SAMPLE PROBLEMS FOR FINAL EXAM

MATH 526 Appl. Math. Stat., 12/18

1. Find the probability that a hand of seven cards from a deck of 52 cards contains exactly two distinct pairs.

2. Let $X_1, \ldots, X_{16}$ be a random sample from a normal population. The sample mean is 21.2 and the sample variance is 3.9. Find a 98% confidence interval for the population mean.

3. Let $(X, Y)$ be a pair of random variable with the probability density function $f_{XY}$ given by

$$f_{XY}(x, y) = \begin{cases} 6x & 0 < x < 1, \quad 0 < y < 1-x \\ 0 & \text{otherwise} \end{cases}$$

Find the conditional probability $P(X \geq .3|Y = .5)$.

4. Given an urn that contains 8 red balls, 7 green balls, and 10 white balls. Six balls are randomly selected with replacement. Let $X_R$ be the number of red balls, $X_G$ be the number of green balls and $X_W$ be the number of white balls in the selection. Compute the conditional probability $P(X_R = 1, X_G = 3|X_W = 2)$.

5. Let $X$ be a random variable with mean 3 and variance 5. Compute $E[(3X + 2)^2]$.

6. Let $X_1, \ldots, X_{25}$ be a random sample from an $N(\mu, 16)$ population. The sample mean is 4.8 and the sample variance is 5.2. Find a 95% confidence interval for the population mean.

7. Let $X_1, \ldots, X_{16}$ be a random sample from a normal population. Test $H_0 : \mu_X = 2$ against $H_1 : \mu_x > 2$ at the 2% level of significance. Determine the critical region. If the sample mean is $\bar{x} = 2.2$ and the sample variance is $s^2 = 4$ determine if $H_0$ is accepted or rejected.

8. To test the hypothesis $H_0 : \mu = 2$ against the alternative hypothesis $H_1 : \mu > 2$ a random sample of size 16 from an $N(\mu, 4)$ population is taken. The critical region is $\{\bar{x} : \bar{x} > 2.2\}$. If the correct mean is 3 then determine the probability of a type II error.
9. Let $X_1, \ldots, X_{12}$ be a random sample from a normal population. The sample mean is $\bar{x} = 2.3$ and the sample variance is $s^2 = 2$. Find a 98% confidence interval for the variance.

10. Let $X$ be a continuous random variable with the probability density function

$$f_X(x) = \begin{cases} \frac{2}{5}|x| & -2 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Compute $E[4X^3 + 6X^2]$.

11. Let $X$ be a random variable with mean 3 and variance 5. Compute $E[(3X + 2)^2]$.

12. An urn contains 6 red balls and 10 green balls. A ball is randomly chosen from the urn. If a red ball is selected then a fair coin is tossed 8 times and if a green ball is selected then the coin is tossed 12 times. Let $X$ be the number of heads that occur in tossing the coin. Determine $P(X = 5)$.

13. Let $X_1, \ldots, X_{16}$ be a random sample from an $N(\mu, \sigma^2)$ distribution and let $\bar{X}$ be the sample mean for this sample. Find a constant $c$ so that $P(|\bar{X} - \mu| > cS) = .05$ where $S^2$ is the sample variance.

14. Let $X$ be a random variable with probability density function

$$f_X(x) = \begin{cases} c(x + 1) & |x| < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the constant $c$ and $P(X^2 > \frac{1}{4})$. 
