

## SAMPLE PROBLEMS for EXAM II

MATH 526 Applied Math. Stat., 11/18

1. Let  $X_1, X_2, \dots, X_6$  be a random sample from an  $N(3, 4)$  distribution. Find a constant  $c$  such that  $P(S^2 > c) = .95$  where  $S^2$  is the sample variance of this random sample of size 6.
2. Let  $X$  and  $Y$  be two independent normal random variables where  $X$  has an  $N(2, 9)$  distribution and  $Y$  has an  $N(1, 16)$  distribution. Find  $P(X + Y < 5)$ .
3. Let  $X$  be a random variable with an  $F$  distribution with  $(5, 3)$  degrees of freedom. Find a constant  $c$  such that  $P(X < c) = .05$ .
4. Let  $X$  be a random variable with an  $N(2, 4)$  distribution. Find  $P(|X - 2|^2 > .36)$ .
5. Let  $Y$  be a binomial  $(b(300, p))$  random variable. If the observed value of  $Y$  is  $y = 75$  then find a 90 percent confidence interval for  $p$  using the Central Limit Theorem approximation.
6. A random sample of size 15 from a normal distribution  $N(\mu, \sigma^2)$  yields  $\bar{x} = 3.2$  and  $s^2 = 4.24$ . Find a 90 percent confidence interval for the variance  $\sigma^2$  when  $\mu$  is unknown.
7. Two independent random samples of size  $n_1 = 16$  and  $n_2 = 10$  taken from two independent normal distributions  $N(\mu_1, \sigma_1^2)$  and  $N(\mu_2, \sigma_2^2)$  respectively yield  $\bar{x}_1 = 3.6, s_1^2 = 4.14, \bar{x}_2 = 13.6, s_2^2 = 7.26$ . Find a 90 percent confidence interval for  $\sigma_1^2/\sigma_2^2$  when  $\mu_1$  and  $\mu_2$  are unknown.
8. Find a 95 percent confidence interval for the mean  $\mu$  of a normal population  $N(\mu, 4)$  for a random sample of size 16 where  $\bar{x} = 3.2$ .
9. Let  $X_1, X_2, X_3, X_4$  be a random sample of size four from an  $N(1, 16)$  distribution and let  $\bar{X}$  be the sample mean of this random sample. Find  $P(\bar{X} < 1.5)$ .
10. How large a sample is required from an  $N(\mu, 1)$  distribution so that a 95% confidence interval has length  $\leq 1$ ?

11. Let  $X_1, X_2, X_3, X_4$  be four independent random variables each with the probability density function,  $f_X$ , given by

$$f_X(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability that exactly two of these four random variables exceed  $\frac{1}{2}$  and exactly one of these random variables is less than  $\frac{1}{4}$ .