SAMPLE EXAM I

MATH 526 Applied Math. Stat., 9/17

Print Name:

Student Number:

The exam contains ten problems. Show your work with brief explanations to ensure credit for the solution of each problem. Put a box around the answer to each problem. No use of calculators, books or notes is allowed. No use or viewing of electronic devices is allowed.

1. Find the probability that a hand of five cards from a deck of 52 cards contains exactly one triple and exactly one pair. This hand is often called a "full house."

2. How many ways can a group of five men and five women be seated in a row of ten chairs so that no two men or no two women are seated next to each other?

3. Let $A$ and $B$ be independent events with $P(A) = .2$ and $P(B) = .7$. Find $P(A^c \cap B^c)$ where $A^c$ and $B^c$ are the complements of $A$ and $B$ respectively.

4. An urn contains ten white balls and fifteen red balls. If two balls are randomly chosen without replacement from the urn, then find the probability that the second ball chosen is red.

5. Let $X$ be a continuous random variable with probability density function

$$f_X(x) = \begin{cases} 4x^3 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find $\mu_X$ and $\sigma^2_X$.

6. Let $f$ be the probability density function given by

$$f(x) = \begin{cases} cx & 0 < x < 1 \\ \frac{1}{2}(x - 1) & 1 \leq x < 2 \\ 0 & \text{otherwise} \end{cases}$$

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Find $c$.

7. Let $X$ and $Y$ be independent random variables with $E(X) = 2, E(Y) = 3, E(X^2) = 5, E(Y^2) = 6$. Determine $E[(2X + Y)^2]$.

8. One fair die is rolled six times. Find the conditional probability that a 5 occurs at least twice in the six rolls given that exactly one 5 occurs in the first two rolls.

9. Each of ten distinct balls is randomly placed in one of ten cells, denoted 1 to 10, so that any cell may contain 0 to 10 balls. Find the probability that exactly one cell is empty.

10. Urn I contains ten red balls and 15 green balls and Urn II contains 7 red balls and 8 green balls. A fair die is rolled once. If the outcome on the die is from the set $\{1, 2\}$ then one ball is selected from Urn I and if the outcome on the die is from the set $\{3, 4, 5, 6\}$ then one ball is selected from Urn II. Find the probability of choosing a red ball.

11. Each of seven people on an elevator randomly leaves the elevator on one of ten floors. Find the probability that three people leave on one floor and two people leave on each of two other floors.