SAMPLE PROBLEMS FOR EXAM II

MATH 627 Probability, 11/17

1. Let $X$ and $Y$ be random variables with the joint probability density function

$$f_{XY}(x, y) = \begin{cases} \frac{6}{7}(x^2 + \frac{xy}{2}) & 0 < x < 1, \ 0 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

Find $P(X > Y)$ and $P(Y > \frac{1}{2} | X = \frac{1}{2})$.

2. Let $X$ and $Y$ be random variables with the joint probability density function given by

$$f_{XY}(x, y) = \begin{cases} 2 & 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Determine the covariance of $(X, Y)$.

3. Let $X$ and $Y$ be random variables with the joint probability density function given by

$$f_{XY}(x, y) = \begin{cases} 6x & 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find $E[X^2]$.

4. Let $X$ be a standard normal random variable. Define the random variable $Y$ as

$$Y = \begin{cases} X & X > 2 \\ 0 & X \leq 2 \end{cases}$$

Find $E[Y]$.

5. Let $X_1$ and $X_2$ be independent random variables both with the probability density function

$$f(x) = \begin{cases} e^{-x} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

Find the variance of $Y = X_1X_2$. 

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6. Let \((X, Y)\) be a pair of random variables with the joint probability density function

\[
f_{XY}(x, y) = \begin{cases} 
6y & 0 < y < x < 1 \\
0 & \text{otherwise}
\end{cases}
\]

Find the marginal density for \(X\), \(f_X\), and the conditional mean of \(Y\) given \(X = .7\), \(E[Y|X = .7]\).

7. Let \(X\) be a random variable with the probability density function

\[
f_X(x) = \begin{cases} 
x/2 & 0 < x < 2 \\
0 & \text{otherwise}
\end{cases}
\]

Find the probability density function for \(Y = X^3\).

8. Let \(X\) and \(Y\) be jointly normal random variables with the parameters \(\mu_X = 1, \mu_Y = 4, \rho_{XY} = .3, \sigma_X^2 = 9, \sigma_Y^2 = 16\).
Find \(P(Y \leq 3)\) and \(P(2 < Y < 4|X = 2)\).

9. Let \(X\) be a random variable with the probability density function

\[
f_X(x) = \begin{cases} 
c e^{-2x} & 0 < x < 1 \\
1/10 & 1 \leq x \leq 3 \\
0 & \text{otherwise}
\end{cases}
\]

Find the constant \(c\) and evaluate the distribution function at 1.5, \(F_X(1.5)\).

10. Let \(X\) be a random variable with the probability density function

\[
f_X(x) = \begin{cases} 
1 + x & -1 < x < 0 \\
1 - x & 0 \leq x \leq 1 \\
0 & \text{otherwise}
\end{cases}
\]

Find \(P(4X^2 \leq 1)\).