factor B, where

$$B = \frac{L_1 \cosh (\pi L_2/2a)}{l g_o (1 - p) r_d}. \tag{51}$$

For the calculation of the induced charge, we need the detailed behavior of the potential disturbance ΔW as a function of x along the channel for an elementary noise fluctuation $-\Delta W_x$ at position x_o. Due to the new boundary conditions, we find now in Region I (see Appendix II)

$$(1 - w) \Delta W = \frac{\Delta i_d}{l g_o} x \text{ for } 0 < x < x_o \tag{52}$$

$$(1 - w) \Delta W = \frac{\Delta i_d}{l g_o} (x - \gamma L_1) \text{ for } x_o < x < L_1 \tag{53}$$

where $x = 0$ now corresponds to the beginning of Region I. Equation (52) is identical to the corresponding equation in [4], while (53) has now a factor of γ in front of L_1, since ΔW no longer needs to be equal to zero at the end of Region I. There is a discontinuity in ΔW at $x = x_o$. After inserting Δi_d from (50) into (52) and (53), we can determine γ from the fact that the discontinuity in ΔW is equal to $-\Delta W_x$. We find with (20)

$$\gamma = 1 + 2p(1 - p) \frac{W_{oo}}{E_{\text{sat}} L_1} \left\{ 1 - \frac{1}{\cosh (\pi L_2/2a)} \right\}. \tag{54}$$

For $L_2 = 0$, γ becomes unity, and the original van der Ziel expressions are obtained.

From the potential perturbation ΔW given by (52) and (53), van der Ziel calculated the induced charge ΔQ in Region I. By tracing the additional factor of γ, we obtain the new expression

$$\Delta Q = \frac{2a\rho L_1 l \Delta i_d}{I_d} [-k + \gamma w(x_o)] \tag{55}$$

where

$$k = (f_1)^{-1} [-\tfrac{1}{3}(p^3 - s^3) + \tfrac{1}{6}(p^4 - s^4) + (s^2 - \tfrac{2}{3}s^3)(p - s)] + \gamma p. \tag{56}$$

We next have to add the induced charge in Region II which results from a change Δi_d of the current flowing through the conducting channel. Since the velocity of the carriers in Region II is saturated, the additional current Δi_d requires additional carriers with a charge per unit length of $(\Delta i_d)/u$, where u is the saturated velocity. The induced charge on the gate has the opposite sign and is therefore given by $-(\Delta i_d L_2)/u$. This additional charge can be added to (55) to give a revised expression ΔQ_{rev}, as seen by

$$\Delta Q_{\text{rev}} = \frac{2a\rho L_1 l \Delta i_d}{I_d} [-k' + \gamma w(x_o)] \tag{57}$$

where

$$k' = k + \frac{L_2}{L_1} (1 - p). \tag{58}$$

Equations (57) and (58) become evident when one remembers that $I_d = 2a(1 - p) l \rho u$.

Finally, we can proceed to the revised expression for the square of the induced charge. However, we have to remember that the noise temperature is field dependent, as given in (39). Van der Ziel's expression, (21) in [4], can thus be rewritten and extended by multiplying it by B^2

because of our modification of (51), by replacing his factor $[-p + (W_o(x_o)/W_{oo})^{1/2}]$ by $(-k' + \gamma w)$, and by modifying his temperature T by (39), as seen by the equation

$$\overline{\Delta Q_{\text{rev}}^2} = \frac{64kT_o\Delta f}{g_o/L_1}\frac{L_1^2\epsilon^2}{a^2} lB^2(f_1)^{-3} \cdot\left[1 + (f_1)^3\delta\left(\frac{W_{oo}}{E_{\text{sat}}L_1}\right)^3\frac{1}{(1-w)^3}\right] \cdot(k' - \gamma w)^2(1-w)^2 2w\Delta w. \tag{59}$$

The total mean square of the charge fluctuation may be obtained by integrating (59) over w between the limits of s and p. The total mean square of the gate current simply follows by multiplying the resulting expression by ω^2, seen as

$$\overline{i_{g1}^2} = \frac{64kT_o\Delta f}{g_o/L_1}\frac{L_1^2\epsilon^2\omega^2}{a^2} lB^2(R_o + R_\delta). \tag{60}$$

The notation i_{g1} is chosen to designate the total gate noise current due to all sources in Region I. The expressions R_o and R_δ follow from integration of (59) and are given by

$$R_o = (f_1)^{-3}\{(k')^2(p^2 - s^2) - \tfrac{4}{3}k'(k'+\gamma)(p^3 - s^3) + \tfrac{1}{2}[(k')^2 + 4k'\gamma + \gamma^2]\cdot(p^4 - s^4) - \tfrac{4}{5}(k'\gamma + \gamma^2)(p^5 - s^5) + (1/3g^2)(p^6 - s^6)\} \tag{61}$$

$$R_\delta = \delta\left(\frac{W_{oo}}{E_{\text{sat}}L_1}\right)^3\left[-2(k'-\gamma)^2\left(p - s + \ln\frac{p-1}{s-1}\right) + (2k'\gamma - \gamma^2)(p^2 - s^2) - \tfrac{2}{3}\gamma^2(p^3 - s^3)\right]. \tag{62}$$

For an asymmetric transistor, $\overline{i_{g1}^2}$ in (60) first has to be divided by a factor of 2 since there is only one gate. Furthermore, if we introduce $g_o(\text{asym})$, where $g_o(\text{symm}) = 2g_o(\text{asym})$, then the numerical factor of 64 in (60) has to be changed to 16.

VII. DIFFUSION NOISE IN VELOCITY SATURATED PINCH-OFF REGION

In this section, we shall investigate the noise mechanism in the saturated drift region of the transistor. The noise mechanism can be described in several different ways. For example, the impedance field method of Shockley *et al.* [7] is suitable to describe the noise in the pinch-off region. We shall look here at the problem in a somewhat different way.

We are dealing here with a nonequilibrium situation, and the normal Johnson noise formulas are not valid. As has been shown by Shockley [7] and van der Ziel *et al.* [14], [15], the basic noise current density source may be written as

$$\overline{J_n(x)J_n^*(x')} = \frac{4q^2Dn\Delta f}{A}\delta(x - x'). \tag{63}$$

In (63), $J_n(x)$ is the noise current density at x, D is the diffusion constant of the carriers at the high dc electric field existing in the sample, n is the carrier density, A is the cross-sectional area, and $\delta(x - x')$ is the Dirac delta function. By using the Einstein relation $D = (kT/q)\mu$, with μ = mobility, (63) goes into the conventional Johnson noise formula. The Einstein formula, however, is only valid at low fields.

The noise current J_n may be interpreted as a distribution of spatial and time impulses that are mutually uncorrelated. Indeed, (63) can be rewritten as

$$\overline{(AJ_n(x))^2} = \frac{4q^2DnA\Delta f}{\Delta x} \tag{64}$$

for the autocorrelation of the current density at the point x, where we have used the fact that $\delta(x - x') = 1/\Delta x$ for $x = x'$. We now compare (64) with the shot noise expression

$$\overline{i^2} = 2qI_o\Delta f. \tag{65}$$

The shot noise spectrum is the result of current impulses of content q occurring at a rate $r = I_o/q$.

Similarly, (64) may be interpreted as a sequence of current impulses occurring at the rate

$$r = \frac{2DnA}{\Delta x} \tag{66}$$

where each current impulse extends over the distance Δx. Each of the current impulses thus results in a displacement of a charge q over a distance Δx, or, in other words, each current impulse leads to the formation of a dipole layer of strength $(q\Delta x)/A$. Since these current impulses occur in the saturated velocity regime, where we neglect mobility and diffusion effects, the resulting dipole layers are unable to relax, and they drift unchanged to the drain contact. We shall use this interpretation for the analysis of the diffusion noise in the field-effect transistor. The interpretation of high-field noise in FET's as diffusion noise also has been suggested by van der Ziel [14].

VIII. THE DIFFUSION-NOISE INDUCED OPEN-CIRCUIT DRAIN VOLTAGE

We have identified the diffusion noise as spontaneous generation of dipole layers $(q\Delta x)/A$, generated at the rate $r = (2DnA)/\Delta x$, which drift from their generation point $x_0 > 0$ to the drain contact at $x = L_2$. In this section, we take the origin of x at the pinch-off point. Under ac open-circuit drain conditions, fluctuating fields and potentials at the pinch-off point, generated by the drifting dipole layers, must be nullified by drain voltage fluctuations so that a constant dc current is maintained.

We first calculate the potential and field of a dipole layer in the channel. For this purpose, it is advantageous to consider first the potential due to a charge layer of density σ at $x = x_o$ extending from $y = -b$ to $y = +b$. As may be verified, the corresponding potential Φ is given by

$$\Phi = \frac{\sigma}{\epsilon a} \sum_{n=1,3,5}^{\infty} \left(\frac{2a}{n\pi}\right)^2 \sin \frac{n\pi}{2a} b \cos \frac{n\pi}{2a} y \exp \pm \frac{n\pi}{2a} (x - x_o). \tag{67}$$

At the metal gate electrodes, the potential is zero as required by the boundary conditions. The plus sign applies to the region $x < x_o$ and the minus sign to the region $x > x_o$. The potential Ψ due to a dipole layer consisting of a charge density $+\sigma$ at x_o and $-\sigma$ at $x_o + \Delta x_o$ is obtained from the preceding by differentiation, as seen by

$$\Psi = -\Delta x_o \frac{\partial \Phi}{\partial x_o} = \pm \frac{\sigma \Delta x_o}{\epsilon a} \sum_{n=1,3,5}^{\infty} \frac{2a}{n\pi} \sin \frac{n\pi}{2a} b \cos \frac{n\pi}{2a} y \cdot \exp \pm \frac{n\pi}{2a} (x - x_o). \tag{68}$$

In (68), the plus sign applies for $x < x_o$ and the minus sign for $x > x_o$. The x component of the electric field due to the dipole layer becomes

$$E_x = -\frac{\sigma \Delta x_o}{\epsilon a} \sum_{n=1,3,5}^{\infty} \sin \frac{n\pi}{2a} b \cos \frac{n\pi}{2a} y \exp \pm \frac{n\pi}{2a} (x - x_o). \tag{69}$$

Next, we match boundary conditions at the pinch-off point. In keeping with the approximations already made in evaluating the small-signal parameters of the transistor, we retain only the fundamental components of potential and field. The potential at the center of the pinch-off point in Region I is $-W_{oo}(p^2 - s^2)$. This potential is consistent with the solution of Poisson's equation in the transverse direction taking into account the space charge of the ionized acceptors. In Region II at $x = 0$, we also have the potential $-W_{oo}(p^2 - s^2)$ due to the particular solution of Poisson's equation taking into account the ionized acceptor and mobile hole space charges. In addition, we have the potential due to charges or voltages on the drain contact. As before, they are of the form $\alpha \cos (\pi y/2a) \exp (\pi x/2a)$ and $\beta \cos (\pi y/2a) \exp -(\pi x/2a)$. Finally, we have the potentials and fields of the dipole layer at $x = x_o$. Thus matching of the potential at the pinch-off point $y \approx 0$, $x = 0$ gives

$$-W_{oo}(p^2 - s^2) = -W_{oo}(p^2 - s^2) + \frac{\sigma \Delta x_o}{\epsilon} \frac{2}{\pi} \sin \frac{\pi b}{2a} \cdot \exp \frac{-\pi x_o}{2a} + \alpha + \beta. \tag{70}$$

Similarly, matching the electric field gives

$$E_{\text{sat}} = \frac{-\sigma \Delta x_o}{\epsilon a} \sin \frac{\pi b}{2a} \exp \left(-\frac{\pi x_o}{2a}\right) - \frac{\pi}{2a} \alpha + \frac{\pi}{2a} \beta. \tag{71}$$

As stated, we are calculating open-circuit noise voltages. The quantities α and β adjust themselves to cancel out the effect of the dipole layer. Calling the $\delta\beta$ and $\delta\alpha$ the perturbations of α and β due to the dipole layer, we obtain from (70) and (71)

$$\delta\alpha = -\frac{\sigma \Delta x_o}{\epsilon} \frac{2}{\pi} \sin \frac{\pi b}{2a} \exp - \frac{\pi x_o}{2a}$$

$$\delta\beta = 0. \tag{72}$$

The potential perturbation at the drain contact $x = L_2$ due to the dipole layer is, therefore,

$$\Delta v_{d2} = -\frac{\sigma \Delta x_o}{\epsilon} \frac{2}{\pi} \sin \frac{\pi b}{2a} \exp \frac{\pi (L_2 - x_o)}{2a}. \tag{73}$$

In (73), we have omitted the contribution of the potential from (68) for $x > x_o$. Due to an image dipole layer at $x = L_2 + (L_2 - x_o)$ beyond the drain contact, this portion of the potential approximately gets cancelled out. The notation v_{d2} again signifies that the drain voltage is due to a source in Region II.

Now, the dipole layer drifts with a constant speed u from x_o, starting at its instant of generation t_o. Therefore, (73) gives a time-dependent induced voltage

$$\Delta v_{d2}(x_o, t - t_o) = -\frac{\sigma \Delta x}{\epsilon} \frac{2}{\pi} \sin \frac{\pi b}{2a} \exp \frac{\pi}{2a} \cdot [L_2 - x_o - u(t - t_o)] \tag{74}$$

where

$$0 \leq t - t_o \leq (L_2 - x_o)/u.$$

Next, we want to calculate the spectrum due to the drifting dipole layers. A random process of uncorrelated events, occurring at the rate r, each having a response $h(t)$, produces a spectral density

$$\phi(\omega) = \frac{1}{2\pi} r \mid H(\omega) \mid^2 \tag{75}$$

where $H(\omega)$ is the Fourier transform of $h(t)$, as seen by

$$H(\omega) = \int_{-\infty}^{+\infty} \exp (-i\omega) \; h(t) \; dt. \tag{76}$$

Since we are not interested in frequencies where the transit time through the pinch-off channel becomes comparable to $1/\omega$, we can evaluate (76) in the limit of $\omega \rightarrow 0$.

We obtain with (74), using $\sigma = q/A = q/(2bl)$,

$$H(\omega)_{\omega \to 0} = -\frac{1}{\pi} \frac{q \Delta x_o}{\epsilon b l} \sin \left(\frac{\pi b}{2a}\right) \cdot \frac{2a}{\pi u} \left[\exp \frac{\pi}{2a}(L_2 - x_o) - 1\right]. \tag{77}$$

Finally, we may obtain the drain voltage fluctuations due to dipole layers starting between x_o and $x_o + \Delta x_o$ by inserting (77) and (66) into (75) as seen by the equation

$$\overline{\Delta v_{d2}^2(x_o)} = \frac{32a^2}{\pi^4 u^2}\frac{q^2 Dn\Delta f}{\epsilon^2 bl}\sin^2\frac{\pi b}{2a} \cdot \left[\exp\frac{\pi}{2a}(L_2 - x_o) - 1\right]^2 \Delta x_o. \quad (78)$$

Equation (78) contains an additional factor of 2, since we do not distinguish between positive and negative frequencies as in (76) but consider the total fluctuations in a bandwidth $\Delta f = \Delta\omega/2\pi$. The total drain voltage fluctuations due to dipole layers in Region II follows from an integration over x_o from 0 to L_2. If we introduce the dc current I_d, seen by

$$I_d = qun2bl \quad (79)$$

then, we can finally write

$$\overline{v_{d2}^2} = I_d \frac{16}{\pi^5}\sin^2\left(\frac{\pi b}{2a}\right)\frac{qD\Delta f a^3}{\epsilon^2 b^2 l^2 u^3}\left[\exp\frac{\pi}{a}L_2 - 4\exp\frac{\pi}{2a}L_2 + 3 + \frac{\pi L_2}{a}\right]. \quad (80)$$

For the asymmetric transistor with I_d referring to the current flowing in the one-sided transistor, the numerical factor of 16 in (80) has to be replaced by 64.

IX. DIFFUSION NOISE INDUCED DRAIN AND GATE CURRENTS

As outlined in the previous two sections, the diffusion-noise contribution can be attributed to dipole layers drifting through Region II. Since the dipole layers carry no net charge, there is no directly induced charge on the gate. There is, however, an indirect contribution. To be consistent, we have to evaluate the induced gate currents under short-circuit drain conditions. The noise voltage calculated in Section VIII produces under these short-circuit conditions an induced drain current. This drain current causes a breathing of Channels I and II and, therefore, an induced charge on the gate.

Let us call the induced drain current i_{d2}, where

$$i_{d2} = \frac{v_{d2}}{r_d}. \quad (81)$$

We have already calculated, in essence, the induced charge in the gate due to a current change Δi_d. We can adapt (52)–(57) by simply omitting the potential jump at x_o not needed in the present context. As may be seen from (52) and (53), there is no potential jump or discontinuity at $x = x_o$, if we set the parameter γ equal to zero. We can, therefore, take over (57) with the proviso that γ is set equal to zero also within the parameter k', defined in (56) and (58). We thus obtain from (57), for the induced charge Q_2,

$$Q_2 = -\frac{2a\rho L_1 l i_{d2}}{I_d}k'(\gamma = 0). \quad (82)$$

We obtain, finally, the mean-square gate noise current $\overline{i_{g2}^2}$, due to fluctuations in Region II, by taking the mean square of (82), multiplying it by ω^2, and inserting (80) and (81), as seen by

$$\overline{i_{g2}^2} = \frac{1}{r_d^2}\frac{16}{\pi^5}\sin^2\left(\frac{\pi b}{2a}\right)\frac{qI_d D\Delta f a^3 L_1^2\omega^2}{\epsilon^2 b^2 l^2 u^5}\frac{(k'(\gamma = 0))^2}{(1-p)^2} \cdot \left[\exp\frac{\pi}{a}L_2 - 4\exp\frac{\pi}{2a}L_2 + 3 + \frac{\pi L_2}{a}\right]. \quad (83)$$

For the asymmetric transistor, the numerical value of $\overline{i_{g2}^2}$ is half as large as given by (83). If we simultaneously introduce into (83) the drain resistance and the drain current of the asymmetric transistor, then the numerical factor $16/(\pi^5)$ has to be changed to $64/(\pi^5)$.

X. EVALUATION OF CORRELATION COEFFICIENT

The gate noise currents and the drain noise currents are correlated to some extent, since any elementary noise voltage produced in the channel gives a contribution to both the drain and the gate currents. As before, let us call i_{g1} and i_{d1} the gate and drain currents produced by a fluctuation in the ohmic Region I, and, similarly, let us call i_{g2} and i_{d2} the currents produced by a fluctuation in Region II. Obviously, there is no cross correlation between i_{d1} and i_{g2} or i_{d2} and i_{g1}. The derivation in Section IX shows that there is a very strong correlation between i_{g2} and i_{d2}. Obviously, a drain voltage fluctuation produced by Region II gives rise to a drain current which can be evaluated using Ohm's law and the value of the drain resistance. Similarly, the gate current follows from the breathing of the channel due to drain current. There is a capacitative 90° phase shift between gate and drain currents with full correlation. In other words,

$$\overline{i_{g2}^* i_{d2}} = j[\overline{i_{g2}^2}\cdot\overline{i_{d2}^2}]^{1/2}. \quad (84)$$

Similarly, we can evaluate the correlation between i_{g1} and i_{d1} following closely the treatment by van der Ziel [4]. Because of our boundary conditions at the end of Region I, we have to multiply van der Ziel's equation (20) in [4] by B^2; furthermore, we have to replace his expression $p - u^{1/2}$ by our $k' - \gamma w$ in exact analogy to (57). Finally, we have to insert the field dependent noise temperature of (39) in full analogy with our treatment in Section V. After integration over the reduced channel potential, we obtain the expression

$$\frac{\overline{i_{g1}^* i_{d1}}}{[\overline{i_{g1}^2}\cdot\overline{i_{d1}^2}]^{1/2}} = j\frac{S_o + S_\delta}{(R_o + {}_\delta)^{1/2}(P_o + P_\delta)^{1/2}} \quad (85)$$

where

$$S_o = (f_1)^{-2}\{k'[(p^2 - s^2) - \tfrac{4}{3}(p^3 - s^3) + \tfrac{1}{2}(p^4 - s^4)] + \gamma[-\tfrac{2}{3}(p^3 - s^3) + (p^4 - s^4) - \tfrac{2}{5}(p^5 - s^5)]\} \quad (86)$$

and

$$S_\delta = 2\delta \left(\frac{W_{oo}}{E_{sat}L_I}\right)^3 f_1 \left[(k' - \gamma)\left(s - p + \ln\frac{1-s}{1-p}\right) + \tfrac{1}{2}\gamma(p^2 - s^2)\right]. \quad (87)$$

The overall correlation coefficient defined in (28) can then finally be written as

$$C = \frac{1}{j}\frac{\overline{i_g{}^* i_d}}{[\overline{i_g{}^2}\,\overline{i_d{}^2}]^{1/2}} = \frac{S_o + S_\delta}{(R_o + R_\delta)^{1/2}(P_o + P_\delta)^{1/2}} \cdot \left(\frac{\overline{i_{g1}{}^2}}{\overline{i_g{}^2}}\right)^{1/2}\left(\frac{\overline{i_{d1}{}^2}}{\overline{i_d{}^2}}\right)^{1/2} + \left(\frac{\overline{i_{g2}{}^2}}{\overline{i_g{}^2}}\right)^{1/2}\left(\frac{\overline{i_{d2}{}^2}}{\overline{i_d{}^2}}\right)^{1/2}. \quad (88)$$

In (88), we understand by i_g and i_d the total gate and drain noise currents produced by fluctuations in Regions I and II.

XI. NOISE FIGURE, EXPERIMENTAL DATA, AND DISCUSSION

A test of our noise theory may be made by applying it to a practical device for which experimental data are available. A proper test of the theory, however, must include the thermal noise contributions of the parasitic elements associated with a practical device. Unfortunately, the experimental noise data on GaAs FET's are often not accompanied with sufficient information about the parasitics, so that we are limited at this time to applying our theory to one specific device design for which the published information is adequate.

The equivalent circuit model for the FET which includes the noisy parasitic elements is shown in Fig. 4. The intrinsic device parameters are included in the shaded box. The capacitances C_{sg} and C_{dg} are, respectively, the gate–source and gate–drain capacitances. The latter is of the order of a few percent of the former and will be neglected in our noise analysis. The parameter R_i is an intrinsic resistance in the undepleted region of the channel under the source side of the gate and represents the ohmic path for the displacement current between the gate and source. Its ohmic losses are included in the gate noise source i_g [16]. The parameter r_d is the drain resistance evaluated in Section III. Shunting it is the drain noise source. The transadmittance y_m of the current generator of the intrinsic transistor shown in Fig. 4 can be approximated by the form $y_m = g_m \exp(-j\omega\tau)$, where $\omega\tau$ is a small phase shift [2], [17]. We shall neglect this phase shift in our analysis.

The extrinsic thermal noise sources of importance are those associated with the gate metallization resistance R_m and the source–gate resistance R_f which includes the source contact resistance and the bulk resistance of the epitaxial layer between the gate and source contacts. Both R_m and R_f show full thermal noise as indicated in Fig. 4.

The gate and drain intrinsic noise sources will be represented in terms of equivalent noise conductances g_{gn} and g_{dn}, respectively, where

$$g_{gn} = \frac{\overline{i_g{}^2}}{4kT_o\Delta f} \quad (89)$$

$$g_{dn} = \frac{\overline{i_d{}^2}}{4kT_o\Delta f}. \quad (90)$$

The noise figure can be evaluated by short circuiting the output (drain-ground) terminals and evaluating the noise current contributions of each noise generator, including that of the source e_s. Using the expression for the noise generators in Fig. 4, we obtain the expression for the noise figure, as seen by

$$F = 1 + \frac{1}{R_s}[r_n + g_n |Z_s + Z_c|^2] \quad (91)$$

where $Z_s = R_s + jX_s$ is the source impedance and r_n, g_n, and Z_c are noise parameters whose expressions will be given later. As a function of the source impedance, F exhibits a minimum when Z_s has its optimum value $Z_{s,\,\mathrm{opt}}$ given in terms of its components, by

$$R_{s,\,\mathrm{opt}} = \left((\mathrm{Re}\,Z_c)^2 + \frac{r_n}{g_n}\right)^{1/2} \quad (92)$$

$$X_{s,\,\mathrm{opt}} = -\mathrm{Im}\,Z_c. \quad (93)$$

The minimum value of F can be expressed as

$$F_{\min} = 1 + 2g_n(\mathrm{Re}\,Z_c + R_{s,\,\mathrm{opt}}). \quad (94)$$

This expression will serve as the basis of our discussion which follows. The noise quantities g_n, R_n, and Z_c are expressible in terms of the equivalent circuit parameters as

$$g_n = \left|\frac{y_{11}}{y_{21}} g_{dn} - jC(g_{gn})^{1/2}\right|^2 + (1 - C^2)g_{gn} \quad (95)$$

$$r_n = R_m + R_f + \frac{g_{dn}}{|y_{21}|^2}\frac{(1-C^2)g_{gn}}{g_n} \quad (96)$$

$$Z_c = R_m + R_f + \frac{g_{dn}y_{11}{}^*/|y_{21}|^2 + jC(g_{gn}g_{dn})^{1/2}/y_{21}}{g_n}. \quad (97)$$

Here

$$y_{11} = \frac{j\omega C_{sg}}{1 + j\omega C_{sg}R_i}, \qquad y_{21} \approx \frac{g_m}{1 + j\omega C_{sg}R_i} \quad (98)$$

where y_{11} and y_{21} are, respectively, the short-circuit input and transfer admittance of the intrinsic device. (We have neglected the small effort of the drain-gate capacitance C_{dg} on y_{11} and y_{21}.) Note that for vanishing values of R_i, R_m, and R_f, the expression for $F_{\min}$ and $Z_{s,\,\mathrm{opt}}$ reduce to those given in Section IV when one sets $Y_{s,\mathrm{opt}} = Z_{s,\mathrm{opt}}{}^1 - 1$.

The parasitic resistances R_m and R_f appear as a series

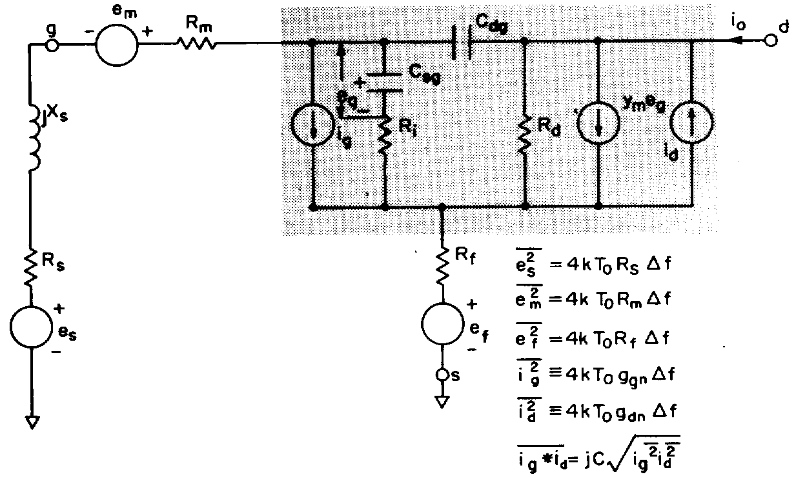

Fig. 4. Equivalent circuit for minimum noise figure calculation with parasitic elements included.

combination in both the equivalent noise resistance r_n and the correlation impedance Z_c, but not in the equivalent noise conductance g_n. One may show that the noise contributions of the parasitics enter in r_n, whereas the ohmic properties of R_m and R_f are relevant in Z_c. Since both Z_c and r_n appear in the expression for $R_{s,\text{opt}}$, the optimum source resistance is a strong function of the parasitic resistances. The ohmic property of the intrinsic resistance R_i also plays a role in $R_{s,\text{opt}}$ via the real part of Z_c.

We have applied these expressions to a practical n-type GaAs FET with a 2-μm gate length whose published design values stated by Brehm and Vendelin are [18], [19]: $a = 2 \times 10^{-5}$ cm, $l = 2.85 \times 10^{-2}$ cm, $L = 2 \times 10^{-4}$ cm, $g_o = \sigma_o a = 1.4 \times 10^{-3}$ mhos (corresponding to a doping level $N_d = 10^{17}$ cm^{-3}), and $W_{oo} = 2.9$ V. The values of the parasitic resistances used are $R_f = 15$ ohms and $R_m = 0.8$ ohms [19]. This design closely parallels geometries studied at other laboratories including our own. At this doping level, the saturation field is approximately 3 kV/cm. Using a value slightly lower, 2.9 kV/cm, and a low field mobility of 4500 cm^2/V·s, the piecewise-linear approximation to the velocity field characteristic in Fig. 2 requires a saturation velocity $u = 1.3 \times 10^7$ cm/s, only slightly higher than the actual value [8].

Evaluation of the noise conductances g_{gn} and g_{dn}, (89) and (90), appearing in the noise figure expression requires a knowledge of the dummy variables (s,p). To obtain solutions for this pair as a function of the external drain-source and gate-source potentials, it was necessary for us to take into account all of the parasitic dc voltage drops in the source–drain region, as well as the self-biasing of the gate due to the voltage drop across R_f produced by the dc drain current. Using our dc and small-signal analysis outlined in Section II and III, and including the parasitic resistances in the source–drain circuit, we have written a computer program which yields values of the variables (s,p) as a function of gate and drain potentials. As a byproduct, I–V characteristics and small-signal parameters were also obtained. For example, at a drain–source bias of 3 V, and a gate-source bias of 0 V, we obtain for the saturation drain current I_{dss}, a value of 28.3 mA and for the transconductance a value of 19 mmhos, which are in very good agreement with Brehm's data [19]. We have also obtained excellent agreement with measurements taken on a variety of device geometries fabricated at our laboratory.

It might be of interest to point out that our computer results show for this device design that for drain–source bias values corresponding to current saturation, i.e., above the "knee" of the I_d–V_d curves, the extent of the saturated velocity region L_2 is of the order of 30 percent of the gate (channel) length l. For shorter gate lengths (of the order of 1 μm), this fraction can reach as high as 90 percent. The extent of this region increases with the source–drain potential but is nearly independent of the source–gate bias. Furthermore, the channel opening under current saturated conditions is almost uniform along the entire length of the gate, that is $s \approx p$, the difference being of the order of 10 percent or less. This means that the longitudinal drop along the channel in Region I is negligible compared to the corresponding drop in Region II. The values (s,p) are strong functions of the gate–source potential, but nearly independent of the source–drain bias. For the device under consideration, typical values of s and p range from 0.65–0.75 at zero gate bias, increasing towards unity at low drain currents.

Knowledge of the (s,p) pairs, obtained from the numerical solutions, also allows us not only to calculate the small-signal parameter g_m, which appears in the noise figure expression, but also the small-signal parameter, C_{sg}, which was obtained by a small-signal analysis of the charge induced on the gate by the field of the depleted impurity charges in the channel and by the Laplace field given by (14). Using the approximation $s = p$, we obtain the simplified expression for $L_2 \approx L_s$

$$C_{sg} \approx \frac{\epsilon l L}{ap} + 1.56\epsilon l. \tag{99}$$

The first term is simply that of a "parallel plate" capacitor of thickness ap. The second term is a fringing field correction [20] for either edge of the gate. The fringing accounts for about 10 percent of the total capacitance for the device being considered but can be as high as 25 percent of the total capacitance for a shorter gate.

We have postulated for the gate charging resistance R_i a bias dependence inverse to that of C_{sg}, so that the product R_iC_{sg} is approximately constant. There are good theoretical grounds for this assumption since it can be shown that this time constant is proportional to (though smaller than) the channel transit time $\tau \approx L/u$. We have set this time constant equal to 4.0 ps corresponding to the published data [19].

As a preliminary to our noise figure presentation, we illustrate in Fig. 5 the calculated dependence of the noise conductances g_{gn} and g_{dn} on the gate bias, or more precisely on the normalized drain current I_d/I_{dss} for a particular value of the high field diffusion constant, D. For other values of D our computations show that these conductances are proportional to D except at the low end of the drain current scale. Also shown in Fig. 5 is the magnitude of the correlation coefficient, C. Note that C is nearly constant in value. (For a 1-μm gate, C approaches unity in value over most of the current range.) This fact (which is the result of the strong correlation between i_{g2} and i_{d2} in the expression for C, as seen in (88) and the linear dependence of the noise conductances on D show that the dominant source of noise in the device, at least in the saturated current regime, is the diffusion noise of the saturated velocity region. We have used for the hot electron temperature parameter δ, a value of 1.2 based on Baechtold's experiments, and our choice of E_{sat}.

Fig. 6 illustrates the dependence of $F_{\min}$ on the normalized drain current for a particular value of source–drain potential and several values of the high-field diffusion constant. Also shown are the experimental results for the two devices reported by Brehm and Vendelin [19]. Notice that our theoretical results exhibit all of the features of the experimental data, in particular, the rapid rise of $F_{\min}$ at high drain currents, a feature not shared by other noise theories. This rapid rise in $F_{\min}$ at high drain currents and the strong dependence on the diffusion constant is a reflection of the influence of diffusion noise on the noise conductances, as illustrated in Fig. 5. The fast increase in $F_{\min}$ to the left of the minimum is a consequence of the drop-off in transconductance at low drain currents.

We also show in Fig. 6 the value of $F_{\min}$ in the absence of parasitics ($R_m = R_f = 0$) for one of the D values. The low level derives from the strong cancellation of noise due to correlation as predicted by (33). The effectiveness of this noise cancellation is diminished drastically when parasitic elements are introduced, hence the large increase in $F_{\min}$. It is obvious then that parasitic resistances should be kept at a minimum, not only because they introduce noise of their own, but also because they indirectly cause an increase in the contributions of the noise sources associated with the intrinsic device.

The minimum noise figure is only a mild function of the source–drain potential in the current saturation region as

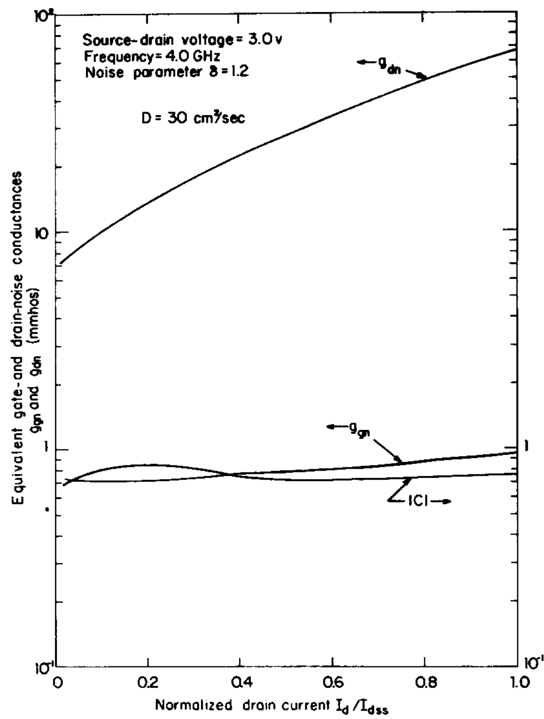

Fig. 5. Equivalent drain noise conductance g_{dn} and equivalent gate noise conductance g_{gn} and correlation coefficient as a function of normalized drain current.

illustrated by the experimental results in Fig. 7 [18], [19]. Our theory also shows this same dependence in good agreement with the data.

The important feature to observe in Figs. 6 and 7 is that to obtain noise figures corresponding to experimental values, one must use a diffusion coefficient in the vicinity of D = 30–40 cm^2/s, substantially lower than the low-field value of D = 110 cm^2/s. This observation conforms to the fact that theoretically, D shows a sharp drop off at high electric fields to levels in general agreement with the range indicated above [9]. More experimental data are necessary, of course, to permit us to make a more specific choice of D. Note that for $D \approx 35$ cm^2/s our theory provides excellent agreement with experiment.

We have also applied our theory to devices with shorter gate lengths (1 μm) operating at X band. Although our agreement with the experimental results is not quite as good (we feel because of unaccounted circuit losses and some still inaccurate device parameter values), nevertheless diffusion coefficients in the range above are indicated.

Summarizing, the present paper shows that the saturated velocity region in field-effect transistors cannot be neglected. To date, only limited experimental data exist on the noise behavior of GaAs FET's, but it apears that these data are in good agreement with our calculations. While we have not shown plots of $R_{s,\text{opt}}$ and $X_{s,\text{opt}}$, nevertheless, their calculated values are also in the range of the observations. It will be necessary to compare more extensive experimental measurements with our theory to establish its validity and limitations. Extensive definitive experiments are now in progress at our laboratory to obtain the necessary data.

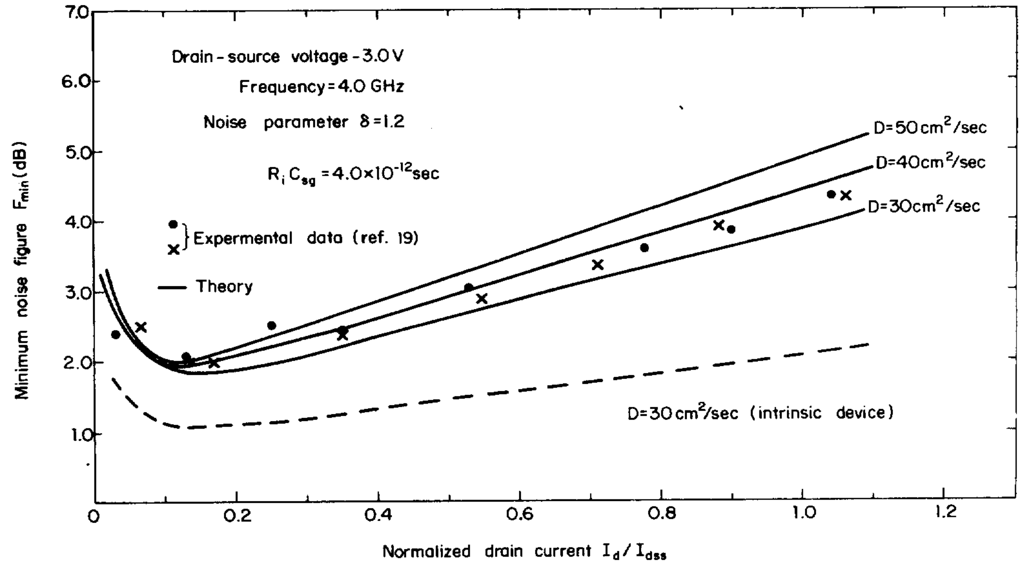

Fig. 6. Minimum noise figure as a function of normalized drain current for various diffusion constants.

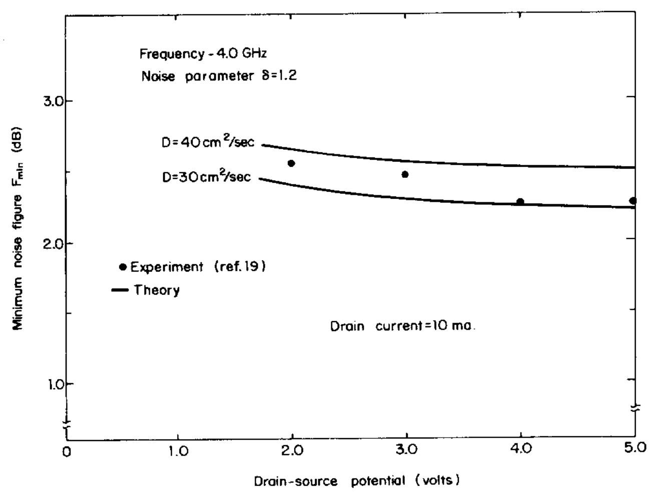

Fig. 7. Minimum noise figure as a function of source–drain potential for a particular diffusion constant.

APPENDIX I

In this Appendix, we derive the noise voltage at L_1 due to Johnson noise within an infinitesimal section of channel. We shall follow van der Ziel's analysis. The width of the ohmic channel, $2b$, is given by

$$2b = 2a[1 - (W/W_\infty)^{1/2}]. \tag{I.1}$$

Taking a perturbation, one obtains

$$\Delta W = -\frac{2}{a}(W_\infty W)^{1/2}\Delta b. \tag{I.2}$$

Noting that the current is kept constant and the longitudinal field is dW/dx, one has

$$I_d = 2\sigma_o b\frac{dW}{dx} = 2\sigma_o(b_o + \Delta b)\left(\frac{dW_o}{dx} + \frac{d\Delta W}{dx}\right)$$

$$= 2\sigma_o b_o\frac{dW_o}{dx}. \tag{I.3}$$

One obtains to first order in the variation

$$\Delta b\frac{dW_o}{dx} + b_o\frac{d\Delta W}{dx} = 0. \tag{I.4}$$

Substituting (I.2) into (I.4) and introducing the normalized dc voltage seen in (36), one obtains

$$\frac{d\Delta W}{\Delta W} = \frac{dw}{1-w}. \tag{I.5}$$

Integration between the limits x and L_1 gives

$$\Delta W_p = -\frac{1-w(x)}{1-p}\Delta W_x \tag{I.6}$$

where p is the value of w at the end of the ohmic region, $-\Delta W_x$ is the fluctuation in the section Δx, and ΔW_p is the resulting fluctuation at the pinch-off point L_1. (I.6) is equation (10) of [3]. ΔW_x is caused by thermal noise and its mean-square value is (as seen by (11), [3])

$$\Delta W_x^2 = 4kT\Delta f\frac{\Delta x}{2\sigma_o bl} = \frac{4kT\Delta f}{I_d}\Delta W_o \tag{I.7}$$

where

$$I_d = 2\sigma_o bl\frac{dW_o}{dx} = 4\sigma_o al(1-w)W_{\infty}wdw \tag{I.8}$$

and dW_o/dx is the dc electric field at x. Introducing (I.7) into the square of (I.6), one obtains (35) of the text.

APPENDIX II

The induced charge on the gate is obtained in the same way as the one employed by van der Ziel, part of whose derivation is reproduced in the following equations. We supplement van der Ziel's equations to account for the presence of Region II.

The distributed negative charge stored in the depleted region above and below the ohmic channel segment of length dx is

$$Q'\,dx = -2(a-b)\rho_o\,dx = -2a\rho_o\left(1-\frac{b}{a}\right)dx. \quad \text{(II.1)}$$

This charge is compensated by an equal and opposite charge $Q\,dx = -Q'\,dx$, on the gate contact. Therefore, the fluctuation Δb in b between x and $x + \Delta x$ produces a fluctuating charge

$$d\Delta Q = -2a\rho_o\left(\frac{\Delta b}{a}\right)dx. \quad \text{(II.2)}$$

The problem is now to compute the perturbation of ohmic channel width $\Delta b/a$ as a function of position. This perturbation exists ahead of $(x > x_o)$ of the perturbing potential ΔW_x at x_o as well as behind it. Ahead of x_o there is the effect of the change in current Δi_d; behind x_o there is the combined influence of Δi_d and ΔW_x. Because the current is given by (I.3), we obtain

$$I_d + \Delta i_d = 2\sigma_o l(b_o + \Delta b)\,\frac{d}{dx}\,(W_o + \Delta W). \quad \text{(II.3)}$$

Substituting (I.1), (I.2), and (36) leads to (compare equation (8) of [4])

$$\Delta i_d = 2\sigma_o al\,\frac{d}{dx}\,(1-w)\Delta W. \quad \text{(II.4)}$$

Integrating this equation over x, we obtain (52) of the text after determining the integration constant such that $\Delta W = 0$ for $x = 0$. In (53), the integration constant is chosen for convenience in the form $-\Delta i_d \gamma L_1/(lg_o)$. γ is then determined in the text. The induced charge can now be calculated in Region I by inserting Δb from (I.2) into (II.2). The detailed dependence of ΔW on x is given in (52) and (53). For further details, the reader should consult [4], equations (12)–(17).

ACKNOWLEDGMENT

The authors wish to thank Mrs. Laura Spiniello and Mrs. Janet Newell who performed the extensive and, at times, exasperating computations of the various small-signal parameters and noise functions which were so helpful in our theoretical analysis.

REFERENCES

[1] W. Baechtold, K. Daetwyler, T. Forster, T. O. Mohr, W. Walter, and P. Wolf, "Si and GaAs 0.5 μm gate Schottky barrier field-effect transistors," *Electron. Lett.*, vol. 9, pp. 232–234, May 17, 1973.

[2] C. A. Liechti, E. Gowen, and J. Cohen, "GaAs microwave Schottky gate FET," *ISSCC Digest Tech. Papers*, p. 158, 1972.

[3] A. van der Ziel, "Thermal noise in field-effect transistors," *Proc. IRE*, vol. 50, pp. 1808–1812, 1972.

[4] ——, "Gate noise in field-effect transistors at moderately high frequencies," *Proc. IEEE*, vol. 51, pp. 461–467, Mar. 1963.

[5] W. Baechtold, "Noise behavior of Schottky barrier gate field-effect transistors at microwave frequencies," *IEEE Trans. Electron Devices*, vol. ED-18, pp. 97–104, Feb. 1971.

[6] ——, "Noise behavior of GaAs field-effect transistors with short gate lengths," *IEEE Trans. Electron Devices*, vol. ED-19, pp. 674–680, May 1972.

[7] W. Shockley, J. A. Copeland, and R. P. James, "The impedance field method of noise calculations in active semiconductor devices," in *Quantum Theory of Atoms, Molecules, and the Solid State*, Per-Olov Lowdin, Ed. New York: Academic Press, 1966.

[8] Y. G. Ruch and G. S. Kino, "Transport properties of gallium arsenide," *Phys. Rev.*, vol. 174, pp. 921–931, Oct. 1968.

[9] W. Fawcett and H. D. Rees, "Calculation of the hot electron diffusion rate for GaAs," *Phys. Lett.*, vol. 29 A, pp. 578–579, Aug. 11, 1969.

[10] W. Shockley, "A unipolar field-effect transistor," *Proc. IRE*, vol. 40, pp. 1365–1376, Nov. 1952.

[11] A. B. Grebene and S. K. Ghandhi, "General theory for pinched operation of the junction-gate FET," *Solid-State Electron.*, vol. 12, pp. 573–589, July 1969.

[12] F. M. Klaassen, "Comments on hot carrier noise in field-effect transistors," *IEEE Trans. Electron Devices*, vol. ED-18, pp. 74–75, Jan. 1971.

[13] A. van der Ziel, "Noise resistance of FET's in the hot electron regime," *Solid-State Electron.*, vol. 14, pp. 347–350, Apr. 1971.

[14] ——, "Thermal noise in the hot electron regime in FET's," *IEEE Trans. Electron Devices*, vol. ED-18, p. 977, Oct. 1971.

[15] A. van der Ziel and K. M. van Vliet, "HF thermal noise in space-charge limited solid-state diodes—II," *Solid-State Electron.*, vol. 11, pp. 508–509, 1968.

[16] A. van der Ziel and J. W. Ero, "Small-signal high-frequency theory of field-effect transistors," *IEEE Trans. Electron Devices*, vol. ED-11, pp. 128–135, Apr. 1964.

[17] P. Wolf, "Microwave properties of Schottky-barrier field-effect transistors," *IBM J. Res. Develop.*, vol. 14, pp. 125–141, Mar. 1970.

[18] G. E. Brehm, "Variation of microwave gain and noise figure with bias for GaAs FET's," 1973 Cornell Conf. Microwave Semiconductor Devices, Circuits, and Applications, Cornell University, Ithaca, N. Y.

[19] G. E. Brehm and G. D. Vendelin, "Biasing FET's for optimum performance," *Microwaves*, vol. 13, pp. 38–44, Feb. 1974.

[20] E. Wasserstrom and J. McKenna, "The potential due to a charged metallic strip on a semiconductor surface," *Bell Syst. Tech. J.*, vol. 49, pp. 853–877, May–June 1970.