1. Find the probability that a hand of five cards from a deck of 52 cards contains exactly two distinct pairs and three hearts.

2. Let $X$ and $Y$ be jointly normal random variables with parameters
   \[ \mu_X = 5.8, \quad \mu_Y = 5.3, \quad \rho_{XY} = .6, \quad \sigma_X = \sigma_Y = .2 \]
   Find $P(5.28 < Y < 5.92 | X = 6.3)$.

3. Let $X$ and $Y$ be random variables of continuous type with the joint probability density function
   \[ f_{XY}(x, y) = \begin{cases} e^{-y} & 0 < x < y < \infty \\ 0 & \text{otherwise} \end{cases} \]
   Find the covariance of $(X, Y)$.

4. Let $X$ and $Y$ be random variables with the joint probability density function
   \[ f_{XY}(x, y) = \begin{cases} 8xy & 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases} \]
   Find $P(2Y + X < 2)$.

5. Let $X$ and $Y$ be random variables with the joint probability density function
   \[ f_{XY}(x, y) = \begin{cases} 6y & 0 < y < x < 1 \\ 0 & \text{otherwise} \end{cases} \]
   Determine $P(0 < Y < \frac{1}{2} | X = \frac{3}{4})$.

6. Let $X$ be a random variable with the probability density function
   \[ f_X(x) = \begin{cases} \frac{x + 1}{2} & |x| < 1 \\ 0 & \text{otherwise} \end{cases} \]
   Find the probability density function of $Y = X^2$ in the interval $(0, 1)$. 

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7. Let $X$ and $Y$ be random variables with the joint probability density function

$$f_{XY}(x, y) = \begin{cases} 6x & 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Let $Z = 2X + Y$. Compute $P(Z < 1)$.

8. Let $X_1, X_2, ..., X_{25}$ be independent, identically distributed random variables each with the uniform probability density function on the interval $[0, 2]$. Use the Central Limit Theorem approximation to compute approximately $P(.9 \leq \bar{X} \leq 1.1)$ where $\bar{X} = \frac{1}{25} \sum_{i=1}^{25} X_i$.

9. An urn contains 6 red balls and 10 green balls. A ball is randomly chosen from the urn. If a red ball is selected then a fair coin is tossed 8 times and if a green ball is selected then the coin is tossed 12 times. Let $X$ be the number of heads that occur in tossing the coin. Determine $P(X = 5)$.

10. Let $X$ and $Y$ be random variables with the joint probability density function $f_{XY}$ given by

$$f_{XY}(x, y) = \begin{cases} e^{-y} & 0 < x < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

Find the moment generating function of $Z = X + Y$.

11. Let $X$ be a random variable with the probability density function

$$f_X(x) = \begin{cases} \frac{x + 1}{2} & |x| < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability density function of $Y = X^2$ in the interval $(0, 1)$.

12. Each of ten balls numbered 1 to 10 is randomly placed in one of ten boxes that are numbered 1 to 10 so that each box contains exactly one ball in it. Find the probability that the odd numbered balls are all placed in the boxes numbered 1 to 5.
13. Let $X_1, \ldots, X_{25}$ be independent, identically distributed $N(1, 16)$ random variables and let $\bar{X} = \frac{1}{25} \sum^{25}_{i=1} X_i$ be the sample mean. Determine $P(.5 < \bar{X} < 1)$.

14. Let $X, Y$ be independent $N(0, 1)$ random variables. Compute $P(X - Y > 2)$. 