The exam contains eleven problems with the last problem being extra credit. Show your work with brief explanations to ensure credit for the solution of each problem. Put a box around the answer to each problem. No use of calculators, books or notes is allowed. No use or viewing of electronic devices is allowed.

1. Fifteen distinct pairs of shoes are in a box. Eight shoes are randomly chosen from the box without replacement. What is the probability that among the eight shoes in the sample there are exactly two pairs of shoes?

2. A fair die is rolled once. If 1 or 2 occurs on the die then a fair coin is tossed five times and if 3, 4, 5, or 6 occurs a fair coin is tossed eight times. Determine the probability that exactly three heads occur in the tosses of the coin.

3. A fair coin is tossed once. If heads occurs then a die is rolled twice and if tails occurs a die is rolled once. Find the probability that 4 occurs at least once among the outcomes of the rolls of the die.

4. A committee contains ten people containing a board consisting of a president, a vice president, a secretary and a treasurer from among 20 males and 20 females. Find the probability that the committee contains a board consisting of two females and two males.

5. Given that a hand of five cards contains at least two kings, what is the conditional probability that the hand has exactly three kings?

6. Let $X$ be a random variable with moment generating function $M_X(t) = \left( \frac{1}{6}e^t + \frac{2}{6}e^{2t} + \frac{3}{6}e^{3t} \right)^2$. Determine $E(X)$. 

7. Each of ten sticks of the same length is broken into one long and one short stick and the resulting sticks are arranged randomly in pairs. Find the probability that each of the pairs contains a long and a short stick.

8. Let A and B be independent events with \( P(A) = .5 \) and \( P(B) = .4 \). Find \( P(A^c \cup B^c) \) where \( A^c \) and \( B^c \) are the complements of A and B resp.

9. Let \( X \) be a discrete random variable such that \( P(X = 1) = p, P(X = 0) = q, P(X = 2) = r \) where \( p + q + r = 1 \). Find the variance for \( X \).

10. From a population of three distinct symbols denoted \( (x, y, z) \) a sample of size 15 is randomly chosen with replacement. Find the probability that the sample contains 5 symbols from each of \( x, y, z \).

11. An elevator contains seven people. If each person randomly departs from the elevator on one of the ten floors numbered 1 to 10 where the elevator stops, then find the probability that exactly 5 of the people depart on either floor 2 or floor 3.