

MATH 630 Homework #10 (April 2020)

Binomial asset pricing model, Black-Scholes equation

1. Let a stock have the price 75 at $t = 0$. Assume that the risk free force of interest is 6% per unit period. Find the risk neutral probabilities denoted \tilde{p}, \tilde{q} for one unit of time. In one unit of time the stock can achieve 95 or 63. Determine the call option value at $t = 0$.
2. Let $V(S, t) = aS + be^{rt}$. Verify that for any choice of constants a and b , V satisfies the Black-Scholes equation. Analyze the boundary behavior of V at expiration and under the condition $S = 0$ and $S \rightarrow \infty$.
3. Let $V(S, t) = e^{at}S^2$. Verify that for a certain choice of a , V satisfies the Black-Scholes equation.
4. Let $G(S, t)$ satisfy the following equation

$$G_t + \frac{1}{2}\sigma^2 S^2 G_{SS} + rSG_S = 0$$

This is not the Black-Scholes equation because it is missing the final term, $-rG$. Verify that $V(S, t) = e^{rt}G(S, t)$ does satisfy the Black-Scholes equation.