SAMPLE PROBLEMS FOR FINAL EXAM

MATH 526 Appl. Math. Stat., 12/18

1. Find the probability that a hand of seven cards from a deck of 52 cards contains exactly two distinct pairs.

2. Let $X_1, ..., X_{16}$ be a random sample from a normal population. The sample mean is 21.2 and the sample variance is 3.9. Find a 98% confidence interval for the population mean.

3. Let (X, Y) be a pair of random variable with the probability density function f_{XY} given by

$$\begin{aligned} f_{XY}(x,y) &= 6x \qquad 0 < x < 1, \qquad 0 < y < 1 - x \\ &= 0 \qquad\qquad \text{otherwise} \end{aligned}$$

   Find the conditional probability $P(X \geq .3 | Y = .5)$.

4. Given an urn that contains 8 red balls, 7 green balls, and 10 white balls. Six balls are randomly selected with replacement. Let X_R be the number of red balls, X_G be the number of green balls and X_W be the number of white balls in the selection. Compute the conditional probability $P(X_R = 1, X_G = 3 | X_W = 2)$.

5. Let X be a random variable with mean 3 and variance 5. Compute $E[(3X + 2)^2]$.

6. Let $X_1, ...X_{25}$ be a random sample from an $N(\mu, 16)$ population. The sample mean is 4.8 and the sample variance is 5.2. Find a 95% confidence interval for the population mean.

7. Let $X_1, ..., X_{16}$ be a random sample from a normal population. Test $H_0 : \mu_X = 2$ against $H_1 : \mu_x > 2$ at the 2% level of significance. Determine the critical region. If the sample mean is $\bar{x} = 2.2$ and the sample variance is $s^2 = 4$ determine if H_0 is accepted or rejected.

8. To test the hypothesis $H_0 : \mu = 2$ against the alternative hypothesis $H_1 : \mu > 2$ a random sample of size 16 from an $N(\mu, 4)$ population is taken. The critical region is $\{\bar{x} : \bar{x} > 2.2\}$. If the correct mean is 3 then determine the probability of a type II error.

9. Let $X_1, ...X_{12}$ be a random sample from a normal population. The sample mean is $\bar{x} = 2.3$ and the sample variance is $s^2 = 2$. Find a 98% confidence interval for the variance.

10. Let X be a continuous random variable with the probability density function

$$\begin{aligned} f_X(x) &= \frac{2}{5}|x| \qquad -2 < x < 1 \\ &= 0 \qquad \text{otherwise} \end{aligned}$$

Compute $E[4X^3 + 6X^2]$.

11. Let X be a random variable with mean 3 and variance 5. Compute $E[(3X + 2)^2]$.

12. An urn contains 6 red balls and 10 green balls. A ball is randomly chosen from the urn. If a red ball is selected then a fair coin is tossed 8 times and if a green ball is selected then the coin is tossed 12 times. Let X be the number of heads that occur in tossing the coin. Determine $P(X = 5)$.

13. Let $X_1, ..., X_{16}$ be a random sample from an $N(\mu, \sigma^2)$ distribution and let $\bar{X}$ be the sample mean for this sample. Find a constant c so that $P(|\bar{X} - \mu| > cS) = .05$ where S^2 is the sample variance.

14. Let X be a random variable with probability density function

$$\begin{aligned} f_X(x) &= c(x + 1) \qquad |x| < 1 \\ &= 0 \qquad \text{otherwise} \end{aligned}$$

Find the constant c and $P(X^2 > \frac{1}{4})$.