Learning Social Circles in Ego-Networks based on Multi-View Network Structure

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Abstract—In social network analysis, automatic social circle detection in ego-networks is becoming a fundamental and important task, with many potential applications such as user privacy protection or interest group recommendation. So far, most studies have focused on addressing two questions, namely, how to detect overlapping circles and how to detect circles using a combination of network structure and network node attributes. This paper asks an orthogonal research question, that is, how to detect circles based on network structures that are (usually) described by multiple views? Our investigation begins with crawling ego-networks from Twitter and employing classic techniques to model their structures by six views, including user relationships, user interactions and user content. We then apply both standard and our modified multi-view spectral clustering techniques to detect social circles in these ego-networks. Based on extensive automatic and manual experimental evaluations, we deliver two major findings: first, multi-view clustering techniques perform better than common single-view clustering techniques, which only use one view or naively integrate all views for detection; second, the standard multi-view clustering technique is less robust than our modified technique, which selectively transfers information across views based on an assumption that sparse network structures are (potentially) incomplete. In particular, the second finding makes us believe a direct application of standard clustering on potentially incomplete networks may yield biased results. We lightly examine this issue in theory, where we derive an upper bound for such bias by integrating theories of spectral clustering and matrix perturbation, and discuss how it may be affected by several network characteristics.

Index Terms—Social Circle Detection, Privacy Protection, Multi-View Spectral Clustering, Graph Perturbation.

1 INTRODUCTION

Online social network has been rising as a new and very popular platform for modern socialization - Facebook had recorded one billion active user accounts by late 2012, with about 10 million messages posted every hour and 46% of young users checking their Facebook as a first thing in the morning1. What lies behind this tremendous popularity, on the other hand, is a rich source of network information that could be properly integrated and analyzed for better understanding and promoting the modern online socialization, fulfilling the values of social network analysis.

In social network analysis, a fundamental and important task is to detect social circles in a user’s ego-network (or, as we abbreviate as ego-net) [42]. Here, a user’s ego-net is a sub-network that contains only her friends as nodes – the user is called the ego, each friend is called an alter, and a social circle is a subset of the alters who are similar under certain measurement. As suggested in [42], social circle has many potential applications, including content filtering and group recommendation. We also notice its particular application in the privacy and HCI research communities for controlling information boundary [56], [58], in a sense that an ego could have some new posts only visible to friends in designated social circles, which could reduce the risk of revealing her (private) information to untargeted friends. Indeed, it has been shown a user’s information such as location be inferred from her posts that contain local restaurants [38] or location-indicating words like “Time Square” [9], [13].

While the notion of social circle has been commercialized in several products including the Google+ circle and the Facebook custom list, it seems not well-received by users. As argued in [42], a main reason is most products require manual labeling of these circles, which is usually tedious and labor-intensive. To push the practice of social circle, it hence remains an important task to design methods that could automatically and effectively detect them in ego-nets.

Tracing this line of research, we notice the literature has been focused on addressing two questions, namely, how to detect circles that overlap and how to detect circles based on network node attributes (e.g. [8], [43], [69]); there is also an attempt to improve circle detection in a target ego-net by leveraging circle information from other ego-nets [18]. While these studies have advanced the practice of social circle in various directions, they all consider only a single view of the network structure. In reality, however, the ego-net structure may be described by multiple views – one view may show the friend relationship between alters while another may show their interaction frequencies. This simple observation motivates us to ask an orthogonal research question in this paper, i.e. how to effectively leverage the (usually present) multiple views of ego-net structure for better social circle detection?

To investigate the question, we first crawl ego-nets from Twitter and employ classic techniques to model the ego-net structure from six views, namely, two relationship views regarding the friendship and common friends between alters, three interaction views regarding the replies, co-replies and re-tweets of alters, and one content view regarding alters’ post similarities. We do not use alter profiles (e.g. education, age or hobbies) as most studies do, considering alters may

not provide these information due to privacy concerns.

Then, we examine and compare several clustering tech-
niques in their performance of detecting social circles based
on the constructed multi-view ego-net structure. The exami-
nation includes a most common single-view clustering tech-
nique based solely on the friendship view [68], a benchmark
single-view clustering technique that naively integrates all
views into one and performs clustering, a standard multi-
view clustering technique that fully transfers information
across views [33], and our modification of this technique
which now selectively transfers information across views.
Based on extensive experimental evaluations, we have come
to two major findings: first, multi-view techniques gener-
ally outperform single-view techniques in the qualities of
detected circles; second, our modified multi-view clustering
technique outperforms the standard multi-view technique.

The second observation raises our particular interest, as
it suggests more careful interpretation and treatment of the
sparse ego-net structures. Indeed, we have observed that 1)
some views of an ego-net structure are very sparse and 2)
our modified multi-view technique that selectively transfers
information from sparse views to other views outperforms
the standard multi-view technique that fully transfers infor-
mation across views. Our conjecture for the co-occurrence of
both phenomena is that the sparse ego-net structures may
in fact be interpreted as incomplete structures (e.g. due to
the limited time for data collection), and standard clustering
techniques that ignore such ‘hidden’ incompleteness may
output a result which deviates significantly from the optimal
one. Note we are not blaming the sparseness for the perfor-
ance degeneration, but the incompleteness that induces
the sparsity. To better understand the issue, we also derive a
performance deviation upper bound by integrating theories
of spectral clustering and matrix perturbation, and discuss
how it may be affected by several network characteristics.
The obtained implications are supported in simulations.

In summary, the contributions of this paper² are un-
folded in two phases. First, we propose to effectively lever-
age multiple views of the network structure for better
automatic social circle detection in ego-nets. To that end,
we introduce multi-view spectral clustering techniques and
demonstrate they superior circle detection performance, as
compared with common single-view clustering techniques.
Second, we propose to interpret the sparseness of ego-net
structure as incompleteness, and conjecture the ignorance of
such hidden incompleteness may result in performance bias.
To that end, we first derive an upper bound for the per-
formance bias, with implications supported in simulations;
we then propose a modified multi-view clustering technique
which selectively transfers information from sparse views,
and demonstrate its superior circle detection performance
as compared with the standard multi-view clustering tech-
nique which fully transfers information across views. Fi-
nally, extensive experimental evaluations are done based on
the ego-nets we crawled from Twitter.

The rest of this paper is organized as follows: section
2 introduces the general notations and problem setting;
section 3 introduces the multi-view ego-net structure used
in our study; section 4 presents the examined multi-view
spectral clustering techniques as well as our interpretation
of the network sparseness; section 5 presents the experi-
mental evaluations; related works are reviewed in section 6 and
discussions in section 7; section 8 concludes the studies.

2 Notations and Problem Setting

For a matrix $M$, let $M_{ij}$ be its entry at row $i$ and column
$j$, $M_j$ be its $j_{th}$ column and $M_i$ be its $i_{th}$ row; let $M^T$
be its transpose, $| |M| |$ be its operator norm and $| |M| |_{F}$
be its Frobenius norm; when $M$ is associated with view $t$,
we denote it by $M^{(t)}$. Let $I$ be an identity matrix properly sized
by the context. For two matrices $M, M'$ (of the same size), let
$\succ$ and $\succcurlyeq$ be the Loewner partial orders such that $M \succ M'$
if $M - M'$ is positive semi-definite and $M \succcurlyeq M'$ if $M - M'$
is positive definite; let $M \circ M'$ be their Hadamard product.
Finally, define $[\ell] := \{1, 2, ..., \ell \}$ for an integer $\ell > 0$.

Recall the structure of an ego-net could be described
from multiple views, where each view corresponds to one
type of connections between network nodes (i.e. alters).
We characterize the view $t$ of an ego-net structure by a
similarity matrix $K^{(t)}$, such that $K^{(t)}_{ij}$ is some pre-defined
similarity between alter $i$ and alter $j$. (When referring to an
arbitrary view, however, as we do in theoretical analysis, the
superscript $t$ may be omitted in notation.)

Now, given an ego-net consisting of $n$ alters and charac-
terized by multiple views $\{K^{(t)}\}$ where $t \in [T]$, our task is
to automatically detect social circles based solely on $\{K^{(t)}\}$.

3 A Multi-View Ego-Net Structure

3.1 A Motivating Example

An advantage of considering multiple views of the ego-
net structure is that different views may provide comple-
mentary information for more effective discovery of hidden
social circles. Figure 1 shows a sub-sample of the ego-net
structure we crawled from Twitter, which consists of six al-
ters (denoted by A, B, C, D, E, F respectively) and described
from five views – (a) shows two relation views indicating
the friend relations between alters and their common friend
numbers; (b) shows two interaction views indicating the
numbers of replies and retweets between alters; (c) shows
a content view indicating similarities between alters’ posts.

We see different types of views are partly consistent in
suggesting the alters similarities, e.g. alters A and B not
only have strong connections in the relation view, but also
interact frequently based on the interaction view; on the
other hand, although alters C and D are not friend (yet),
it may still be helpful to group them since they have many
friends in common and highly similar posts (i.e. they may
still find a lot to talk with each other and thus promote the
network information flow).

3.2 View Modeling

In this study, we crawl data from Twitter and employ classic
techniques to model six views of its ego-net structures.
These models are explained as below.

Friendship. This view characterizes the friend relation be-
tween alters by a similarity matrix $K^{(1)}$, defined as $K^{(1)}_{ij} = 1$

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² This paper is a journal extension of our previous study [71].
if alters $i$ and $j$ follow each other on Twitter and $K_{ij}^{(1)} = 0$ otherwise. It is a most common view for social circle detection.

**Common Friend**. This view characterizes the number of common friends between alters by a similarity matrix $K^{(2)}$, defined as $K_{ij}^{(2)} = m$ if alters $i$ and $j$ have $m$ friends in common (excluding the alters $i$ and $j$ themselves).

**Reply**. This view characterizes the reply frequency between alters by a similarity matrix $K^{(3)}$, defined as $K_{ij}^{(3)} = m$ if alters $i$ and $j$ reply to one or another by $m$ times in total.

**Co-Reply**. This view characterizes the co-reply frequency of alters by a similarity matrix $K^{(4)}$, defined as $K_{ij}^{(4)} = m$ if alters $i$ and $j$ co-reply to $m$ posts on Twitter.

**Re-tweet**. This view characterizes the re-tweet frequency between alters by a similarity matrix $K^{(5)}$, defined as $K_{ij}^{(5)} = m$ if alters $i$ and $j$ re-tweet each other for $m$ times in total.

**Topic**. This view characterizes the post similarity between alters by a similarity matrix $K^{(6)}$, where $K_{ij}^{(6)}$ is the cosine similarity between the normalized topic vectors of alter $i$ and alter $j$. These vectors are obtained by first getting a topic vector for each alter by uploading his/her posts to the online annotation tool TagMe [20], and then normalizing the returned vectors by the TF-IDF technique.

### 3.3 Cluster Assumption

Our cluster assumption is similar to [43] but extended from its single view setting to a multi-view setting. Specifically, we assume alters in the same social circles should have high similarity (as compared with alters in different circles) from multiple views. This means within-circle alters are more likely to be friends, to share more common friends, to retweet or reply to each other more often, to co-reply to more posts and to post more similarity tweets.

### 4 Clustering on Multi-View Ego-Net

Based on the multi-view ego-net structure presented in the previous section, we propose to detect social circles by multi-view spectral clustering techniques (e.g. [33], [35]), which have been shown effective in clustering multi-view graphs\(^3\). Specifically, we employ co-trained spectral clustering [33], which is briefly reviewed in section 4.1.

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3. In this paper, a network (structure) is viewed as a graph and these two terms are used interchangeably.
sparseness? and how bad could it be when one clusters a graph while ignoring its inherent incompleteness (as done by most studies)? We discuss these issues in section 4.3.

The discussions in this section involve a number of new notations. In Table 1, we summarize the major notations for an arbitrarily fixed view.

4.1 Co-Trained Spectral Clustering: A Brief Review

This section briefly reviews co-trained spectral clustering [33], a popular multi-view spectral clustering technique we propose to apply for social circle detection.

Spectral clustering [50] is a classic technique to group nodes of a graph based solely on the graph topology. The topology is usually characterized by a node similarity matrix, from which a graph Laplacian matrix is constructed. It is shown eigen-vectors of this Laplacian matrix contain discriminative information for node clustering, and spectral clustering uses these vectors as latent node features on which standard attribute-based clustering techniques such as K-means are performed to group nodes.

In general, multi-view spectral clustering is an extension of the classic spectral clustering from the single-view setting to a multi-view setting, where clustering information in one view is used to modify the clustering tasks in other views so that different views would reach some consistency in results. The co-trained spectral clustering technique [33] alternately uses eigen-vectors of an examined view to refine similarity matrices of other views, by first projecting and then reconstructing those matrices in a new space spanned by eigen-vectors of the examined view. Take view \( t \) for example, \( U_{i,k}^{(t)} \) be a matrix whose columns are the \( k \) principal eigen-vectors of the normalized Laplacian matrix of \( K^{(t)} \). Then, the similarity matrix of another view \( t' \) is refined by

\[
K^{(t')} = U_{i,k}^{(t')} \left( U_{i,k}^{(t)} \right)^T K^{(t)}. \tag{1}
\]

Authors showed (1) could encourage consistent clustering across views, by throwing away grouping information within each cluster in each view. Finally, when the alternate update converges, eigen-vectors of all views (or, some dominant view) are concatenated to form latent node features on which standard K-means is performed to group nodes.

4.2 Selective Co-trained Spectral Clustering

While the standard co-trained spectral clustering has been shown effective for graph clustering, it ignores the inherent incompleteness of different ego-net views. This means results on sparse views may not be very reliable (in a sense that alters assigned to different groups may not be truly distance in relations), and fully transferring them to other views may mislead their clustering performance. Since we do not know whether a sparse view is incomplete or not, a safe strategy is to only transfer its assignments on pairs of alters whose connections are observed.

In this section, we present a heuristic which modifies co-trained spectral clustering so that clustering results in sparse views are selectively transferred to refine other views. The heuristic is twofold: a view is considered incomplete if its fraction of observed connections is below some threshold, and (in an incomplete view) alters are considered to have observed connections if their similarities are non-zero.

The algorithm of our proposed technique is presented in Algorithm 1, where the involved functions are defined as:

- \( \text{eig}(K,k) \) returns an \( n \times k \) matrix whose columns are the \( k \) principal eigen-vectors of the normalized Laplacian of the (similarity) matrix \( K \).
- \( \text{clust}(U,k) \) returns an \( n \times n \) matrix \( C \) obtained by performing \( k \)-means clustering on sample matrix \( U \) (where each row is one example), such that \( C_{ij}=1 \) if examples \( i \) and \( j \) are assigned to the same group and \( C_{ij}=-1 \) otherwise.
- \( \text{update}(t) \) is defined for view \( t \) as

\[
\text{update}(t) = \exp \sum_{t'\in[T],t'\neq t} C^{(t')} \circ (1\{K^{(t')} \neq 0\}) \delta_{t'}. \tag{2}
\]

where \( C^{(t')} \) is the output matrix of \( \text{clust}(U^{(t')},k) \), 0 is a matrix of zeros same sized as \( K^{(t')} \), 1 is an element-wise indicator function, and \( \delta_{t'} \) is a binary function outputting 1 if \( K^{(t')} \) is sufficiently sparse (i.e. its fraction of observed entries is below some threshold) and 0 otherwise.

The core of our modified algorithm is \( \text{update}(t) \), which is used to update the similarity matrix of view \( t \) based on information selectively transferred from other views. We slightly elaborate its design in the following, assuming the case of two-view clustering:

For \( C^{(t)} \): if alters \( i, j \) are assigned to the same group in view \( t' \), we have \( C^{(t)}_{ij}=1 \). This could result in \( \text{update}(t)_{ij}>1 \) and consequently the increase of these two alters’ similarity in view \( t \) through \( K^{(t)}_{ij}=K^{(t)}_{ij} \cdot \text{update}(t)_{ij} \). Note, however, \( C^{(t)}_{ij}=1 \) does not guarantee the increase of \( K^{(t)}_{ij} \), because in order to get \( \text{update}(t)_{ij}>1 \), we also need the indicator function to output 1 if view \( t \) is incomplete (i.e. the connection between two alters needs to be observed in view \( t \)).

For \( K^{(t)} \): if alters \( i \) and \( j \) have observed connection, we have \( K^{(t)}_{ij} \neq 0 \) and thus \( 1\{K^{(t')} \neq 0\}_{ij}=1 \). This could allow their clustering result \( C_{ij}^{(t')} \) be transferred to other views through \( \text{update}(t) \) (specifically, \( C_{ij}^{(t')} \cdot (1\{K^{(t')} \neq 0\})_{ij} \)). Of course, the whole selective mechanism is valid only when view \( t' \) is considered incomplete (i.e. \( \delta_{t'}=1 \)).

For \( \delta_{t'} \): if view \( t' \) is considered incomplete, we have \( \delta_{t'}=1 \). This will activate the selective mechanism for \( K^{(t')} \) (as we described above); otherwise, the selective mechanism is de-activated and all clustering results in view \( t' \) will be transferred to modify view \( t \) through \( \text{update}(t) \).

The following example demonstrates how the proposed algorithm may leverage the clustering result in one view to refine another view. Suppose the result in view one is

\[
C^{(1)} = \begin{bmatrix}
1 & -1 & 1 \\
-1 & 1 & -1 \\
1 & -1 & 1
\end{bmatrix}, \tag{3}
\]

which indicates alters 1 and 3 are grouped whereas 2 and 3 are separated. Suppose the similarity matrix of view two is

\[
K^{(2)} = \begin{bmatrix}
1 & 0.4 & 0.1 \\
0.4 & 1 & 0.6 \\
0.1 & 0.6 & 1
\end{bmatrix}. \tag{4}
\]
The algorithm will refine $K^{(2)}$ using $C^{(1)}$ through update(1) and, when results are fully transferred, we have an update

$$K^{(2)} = \begin{bmatrix} 2.7 & 0.2 & 0.3 \\ 0.2 & 2.7 & 0.2 \\ 0.3 & 0.2 & 2.7 \end{bmatrix}.$$ (5)

In the updated $K^{(2)}$, it is clear similarities between alters grouped in view one are increased and vice versa. Note, however, that grouping in view one does not necessarily lead to grouping in view two – for instance, alters 1 and 3 still have low similarity even though they were grouped in one view – the updated matrix is a compromise between results in other views and observations in the current view. When results are selectively transferred, we expect the algorithm would converge faster as less consistency needs to be compromised between views. This seems indeed the case as is evident from our experimental studies. Finally, note since $C^{(1)}$ is symmetric, $K^{(2)}$ would remain symmetric after update (and thus its corresponding Laplacian matrix remains positive semi-definite which admits positive eigenvalues).

### 4.3 When Ego-Net is Inherently Incomplete

As we mentioned before, a sparse ego-net may be inherently incomplete. Such incompleteness distinguishes itself from most previous studies on incomplete graphs which assume prior knowledge on whether the graph is indeed incomplete and which part of the graph is incomplete. None of these is known, however, for an inherently incomplete graph. Then, what could we say about clustering such a graph?

This section is an attempt to answer the above question through the derivation and discussion of an upper bound on the possible performance bias when one performs standard clustering on an inherently incomplete graph while ignoring its potential incompleteness (as most studies do). To derive the bound, we integrate a classic spectral clustering theory [29] with a recent result in matrix perturbation theory [72], and employ several properties of the Loewner partial orders (e.g. [25, Chapter 7]). We then discuss the implications of our bound, with a focus on how it could be affected by various ego-net characteristics; the implications seem supported in later simulations. Our discussion will focus on single-view clustering as it is the backbone of multi-view clustering techniques (e.g. co-trained spectral clustering could be regarded as single-view spectral clustering on a dominant view which has been refined by other views).

#### 4.3.1 Preliminaries

First, we make a free-approximation assumption to simplify discussion. It is well known that spectral clustering is an approximated solution to the optimal normalized cut problem, and there is a rich literature studying the approximation error (e.g. [55], [73]). Although we apply spectral clustering and evaluate results under the optimal cut framework, such approximation is not our focus. We thus assume the approximation error is zero, which could be satisfied if the $k$ principal eigen-vectors of the graph Laplacian matrix are piece-wise constant with respect to the optimal normalized cut result on the graph [29]; when the assumption is not satisfied, our analysis could be generalized by simply adding an error term for the approximation.

Next, recall we have an ego-net consisting of $n$ alters and characterized by multiple $n$-by-$n$ similarities matrices $\{K^{(i)}\}$, each representing one view of the ego-net structure. Since all analysis in this section applies to an arbitrary single view, the superscript $t$ (i.e. view index) will be omitted.

Consider the task of $k$-partitioning the $n$ alters based on a complete ego-net characterized by similarity matrix $K$. Let

$$L = D^{-1/2} KD^{-1/2}$$ (6)

be the normalized Laplacian matrix, where $D$ is an $n$-by-$n$ diagonal matrix where $D_{ii} = \sum_{j\in [n]} K_{ij}$. Let $\sigma_k$ and $\lambda_k$ be the $k$th principal eigenvalues of $D$ and $L$ respectively.

Let $K$ denote an inherently incomplete observation of $K$, with observed entries indexed by set $\Omega$, such that $K_{ij} = K_{ij}$ if $(i,j)\in \Omega$ and $K_{ij} = 0$ otherwise. Note $K_{ij} = 0$ may imply $K_{ij}$ is unobserved or $K_{ij}$ is observed but has value 0. Similar to $L$, let $\tilde{L} = \tilde{D}^{-1/2} \tilde{K} \tilde{D}^{-1/2}$ be the normalized Laplacian of $\tilde{K}$, where $\tilde{D}$ is diagonal matrix with $\tilde{D}_{ii} = \sum_{j\in [n]} \tilde{K}_{ij}$.

The resulted $k$-partition of $n$ alters based on $\tilde{K}$ is represented as an $n$-by-$k$ matrix $V$, defined as $V_{ij} = 1$ if alter $i$ is assigned to cluster $j$ and $V_{ij} = 0$ otherwise. Similarly, the partition result based on $K$ is represented as matrix $\tilde{V}$. We then evaluate the difference between these two results using the metric employed in [2, Formula (2)], i.e.

$$d(\mathcal{V}, \tilde{\mathcal{V}}) = \frac{1}{2} \left\| \sum_{j\in [k]} V_{j} (V_{j})^T - \sum_{j\in [k]} \tilde{V}_{j} (\tilde{V}_{j})^T \right\|_F^2.$$ (7)

Intuitively, metric (7) counts the pairs of alters assigned to different clusters in two results, each weighted by the corresponding cluster size. Indeed, note $(V_{j} (V_{j})^T)_{i_1 i_2}$ equals 1 if alters $i_1$ and $i_2$ are both assigned to cluster $j$ and equals 0 otherwise; and $(\tilde{V}_{j})^T V_{j}$ is the size of cluster $j$. Also note the metric is bounded when each cluster contains at least one alter, which could be easily guaranteed by proper algorithm design. Finally, it is generally hard to give a threshold under which a bias $d(\mathcal{V}, \tilde{\mathcal{V}})$ could be considered acceptable, as it depends on the cluster sizes and the applications. Nevertheless, one could get more sense through simple calculations: for example, suppose $n$ nodes are equally partitioned into $k$ clusters, and results based on complete and incomplete graphs differ on $p$ fraction of node pairs, then $d(\mathcal{V}, \tilde{\mathcal{V}}) = k^2 p$.

5. A classic definition is $L = I - D^{-1/2} KD^{-1/2}$. Ours is from [50], which admits the same eigen-vectors but facilitates discussion.
4.3.2 A Bias Bound and its Implications

Based on notions introduced in the previous section, our derived bias bound is stated as follows.

**Proposition 1.** Let \( V, \bar{V} \) be the \( k \)-partitioning result matrices of the optimal normalized cuts based on \( K, \bar{K} \) respectively. Let \( \sigma_1 \) be the principal eigenvalue of \( D^{-1/2} \) and denote \( \Delta = L - \bar{L} \). Then

\[
d(V, \bar{V}) \leq \frac{\sigma_1}{\sigma_n} \left( \frac{\sigma_1}{\sigma_n} + \frac{2 \min \{ \sqrt{k} \| \Delta \|_2, \| \Delta \|_F \}^2}{\lambda_k - \lambda_{k+1}} \right). \tag{8}
\]

The bound has several implications.

First, note \( \sigma_1, \sigma_n, \lambda_k, \lambda_{k+1} \) are constants when an (underlying and complete) ego-net is given. Their impacts on the bias bound could be interpreted as follows. For \( \sigma_1 \) and \( \sigma_n \), note they respectively describe the overall behaviors of the most and least active alters\(^6\) in the ego-net, since they are respectively the largest and smallest row sums of \( K \). Since \( \sigma_1/\sigma_n \geq 1 \) in (8), we see standard spectral clustering (which ignores the potential graph incompleteness) may suffer less performance bias if alters are equally active in the ego-net (in which case \( \sigma_1/\sigma_n \) would be close to 1).

Second, the bound suggests ways of choosing \( k \) to lower the risk of performance bias. Based on the term \( \sqrt{k} \) in (8), we see detecting fewer social circles could generally reduce the risk of suffering performance bias; based on \( \lambda_k - \lambda_{k+1} \), on the other hand, we see one may choose \( k \) at the ‘steepest’ place of the graph spectrum. For instance, when alters have equally active (normalized) behaviors, the graph spectrum is flat and we may choose a large \( k \) to maximize \( \lambda_k - \lambda_{k+1} \).

Finally, the bound sheds light on how bias may decrease as more connections are observed on the ego-net: as observations increase, it is clear that \( K \rightarrow \bar{K} \) and thus \( \bar{D} \rightarrow D \) and \( \bar{L} \rightarrow L \). The latter further implies \( \sigma_1 \rightarrow 0 \) and \( \Delta \rightarrow 0 \), which based on (8) implies a small bias bound. In particular, if the ego-net is fully observed, we have \( \sigma_1 = 0 \) and \( \Delta = 0 \), which results in a zero bias bound and hence no performance bias.

4.3.3 Proof of Proposition 1

**Notations.** First, recall \( \bar{K} \) is an \( n \times n \) similarity matrix with an associated Laplacian \( L \) and a diagonal \( D \), and \( \lambda_k, \sigma_k \) are the \( k \)-th principal eigenvalues of \( L, D \) respectively. The \( k \)-partitioning result is stored in an \( n \times k \) matrix \( V \).

Let \( L = U \Lambda U^T \) be the eigen-decomposition of \( L \) such that \( \Lambda \) is an \( n \times n \) diagonal matrix with \( \lambda_i = \lambda_i \) (\( \lambda_1 \geq \lambda_2 \geq \ldots \)) and \( U \) is an \( n \times n \) unitary matrix where \( U_{ij} \) is the eigenvector for \( \lambda_i \). Let \( U[k] \) be an \( n \times k \) sub-matrix of \( U \) where \((U[k])_{ij} = U_{ij} \) for \( i = 1, \ldots, k \). Since \( U[k] \) is orthonormal, we have \( \bar{P}_k = U[k]U[k]^T \) as the orthogonal projection onto the range space of \( U[k] \) (e.g. [24, Chapter 2]).

All the above notations apply to \( \bar{K} \) and its associated variables, yet capped with notation ‘\( \bar{\cdot} \)’. For instance, \( \bar{P}_k \) is the orthogonal projection onto the range space of \( U[k] \), which contains the \( k \) principal eigenvectors of the Laplacian \( \bar{L} \).

6. The notion of ‘active’ is open to interpretation: in an interaction view, for example, an active alter is someone who interacts frequently with others; while in a relation view, an active alter is someone who has a lot of friends (an indicator of his active socialization).
7. Notation \( A \geq B \) means \( A - B \) approaches zero matrix.

Finally, let \( \succeq, \succ \) be the Loewner partial orders such that \( A \succeq B \) if \( A - B \) is positive semi-definite (PSD) and \( A \succ B \) if \( A - B \) is positive definite. Note \( A \succeq 0 \) implies \( A \) is PSD.

**Proof Sketch.** The strategy of our proof is as follows: we first bound \( d(V, \bar{V}) \) by two new terms using the triangular inequality; then we bound the first term using a recent result in perturbation theory [72], and bound the second term using several Loewner partial order properties (e.g. [25, Chapter 7]); we also borrow some results from [29].

**Step 1: bound \( d(V, \bar{V}) \).** By [2, Formula (3)] we have

\[
d(V, \bar{V}) \leq \frac{\sigma_1}{\sigma_n} \cdot d(D^{1/2}V, D^{1/2}\bar{V}). \tag{9}
\]

Further, by triangular inequality it follows

\[
d(D^{1/2}V, D^{1/2}\bar{V}) \leq d_w(V, \bar{V}) + d(D^{1/2}\bar{V}, D^{1/2}V), \tag{10}
\]

where we define \( d_w(V, \bar{V}) = d(D^{1/2}V, D^{1/2}\bar{V}) \).

**Step 2: bound \( d_w(V, \bar{V}) \).** It is known that spectral clustering is a relaxation of the optimal normalized cut problem. In [2, Formula (1)], the relaxation error is measured by the difference between the orthogonal projections for the two problems. In our context, these two projections (associated with matrix \( K \)) are \( \bar{P}_k \) and

\[
\Pi_k := \sum_{j \in [k]} D^{1/2}V_j V_j^T D^{1/2}/(V_j^T D V_j). \tag{11}
\]

Then, our free-approximation assumption implies \( \bar{P}_k = \Pi_k \). Similarly, the orthogonal projections associated with \( \bar{K} \) are \( \bar{P}_k \) and \( \bar{\Pi}_k := \sum_{j \in [k]} \bar{D}^{1/2}V_j V_j^T \bar{D}^{1/2}/(V_j^T \bar{D} V_j) \), and that \( \bar{P}_k = \bar{\Pi}_k \). Since by definition \( d_w(V, \bar{V}) = \| \Pi_k - \bar{\Pi}_k \|_F^2 \), we have

\[
d_w(V, \bar{V}) = \| \bar{P}_k - \bar{\Pi}_k \|_F^2. \tag{12}
\]

Now, our task becomes bounding \( \| \bar{P}_k - \bar{\Pi}_k \|_F^2 \) instead. A classic technique is the Davis-Kahan theorem (e.g. [72, Theorem 1]), which would give

\[
\| \bar{P}_k - \bar{\Pi}_k \|_F \leq \frac{\| \Delta \|_F}{\kappa}, \tag{13}
\]

where \( \kappa = \inf \{ |\lambda_i - \lambda_j| : 1 \leq i \leq k, j < k \leq n \} \). While this bound is semisem, it contains an implicit dependency on \( \Delta \) (through parameter \( \lambda \)), whereas we prefer a bound that has more explicit dependency on \( \Delta \) (for easier interpretation). To this end, we employ a recent generalization of the Davis-Kahan theorem, which is stated as follows.

**Theorem 1 ( [72, Theorem 2] ).** Let \( L, \bar{L} \in \mathbb{R}^{n \times n} \) be two symmetric matrices with eigenvalues \( \lambda_1 \geq \ldots \geq \lambda_n \) and \( \lambda_1 \geq \ldots \geq \lambda_n \) respectively. Fix \( 1 \leq r \leq s \leq n \) and assume \( \min \{ \lambda_{s+1} - \lambda_s, \lambda_s - \lambda_{s+1} \} > 0 \), where \( \lambda_0 = \infty \) and \( \lambda_{n+1} = -\infty \). Let \( d := s - r + 1 \), and let \( U[k] = [u_1, u_{r+1}, \ldots, u_s] \in \mathbb{R}^{n \times d} \) and \( U[d] = [u_1, u_{r+1}, \ldots, u_s] \in \mathbb{R}^{n \times k} \) have orthonormal columns satisfying \( Lu_j = \lambda_j u_j \) and \( \bar{L}u_j = \lambda_j \bar{u}_j \) for \( j = r, r + 1, \ldots, s \). Then

\[
\| \bar{P}_k - \bar{\Pi}_k \|_F \leq \frac{2 \min(\| \Delta \|_2, \| \Delta \|_F)\|}{\min \{ \lambda_{s+1} - \lambda_s, \lambda_s - \lambda_{s+1} \}}. \tag{14}
\]

8. The original theorem bounds the angles between subspaces, which equals the difference of their orthogonal projectors, e.g. [17, Page 10].
By (12) and Theorem 1 (with \( r = 1 \) and \( s = k \)), we have
\[
d_w(\mathbf{V}, \tilde{\mathbf{V}}) \leq \frac{4 \min(\sqrt{n}||\mathbf{A}||_2, ||\mathbf{A}||_F)}{(\lambda_k - \lambda_{k+1})^2}.
\]

**Step 3:** bound \( d_+ := d(\tilde{D}^{1/2}\tilde{\mathbf{V}}, \tilde{D}^{1/2}\tilde{\mathbf{V}}) \). First, it is easy to verify the following lemma by algebraic arguments.

**Lemma 1.** For any orthogonal matrix \( M \in \mathbb{R}^{n \times p} \),
\[
\sum_{j \in [p]} M_j M_j^T / (M_j^T M_j) = M (M^T M)^{-1} M^T.
\]

Note \( D^{1/2}\tilde{\mathbf{V}} \) and \( \tilde{D}^{1/2}\tilde{\mathbf{V}} \) are orthogonal. Then, jointly applying Lemma 1 and the alternative expression in [2, Page 6] for \( ||M (M^T M)^{-1} M^T - \tilde{M} (\tilde{M}^T \tilde{M})^{-1} \tilde{M}^T||_F^2 \), we have
\[
d_+ = \text{tr}\{T^{-1/2}(\tilde{T} - \tilde{\mathbf{N}})T^{-1/2}\},
\]
where \( T = \tilde{\mathbf{V}}^T D \tilde{\mathbf{V}} \) and
\[
\tilde{\mathbf{N}} = \tilde{\mathbf{V}}^T (D \tilde{\mathbf{D}})^{1/2}\tilde{\mathbf{V}} (\tilde{\mathbf{V}}^T D \tilde{\mathbf{D}})^{1/2}\tilde{\mathbf{N}} (D \tilde{\mathbf{D}})^{1/2}\tilde{\mathbf{V}}.
\]

It the sequel, we bound \( T^{-1/2} \) and \( T - \tilde{\mathbf{N}} \) separately.

Note \( \mathbf{V} \) and \( D^{1/2}\tilde{\mathbf{V}} \) have linearly independent columns (since \( \tilde{\mathbf{V}} \) indicates a partition of alters and \( D^{1/2} \) does not change such indication). Then, by [25, Theorem 7.2.10]
\[
T = \tilde{\mathbf{V}}^T D \tilde{\mathbf{V}} \succ 0 \quad \text{and} \quad \tilde{\mathbf{V}}^T \tilde{\mathbf{V}} \succ 0.
\]

Further, since \( \sigma_n \) is the smallest diagonal entry of \( D \), it is easy to verify \( D \succeq \sigma_n I \). This implies
\[
T \succeq \sigma_n \tilde{\mathbf{V}}^T \tilde{\mathbf{V}}.
\]

Based on (19) and (20), by [25, Corollary 7.7.4] we have
\[
\sigma_n^{-1/2}(\tilde{\mathbf{V}}^T \tilde{\mathbf{V}})^{-1/2} \succeq T^{-1/2}.
\]

To bound \( T - \tilde{\mathbf{N}} \), notice \( (D \tilde{\mathbf{D}})^{1/2} \succeq \tilde{D} \), which implies
\[
(\tilde{\mathbf{V}}^T D \tilde{\mathbf{V}})^{-1} \succeq (\tilde{\mathbf{V}}^T (D \tilde{\mathbf{D}})^{1/2}\tilde{\mathbf{V}})^{-1}.
\]

Plugging (22) in (18), we have
\[
\tilde{\mathbf{N}} \succeq \tilde{\mathbf{V}}^T (D \tilde{\mathbf{D}})^{1/2}\tilde{\mathbf{V}}.
\]

Combining (23) and (19), we have
\[
\delta_1 \tilde{\mathbf{V}}^T \tilde{\mathbf{V}} \succeq \tilde{\mathbf{V}}^T (D - (D \tilde{\mathbf{D}})^{1/2}) \tilde{\mathbf{V}} \succeq T - \tilde{\mathbf{N}},
\]
where \( \delta_1 \) is the largest diagonal entry of \( D - (D \tilde{\mathbf{D}})^{1/2} \).

Further combining (21) and (24) gives
\[
\sigma_n^{-1} \delta_1 I \succeq T^{-1/2}(T - \tilde{\mathbf{N}})T^{-1/2},
\]
and taking trace on both sides yields
\[
\sigma_n^{-1} \delta_1 \succeq d_+.
\]

**Step 4:** wrap up. Finally, combining (9), (10), (15) and (26) proves the proposition.

### 5 Experimental Study

In this section, we perform extensive experimental evaluations, with a focus on examining our set of hypotheses that better social circles in ego-nets could be detected (1) based on multiple views of the ego-net structure (as opposed to current studies that consider only a single view), and (2) by applying multi-view spectral clustering (as opposed to single-view clustering), and (3) by selectively transferring information from sparse views to others in multi-view clustering (as opposed to standard multi-view techniques that fully transfer those information). Implications obtained from the bias bound are also examined.

#### 5.1 Data Preparation

We will experiment on Twitter, which is one of the most popular online social network platforms. To this end, we first implemented a crawler to collect Twitter data using its API, which can return any user’s profile, follower/following lists and tweets. The user profile consists of user name, screen name, user id, profile create time, description (a personal statement), location and time zone. The tweets information consists of tweet id, post time, tweet location, in-reply-to user id, in-reply-to status id, list of re-tweets (user id and tweet id) and tweet content. For each user, we only collected his/her most recent 2000 tweets due to many constraints. It is also noted not all the attributes are available and accurate for all users. For example, user location in user profiles is self-generated textual description, where we have seen “Worldwide”, and “Coming Soon Everywhere” etc. Meanwhile, tweet locations are accurate latitudes and longitudes, but they are missing from most of the tweets. Finally, Twitter has enforced mandatory limits on the crawling rate, especially for crawling account-specific information. We have collected 92 data sets – 92 seed users and all their friends. In our data set, each seed user has 245 friends on average. In total, we have collected information of more than 22K users, with approximately 3 million friendship links, and more than 27 million tweet messages.

A crawled Twitter ego-net structure was modeled by six views, as introduced in section 3.2. Specifically, each view was implemented as follows: for the **friendship** view, two alters were marked as friend if each is in both the follower and following lists of the other, and the **common friend** view counted the number of such friends shared by alters; for the **reply** view, the reply number from alter Nancy to alter Bob was obtained by scanning through Bob’s tweets and counting the replies from Nancy, and vice versa; for the **co-reply** view, the co-reply number of two alters was obtained by scanning through all tweets in the crawled ego-net and counting those they both replied; for the **re-tweet** view, the re-tweet number from alter Nancy by alter Bob was obtained by scanning through Nancy’s tweets and counting those re-tweeted by Bob; for the **content** view, we first obtained a topic vector for each alter by uploading her tweets and profile to TagMe [20], with returned topics having relevance scores (between [0, 1]) below 0.2 removed by the Pareto principle, and then normalized all topic vectors by TF-IDF. Finally, after a similarity matrix is obtained, we normalize it into [0, 1] by dividing all entries by the maximum entry and fix diagonal entries to 1, indicating self-similarity is always
the largest. Note when the maximum entry is zero, we do not perform normalization. (This, however, rarely occurred in experiments.) Also note the normalization may slightly change the interpretation of a similarity matrix, but should ideally not affect the clustering result based on it.

5.2 Experimented Techniques

In experiment, we examined the performance of four clustering techniques that rely only on the ego-net structure.

SCAN [68]: this is a classic and popular clustering technique which detects social circles in a general social network based solely on a single view of the network structure (typically the friendship view in our model). We employed SCAN as a representative of the classic single-view clustering technique for social circle detection, and applied in on the friendship view in experiments.

Spectral Clustering (cs) [50]: this is a classic single-view clustering technique which groups instances based solely on their similarities. Although spectral clustering has not been specifically applied for social circle detection, we employed it in experiments as another representative of the single-view clustering techniques. Specifically, we first separately applied this technique on dominant views to learn their latent feature matrices \( U(t) \)'s (i.e. eigenvectors of the normalized Laplacian matrix of each view), and then concatenated these matrices in a column-wise manner to form an integrated latent feature matrix on which standard k-means clustering was applied to obtain the final grouping result. This approach could be interpreted as the standard multi-view spectral clustering but without cross-view information transfer, a common design to evaluate the effectiveness of multi-view learning techniques.

Co-Trained Spectral Clustering (scs) [33]: this is a popular multi-view spectral clustering technique which we have reviewed in section 4.1 and employed in experiments as a representative of the multi-view clustering technique. Similar to the previous application of (single-view) spectral clustering, when the co-trained spectral clustering algorithm converged, we concatenated the obtained latent feature matrices of dominant views to form an integrated feature matrix, on which K-means was performed to cluster alters.

Selective Co-Trained Spectral Clustering (scsc): this is the modified multi-view spectral clustering algorithm we proposed in Algorithm 1, which selectively transfers clustering results in sparse views to refine other views.

It should be mentioned we considered the friendship, common friend and topic as three dominant views, not only because they were generally denser (hence more complete) but also because they demonstrated stable and better performance in experiments. Note, however, although the non-dominant views were not directly used (as part of the integrated latent feature matrices) for clustering, they were helpful in that their information had been transferred to the dominant views by the multi-view algorithms. In addition, we fixed \( k=5 \) for examined techniques (except SCAN which automatically determines \( k \)), since we had observed similar trends in their performance as \( k \) increased from 3 to 10. Hyper-parameters of SCAN were set as default.

5.3 Evaluation 1: Cluster Compactness

We first evaluated the quality of detected clusters based on its compactness, which is a most common measurement.

5.3.1 Evaluation Metric

In our problem, the unavailability of both cluster ground truth and alter feature matrix has precluded the use of most standard cluster evaluation metrics, including both external ones such as the random index and F-measure and internal ones such as the Davies-Bouldin index and Dunn index. We hence presented and used the following metric to evaluate the compactness of a clustering result (recall in section 4.2 we introduced an \( n \)-by-\( n \) indicator matrix \( C \) to represent the result such that \( C_{ij} = 1 \) if alters \( i \) and \( j \) are grouped and \( C_{ij} = 0 \) otherwise):

\[
\gamma = \frac{\sum_t S_w^{(t)}}{\sum_t S_b^{(t)}},
\]

where \( t \in [T] \) (as there are \( T \) views in total),

\[
S_w^{(t)} = \frac{\sum_{(i,j)} K_{ij} \cdot 1\{C_{ij} > 0\}}{\sum_{(i,j)} 1\{C_{ij} > 0\}}
\]

and

\[
S_b^{(t)} = \frac{\sum_{(i,j)} K_{ij} \cdot 1\{C_{ij} < 0\}}{\sum_{(i,j)} 1\{C_{ij} < 0\}}.
\]

where \( (i, j) \in [n] \times [n] \) (as there are \( n \) alters in the ego-net).

Taking spirit from the classic discriminant analysis (e.g. [4, Formula (4)]), we name (27) the total similarity ratio, where \( S_w^{(t)} \) is the within-circle similarity that measures the average similarity between clustered alters in view \( t \) and \( S_b^{(t)} \) is the between-circle similarity that measures the average similarity between separate alters in view \( t \). It is clear a compact set of clusters should have high within-cluster similarity yet low between-cluster similarity (thus a large total similarity ratio), consistent with a common argument that alters within a circle should have high similarities and vice versa. Finally, to evaluate results for a single view, we use metric

\[
\gamma^{(t)} = \frac{S_w^{(t)}}{S_b^{(t)}},
\]

and when \( S_b^{(t)}=0 \) (as is often the case when view \( t \) is very sparse), we add a small constant to it in the metric.

While by design our metric is an echo of the Fisher ratio in classic discriminant analysis, it finds itself connected to several cluster quality metrics in the literature. For instance, \( S_w^{(t)} \) and \( S_b^{(t)} \) could be interpreted as the homogeneity index and separation index in [54] respectively, except we directly have alter similarities instead of computing them using alter fingerprints; their ratio is also related to the weighted intra-intra index [60] and Calinski-Harabasz index [7], in a sense that for equally sized clusters \( S_w^{(t)}/S_b^{(t)} \) differs from these indices mainly by constants (depending on the cluster size and sample size). These connections could be easily verified, and would largely remain valid for clusters of different sizes (e.g. by relaxing it to the case of equally-sized clusters).
TABLE 2
The size of five social circles detected by different spectral clustering techniques. std is the standard deviation of the sizes over five circles.

<table>
<thead>
<tr>
<th>Cluster</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>std</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC</td>
<td>8</td>
<td>12</td>
<td>13</td>
<td>25</td>
<td>44</td>
<td>14.6</td>
</tr>
<tr>
<td>CSC</td>
<td>16</td>
<td>17</td>
<td>20</td>
<td>24</td>
<td>25</td>
<td>4.04</td>
</tr>
<tr>
<td>SCSC</td>
<td>10</td>
<td>10</td>
<td>13</td>
<td>15</td>
<td>54</td>
<td>18.9</td>
</tr>
</tbody>
</table>

5.3.2 Results and Discussions
To better understand the performance of spectral clustering techniques, we first experimented on one single ego-net, which contains 386 alters. Recall the techniques are single-view spectral cluster (SC), co-trained spectral clustering (CSC) and selective co-trained spectral clustering (SCSC). For CSC and SCSC, we updated their view refinements for 20 rounds (by which both were observed converged generally) and reported the similarity ratios of their clustering results on each view in Figure 2. We saw CSC generally improved with more rounds of update, which is consistent with the spirit of co-trained style learning algorithms. However, its convergence rate was slow (as compared with that of SCSC), and its performance improvements over SC were not significant on the topic and reply views and were little on the friendship and co-reply views. As we explained before, this may be due to the ignorance of CSC on the inherent incompleteness of sparse ego-net views. Comparatively, our proposed SCSC converged fast (usually within one or two rounds of update) and improved SC consistently and significantly on all views.

Next, we examined the sizes of clusters output by different spectral clustering techniques on an ego-net with 102 alters. For CSC and SCSC, these sizes were reported at the update rounds where they respectively achieved their best clustering performance (i.e. the highest total similarity ratios). The statistics were summarized in Table 2. It appears CSC encouraged more balanced clusters, while both SC and SCSC output a big cluster. This may be because CSC enforced stronger view consistency so that a sparse view, for instance, could require a dense (dominant) view to ‘break down’ its inherently big clusters. It should be pointed out imbalanced clusters make sense in many practices, e.g., a family circle is usually much smaller than a friend circle.

Then, we examined the cluster qualities of SC, CSC and SCSC over 92 ego-nets we crawled from Twitter. The total similarity ratio of each technique on each ego-net is shown in Figure 3. We saw SCSC consistently outperformed the other two techniques, and standard SC had the worst performance. These were consistent with our previous observations in Figure 2, and our earlier discussions on the limitation of CSC: as it bindly transfers results on an inherently incomplete view to other views, the clustering tasks on those views may be misled.

Finally, on the same 92 ego-nets we compared the cluster qualities of all examined techniques on the friendship view (as SCAN [68] was designed based specifically on this view). Since SCAN would remove outliers and reported results based solely on the remaining alters, for fair comparison we also reported performance of other techniques based on the same set of remaining alters on each ego-net. These results were shown in Figure 4. We saw results similar to the previous examination, and that SCAN performed similarly to SC but not as good as SCSC. This results suggest that neither single-view clustering or multi-view clustering with improper (full) cross-view information transfer is sufficient for detecting high quality clusters.

5.4 Evaluation 2: Quality of Boundary Alters
In this experiment, we manually evaluated the qualities of detected circles, with a focus on the performance of SCSC on boundary alters, that is, alters that are most distant from the centroids of their assigned clusters. (We focused on these alters because they were most likely to be mis-clustered, and it was too expensive to manually evaluate all alters).

Given a clustering result, our general idea is to let human evaluator (without any information on this result) assign a boundary alter to one of the following two detected circles: (1) its actual assigned circle and (2) its nearest neighbor circle, defined as the circle (not assigned for the alter) whose members have the smallest distance to the alter on average. To be specific, for each boundary alter we first selected 10 tested alters, with 5 randomly from its assigned circle and another 5 randomly from its nearest neighbor circle. Then, a human evaluator would score from 1 to 5 on how much he/she agreed the boundary alter should be clustered with each of these tested alters (based on their profiles and tweets) – 1 for strongly disagree, 2 for somewhat disagree, 3 for neutral, 4 for somewhat agree and 5 for strongly agree – and the two scores averaged over both tested alters from the assigned circle and tested alters from the neighbor circle were separately reported.

We had summarized the above two scores for 60 randomly selected boundary alters, and found the averaged results were 2.63 for alters from the actually assigned circles and 2.52 for alters from the nearest neighbor circles. The observation that the assigned-circle score was greater than the neighbor-circle score suggested SCSC had relatively consistent performance with human – it managed to assign boundary alters to circles where they had tighter connections. The observation that both scores were low, on the other hand, suggested the intrinsic difficulty of social circles detection – it could be circles were generated based on less visible profiles, or they overlapped by nature which could confuse human evaluator.

5.5 Evaluation 3: Keywords of Detected Clusters
To better interpret the detected social circles, we extracted and examined their related tags (as returned by the TagMe tool) in this experiment. Our examination focused on tags that were discriminative across circles and, most importantly, representative of the content posted in each circle.

The representative tags for each circle were extracted as follows. Let $T_i$ be the set of tags returned by TagMe for alter $i$, and $tf(i, t)$ be the frequency of tag $t$ appeared in the posts of alter $i$. The representativeness of tag $t$ for a circle $S$ (which is an index set of its assigned alters) was measured by

$$\Pr(t | S) = \frac{1}{|S|} \sum_{i \in S} \tilde{tf}(i, t),$$

(31)
Fig. 2. The similarity ratios of examined spectral clustering techniques on six views of a Twitter ego-net. In each figure, the vertical axis represents the similarity ratio and the horizontal axis represents the round of algorithm updates. The blue and dotted lines represent the performance of \( sc \), the green and dash lines represent that of \( csc \), and the red and solid lines represent that of the proposed \( scsc \).

Fig. 3. The total similarity ratios of different spectral clustering techniques on 92 Twitter ego-nets. The ratios averaged over these ego-nets are: 108.8 for \( sc \), 24.8 for \( csc \) and 187.4 for \( scsc \).

Fig. 4. The similarity ratios of all examined clustering techniques on the friendship view on 92 Twitter ego-nets. The ratios averaged over these ego-nets are: 71.9 for SCAN, 79.5 for \( cs \), 20.9 for \( csc \) and 100.6 for \( scsc \).

\[
\tilde{f}(i, t) = \frac{ff(i, t)}{\max \{ff(i, t) | t \in T_i\}}.
\]  

Roughly speaking, \( \Pr(t|S) \) is the averaged frequency of tag \( t \) appeared in circle \( S \). Then, tags with largest \( \Pr(t|S) \) were deemed most representative and extracted for examination.

The discriminative tags across circles were extracted as follows. Let \( S_k \) be the circle indexed by \( k \), and \( K = \{k\} \) be the set of all circle indices. The discriminative degree of a tag \( t \) for circle \( k \) was measured as the KL-divergence

\[
\mathbb{D}(t) = \sum_{k \in K} \left( P(t; k) \ln \frac{P(t; k)}{Q(t; k)} \right),
\]  

where

\[
P(t; k) = \frac{\Pr(t|S_k)}{\sum_{k \in K} \Pr(t|S_k)}
\]  

and

\[
Q(t; k) = 1/|K|.
\]

Intuitively, both \( P(t; k) \) and \( Q(t; k) \) could be interpreted as the probability mass of tag \( t \) in circle \( k \) (as over all detected circles) – \( P(t; k) \) was the actually mass and \( Q(t; k) \) was an ideal mass when \( t \) was uniformly distributed; then, \( \mathbb{D}(t) \) said how much the actual distribution of tag \( t \) deviated from the uniform (thus non-informative) distribution. In our experiment, tags with the highest \( \mathbb{D}(t) \) were deemed most discriminative and selected for examination.

The two types of tags extracted from circles detected by \( scsc \) on a randomly selected ego-net were summarized in Table 3. They were obtained by first applying (33) to extract most discriminative tags over all circles, and then applying (31) among these tags to extract most representative ones for each circle. It was clear different circles had different semantic focuses: for instance, circle 2 had more interest in entertainment, while circle 4 seemed more concerned about health care and circle 5 talked about technology frequently. (We also skimmed through the tweets posted in these circles and had consistent findings.) This suggested circles detected by \( scsc \) could be pretty interpretable in terms of topics.

5.6 Evaluation 4: Cluster Inherently Incomplete Graph

In this section, we examined several implications obtained from Proposition 1. Since in reality it is impossible to know whether a social network has been fully observed, we presented simulations for examination.

Consider a set of 200 nodes partitioned into \( k_\ast \) groups, each containing \( \lceil 200/k_\ast \rceil \) nodes. (The last group also contains the residual nodes.) We constructed a binary graph by
building an edge between each pair of nodes, with probably $p$ if they were from the same group and with probability $1-p$ otherwise. The resulted graph was considered as the underlying complete graph. Note this graph could be fully characterized by its adjacent matrix $K \in \mathbb{R}^{200 \times 200}$ such that $K_{ij}=1$ with probability $p$ if nodes $i$ and $j$ were from the same group and with probability $1-p$ otherwise.

To simulate inherently incomplete observations of the graph, we randomly hid a portion of its edges by flipping a portion of entry 1’s to 0’s in $K$ (without recording which entries were flipped). Let $\delta$ be the portion of un-flipped 1’s and $K$ be the resulted adjacent matrix. Then, we applied the standard spectral clustering (e.g. [50]) on both $K$ and $K$ and evaluated the difference of their performance based on metric (7). To minimize the performance variation of the K-means clustering method (mostly induced from its selection of initial cluster centers), we fixed one node for each group to form the initial centers.

In Figure 5, we showed the performance deviation as the observations increase with $k = 5$. It is clear the deviation decreases as more observations are obtained, which is consistent with the implication of our bound.

In Figure 6, we showed the performance deviation under different numbers of detected clusters, which was controlled by parameter $k$ in (8). For each choice, the initial cluster centers were chosen by a standard ‘cluster’ setting in Matlab (i.e. 10% of the nodes were randomly sampled to perform clustering first, and centers of the resulted clusters were used as the initial centers for the final clustering algorithm). It is clear that smaller choice of $k$ suffers smaller performance deviation, and this is consistent with the implication of our bias bound in (8) which decreases as $k$ decreases.

Finally, we examined the impact of coefficient $\sigma_1/\sigma_n$ on performance deviation. To this end, we first fixed $k = 5$, $k = 5$ and $\delta = 30\%$ so the graph contained 5 clusters by nature. The edge-generation probability remained largely the same as before, except for the first cluster we used another probability $p'$ which varied from 0.1 to 0.9 with step size 0.1. Note different choices of $p'$ would generate different graphs, and for each choice we applied spectral clustering on both its generated $K$, $K$ and evaluated the performance deviation. Meanwhile, we recorded the coefficient $\sigma_1/\sigma_n$ for each $K$. Finally, the deviation-coefficient pair for each choice of $p'$ was plotted as point in Figure 7.

We had three major observations from the figure. First, in general a larger coefficient $\sigma_1/\sigma_n$ corresponded to larger performance deviation, which was consistent with the implication of our bound. Second, the relation between the coefficient and deviation was roughly linear, which also coincided with our result. It was noted, however, such relation did not cross the origin as suggested by the bound; instead, it was biased by nearly a constant factor. We believed such bias was reasonable and should largely corresponded to the approximation error of ignored for spectral clustering. (Recall this error was ignored by the free-approximation assumption.) We hence do not claim the bound tight. However, its multiple implications were verified and may still provide useful insights for algorithm designs.

### 6 RELATED WORK

#### 6.1 Social Circle Clustering

Identifying social circles from a user’s online social networks is important for the individual to exert appropriate access control on information sharing [15], [57]. However, manually managing groups on social network sites might present a burden for users, which triggered the idea on using automatic socio-centric network clustering algorithms.
The feasibility of this idea has been demonstrated in the findings from [22], [28].

Sociocentric network clustering, which is usually referred to as community detection, aims to divide people into groups within which they are more similar [1] and have more connections [49] or relationships. Unlike traditional personal network studies that used attribute-based data such as age and sex [44], most graph-based community detection methods [21] used only topological structure and linkage information. These methods include graph partitioning [30], hierarchical clustering [49], and more recent methods through maximization of a likelihood like [48] and based on matrix factorization [74]. There is also a trend in recent research based on graphs which combined link information and content or attribute information [51], [70] or interaction information between individuals [77].

Compared with graph-based methods, another class of approaches attach greater importance to content or link context information. [10], [40], [75], [76] use state-of-the-art methods like topic modeling to take full advantage of semantic information, such as email, tweet messages, and documents, in detecting communities from a social network. [63] proposed a method to find like-minded people who share more semantically relevant tags.

A recent research more related to ours is [51]. It proposed generative Bayesian models to utilize not only topics and social graph topology but also nature of user interactions to discover latent communities in social graphs. The difference between their work and ours is that we also used tag annotation method to analyze content information generated by users, which concerns the understanding of the information and is more meaningful in finding similar topics.

Based on privacy concerns and automatic social circle detection, [42] developed a model to discover social circles by using both network structure and user profile information; [58] proposed an approach based on apriori algorithm to identify hidden groups by dynamically detecting grouping criteria, i.e. certain combinations of properties of a user’s contacts, such as relationship, location, hobbies, age, privacy, etc. The difficulty in utilizing this kind of methods is that automatically collecting attributes of users through online social network is a nontrivial task although traditional personal network studies can collect these information through interviews more easily.

6.2 Multi-View Spectral Clustering

Multi-view spectral clustering is a framework that effectively combines information from multiple sources under the view consistency idea. It has demonstrated superior performance in many applications such as document categorization [5], digit classification [33] and image annotation [62]. However, to our knowledge this framework has not been applied for detecting social circles, and our work is the first effort in this direction.

In this paper, we base our analysis on the co-trained spectral clustering algorithm [33] for its direct use of the similarity matrices as input, which is needed in our problem setting. During investigation, we noticed a limitation of this algorithm, i.e. it assumes all views are complete and equally transfer information across views. In our problem, however, some views are very sparse and potentially incomplete. In this case, equally transferring information among different views may mislead clustering and hurt the performance.

We present theoretical analysis on the performance degeneration and propose a modification of the co-trained spectral clustering algorithm. In experiment we show the modified algorithm indeed improves the performance.

6.3 Clustering on Incomplete Graph

Clustering on incomplete graphs is not a new topic. See studies in [11], [37], [53], [61] for example. Most of these works provided only algorithmic solutions and a few theoretical studies assumed prior knowledge on which part of the graph is missing. However, none of them address our concern on the performance deviation between clustering a graph and its incomplete observation.

A work more related to ours is [29], which analyzes how much the spectral clustering solution on a complete graph may deviate from the optimal normalized cut solution. Our analysis focuses on a fundamentally different problem, i.e. how much would two spectral clustering solutions deviate, with one based on a complete graph and the other based on its incomplete observation. Technically, we use the same evaluation metric as [29] and borrow some of its results, while additionally introducing perturbation theories to incorporate the incomplete observation.

Another related work is [26], which analyzes the effect of graph perturbation on the performance of spectral clustering. We study the same research question, but our analysis is fundamentally different from theirs in at least three aspects. First, the problem settings are different: they study only bi-partitioning based on the second principal eigen-vector, while we study multi-partitioning based on the k learning eigen-vectors. Second, the evaluation metrics are different: their metric does not consider the cluster sizes while ours does. Third, the proving techniques are different: they use a water-filling argument whereas we largely rely on the fundamental properties of Loewner partial orders; we also borrow a latest perturbation result from [72] and some results in [29].

7 DISCUSSIONS

While this paper has focused on initiating and verifying the idea that one could exploit multiple views of the ego-net structure (e.g. by multi-view spectral clustering techniques) for better social circle detection in ego-nets, we have realized certain orthogonal directions that could further the study.

7.1 Problem Setting

The presented study focused on detecting disjoint social circles based solely on network structure, while we had mentioned other studies that focused on detecting overlapping circles and using alter profiles.

Detecting disjoint social circles is a common setting in the literature (e.g. [23], [47]). Here, our adoption was particularly motivated from a user privacy protection perspective – a major proposal in the privacy research community is to protect user privacy by drawing and controlling information boundaries in online social networks, so that an ego’s posts
are spread only within designated circles [59]; in this case, if an alter is assigned to multiple circles, then her re-actions (e.g. ‘like’ or ‘re-tweet’) in assigned designated circles may be easily observed in other assigned non-designated circles. We admit, however, social circles may overlap in reality. In that case, first notice the two settings are convertible – two overlapping circles $S_1$ and $S_2$ generally admit three disjoint circles $S_1 \cap S_2$, $S_1 \setminus S_2$, and $S_2 \setminus S_1$; and three disjoint circles $C_1, C_2$, and $C_3$ could be merged into two overlapping ones $C_1 \cup C_2$ and $C_1 \cap C_2$. This allows a direct technical extension of our study for the case of overlapping circles. In addition, one could also extend more sophisticated techniques (e.g. [43]) from their single-view settings to multi-view settings. While the technical extensions may not be particularly difficult, a more challenging question is how to balance circle overlapping and privacy protection (as we mentioned above).

Using network structure to detect social circle is also a common setting [68], and our adoption was again motivated by privacy protection – that alters may be reluctant to share or even fill in their true profiles on online social networks due to privacy concerns. When this is not a serious concern, however, and alter profiles are largely available, our study could be directly extended by building an extra view on the profile similarities between alters (e.g. similarity between two alters is the inner product of their profile vectors).

7.2 Network Modeling

The network modeling techniques presented in this paper were largely based on previous studies and chosen for their simplicity or based on our experience.

As one example, the interaction similarity between two alters was obtained by summarizing their similarities in both directions (e.g. the reply number between Bob and Nancy was obtained by adding the number of replies from Bob to Nancy and that from Nancy to Bob). This is a classic technique that symmetrizes directed social networks into un-directed ones for analysis (e.g. [41], [45], [52]), but ignores the directional information which may be integrated in a finer manner for further performance improvement. In particular, how to effectively integrate direction modeling with multi-view learning remains an open challenge. (For instance, one may think of modeling each direction as one view but would suffer more sparsity in each new view.)

As another example, the topic view was normalized by the classic TF-IDF technique to highlight the importance of tags for each alter. We chose this pre-processing technique for it had been successfully applied in a similar study [51] as well as other tasks of social network analysis based on our own experience (e.g. [12], [19]). However, the technique itself is not without any limitation – in our application, for instance, it may assign a low TF-IDF weight to an alter’s truly interested topic if she only connects with people who talk about that topic. It then remains an open question that whether and how such lower weights may have adverse impact on social circle detection (in particular, under the multi-view clustering framework presented in this paper).

7.3 Circle Clustering

The major social circle detection techniques examined in this paper belong to the family of multi-view spectral clustering (e.g. [33], [35]). We chose this family for it is a union of spectral clustering [50] and multi-view learning [6], two ancestors with high reputations. It is noted, however, each ancestor has its own challenges which could have been inherited by multi-view spectral clustering.

As one example, standard spectral clustering [50] eventually groups instances by directly applying K-means [27], which is a tremendously popular data clustering technique. However, K-means has a well-known challenge in manually choosing a proper group number [32], which is passed down to spectral clustering and now to the multi-view spectral clustering techniques we applied for social circle detection. One could clearly address the problem from the very origin, say, by applying new K-means [32] that could automatically determine the group number. However, it would perhaps be more interesting to integrate such determination with multi-view learning in the context of social circle detection.

As another example, the multi-view learning family has been largely built on the view-consensus assumption, i.e. the label assignments on different instance views should largely agree [67], which has been a key to theoretically justify its success [3], [16] and sample efficiency [36]. However, the assumption may not always hold, say, due the presence of noise [14], in which case enforcing view consistency may result in performance degradation – an effect usually referred as negative transfer. Negative transfer has been broadly studied in other contexts such as multi-task learning and collective matrix factorization (see studies in [31], [34], [46] and a more rigorous theoretical analysis in [36]), but its discussion in multi-view learning seems pretty scarce. The selective transfer mechanism presented in this paper was an attempt to mitigate the problem, but we believe there remains spaces for improvements. For instance, both [14] and our mechanism ignored results on noisy observations while transferring information across views (under different problem settings), and it remains an open question how these noisy observations may be leveraged by more sophisticated techniques to enhance the performance gain.

7.4 Application

A potential application of social circle is to draw information boundary between different circles, so that a message could be delivered only to designated circles (e.g. [58], [59]). Note the final social circle construction does not have to be entirely automatic, and the ego may manually modify the detected groups. In this case, circle detection still significantly reduces the efforts of human labeling. Another issue is the information boundaries may not be completely secure if the social networking sites allows breaches in privacy protection (e.g. alters could ‘re-share’ their received private posts). These are, however, beyond the scope of the paper.

The discovered social circles could also be used to improve the efficiency of ad delivery, targeted advertising, and opinion mining in social groups. (See [42] for more discussions.) Social circles could also be used to study users’ socialization behavior and social network information flow. When the temporal information of data is available, our methods may be further extended to detect circles in evolving ego-nets. (See some latest progress on dynamic social network analysis in [64], [65] for instance.) In addition, when
information of the ego-net is available from other domains (e.g., [39], [66]), it is possible to further improve our work by considering cross-domain cross-view social circle detection.

8 Conclusion

In this paper, we proposed to automatically detect social circles of an ego-net based on its multi-view network structure. We crawled and constructed multi-view ego-nets from Twitter, and introduced multi-view spectral clustering techniques to automatically detect social circles based on them. Based on extensive experimental evaluations, we demonstrated the effectiveness and superiority of multi-view based social circle detection; we also demonstrated that, by treating sparse views as inherently incomplete and selectively transferring information across views, our modified multi-view clustering technique outperformed the standard multi-view clustering technique. The issue of clustering on inherently incomplete networks was briefly discussed in theory and verified in simulations.

References


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